Making an I.M.P.A.C.T. in Mathematics with Lesson Study:

The Teachers’ Journeys in Their Own Words

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 I.M.P.A.C.T., Improving Mathematical Practices via Algebraic Connections and Technology, is a program at George Mason University focused on creating a collaborative network for knowledge development in mathematics teaching and learning, funded by the State Council of Higher Education in Virginia. It provides a forum for teachers to collaboratively plan lessons, exchange best instructional practices and effective uses of “tech-knowledgy” tools to design instructional tasks that promote algebraic conceptual thinking.

As a PhD student at GMU in Mathematics Education Leadership, Dr. Jennifer Suh, both my professor and the leader of I.M.P.A.C.T., offered me the opportunity to teach a class to elementary and middle school teachers in an area of designated need in Petersburg, VA in collaboration with the district mathematics specialist there. The goals of the class were to improve algebraic reasoning and productively integrate technologies while teaching and learning mathematics. To reach these goals, the teachers collaborated to participate in several cycles of research, design, implementation, observation, debrief and revision - modeled after tradition Japanese lesson study cycles.

This article offers a narrative from the teachers in this class, in their own words, as told through excerpts of archives I collected throughout their journeys - including online surveys, blog posts, critiques of technology tools, responses to vignettes, lesson plans, written observations, and reflection papers. The narrative demonstrates both their progressions of learning and their growing appreciation for lesson study as a “more productive” model of professional development.

**Review of Related Literature**

**Why Focus on Algebra?**

Given its important role in mathematics as well as its role as a gatekeeper to future educational and employment opportunities, algebra has become a focal point of both reform and research efforts in mathematics education (Knuth, 2006). A recent review of educational research calls us to turn our attention to “questions of how *all* students can prosper in algebra courses and how to better prepare students who are in the pipeline moving toward algebra (Stein, 2011).”

Although algebra is now typically a course taken in 8th or 9th grade, the following report from the Mathematical Association of America (MAA) summarizes how early algebra actually begins in elementary school:

It is now widely understood that preparing elementary students for the increasingly complex mathematics of this century requires an approach different from the traditional methods of teaching arithmetic in the early grades, specifically, an approach that cultivates habits of mind that attend to the deeper, underlying structure of mathematics and that embeds this way of thinking longitudinally in students' school experiences, beginning with the elementary grades.  This approach to elementary grades mathematics has come to be known as early algebra.  There is general agreement that early algebra comprises two central features: (1) generalizing, or identifying, expressing and justifying mathematical structure, properties, and relationships and (2) reasoning and actions based on the forms of generalizations. (2006)

Further, the MAA contends that the payoff for early algebra is three-fold: (1) addresses the five competencies needed for children’s mathematical proficiencies, (2) creates more preparedness for advanced mathematics, and (3) democratizes access to mathematical ideas and ultimately lifelong success (MAA, 2006).

With such tremendous payoffs, there is no wonder why there is such a current focus on algebraic reasoning as a study of patterns, functions and relationships in the elementary grades. There are a variety of recent studies focused on particular components of early algebra, such as: understanding the equal sign (Knuth, 2006); using multiple representations (Richardson, 2009); explicitly stating generalizations and finding examples, counterexamples, and proofs to support them (Schifter, 2009); and integrating ‘algebrafied’ tasks across the curriculum (Soares, 2006).

There are also a variety of book publications aimed at helping teachers support their students in constructing strategies and understanding big ideas of early algebra. The books often offer both practical classroom ideas as well as opportunities for teachers to deep their own algebraic understandings. For example, *Young Mathematicians at Work: Constructing Algebra* (Fosnot, 2010), provides a “landscape for learning” – a trajectory of big ideas, strategies, and models for algebra. This book tells the story of students and teachers mathematizing together through a sequence of investigations and mini-lessons. A second book, *Patterns, Functions, and Change* (Schifter, 2008), serves as a casebook for professional development as teachers try to understand the complexities of students’ thinking while also building their own understandings. This book comes with a facilitator’s guide as well and videos of children engaged in mathematics in real classroom settings. I mention these two particular books here but will describe their significance to this particular pilot study in a later section.

**Why Focus on Technology?**

Students are living and learning in an age of new media – where they give constant attention to the latest scoop on TV, the hottest music for their iPods, newest games for their game systems, instantaneous updates in their online communities and social networks, and they have mobile apps that manage all of these interests simultaneously. Students aged 8-18 are constantly (an average of 7.5 hours a day!) interacting with media – more than ANY other activity besides (maybe) sleeping – according to a popular report, compiled by the Kaiser Family Foundation (2010). Further, 90% of young students aged 5-8 have used computers and 52% have used smartphones, iPods, tablets or the like to play games, watch videos, or use apps, according to a follow-up study conducted by Common Sense Media (2011).

This age of new media implies implications for teaching and learning. Traditional methods of teaching may not be engaging today’s learners who are used to these dynamic and interactive platforms. Since these new media forms have altered how youth socialize and learn, how are we altering how we teach?

To react to this age of new media, the commercial industry has capitalized by providing a tremendous variety of video games and mobile apps to pique the interest of all types and ages. There are both benefits and dilemmas that stem from this influx of opportunity to engage and interact digitally.

Research in the field of mathematics education is not progressing nearly as quickly as the changes in technology. But, instead of becoming discouraged, NCTM’s Principles and Standards indicate that teachers should be encouraged to use technology to extend the mathematics that can be taught and enhance students’ learning (2000). Still, teachers eager to find ways to use technology for teaching, learning, and assessing find it to be a daunting task (Pierce, 2009).

The sixty-seventh NCTM Yearbook is dedicated to ‘Technology-Supported Mathematics Learning Environment’ and the first eight chapters address how research informs practice in regards to incorporating technology. First and foremost, research describes teaching strategies for developing judicious technology use. Teachers should learn to make informed decisions about the appropriate implementation of technologies in a coherent instructional program (Ball, 2005). For example, using virtual manipulatives and other forms of mathematical representations have shown to be effective in young children’s understanding of number and operation and geometric reasoning (Moyer, 2005). Students have also shown to develop elementary number theory concepts quicker and retain them longer when using calculators (Kieran, 2005). Knowing when and how to employ the technology appropriately is key to its effectiveness in learning (Clements, 2005).

 In twenty-first century classrooms, teachers are expected to be flexible in catering to students’ interests by offering nontraditional venues of learning. This may require a shift in practice and professional development because they must learn about the new tools themselves before they are expected to implement them. Teachers need to carefully select and design learning opportunities for students where technology is an essential component in developing students’ understanding, not where it is simply an appealing alternative to traditional instructional routines (Hollenbeck, 2008).

New and emerging technologies will continually transform the mathematics that is available to students and redefine ways that it can be taught, and so it follows that it is a high priority to study and experiment with technology in the field (Fey, 2010). The call is urgent because students are already entering and living in a technologically sophisticated society and workplace.

Recent research on the intersection of early algebra and technology focuses in on technology’s capability to generate and manipulate multiple representations of big ideas in early algebra. Consequently, students, even in elementary school, are able to concentrate on developing higher-order thinking skills by using technologies to complete complex tasks that require them to make connections and analyze mathematical concepts, evaluate mathematical processes and algorithms, and create representations that support task completion (Polly, 2011). In his article, Polly describes how third-graders used the Pan Balance applet from the Illuminations to develop algebraic reasoning while exploring and developing understanding of equality, a fundamental concept of early algebra. Another article also highlights how technology frees students from tedious and repetitive computations to encourage the use of multiple representations, supporting the NCTM’s vision for technology (NCTM 2000, p.25). Erbas claims that “when supported by the teacher, these tools of technology provide students with opportunities to investigate and manipulate mathematical situations to observe, experiment with, and make conjectures about patterns, relationships, tendencies, and generalizations (2004)” – the defining features of early algebra, as defined by the MMA (2006).

**Why Use Lesson Study as the Framework for Learning?**

Lesson study is a form of professional development that originated in Japan and is regarded as a key ingredient in the improvement of mathematics there (Stigler & Hiebert, 1999). Since the Trends in International Mathematics and Science Study (TIMSS) in 1999, lesson study has continued to gained popularity in America as a potentially promising way to alter the professional roles of teachers and improve teaching and learning as a whole (Chokshi & Fernandez, 2005; Takahashi & Yoshida, 2004). Lesson study responds to the concern that traditional forms of professional learning, such as in-service workshops and conferences, continue to be “intellectually superficial, disconnected from deep issues of curriculum and learning, fragmented, and noncumulative” (Ball & Cohen, 1999) by situating the learning in the practice itself, “affording professional learning opportunities intricately connected to classroom practice” (Post, 2008).

Lesson study was introduced to America by Catherine C. Lewis in her 1998 article “A Lesson Is Like a Swiftly Flowing River” and gained more attention from James W. Stigler and James Hiebert in their book *The Teaching Gap* (1999). Lesson study is a collaborative and cyclical process that brings teachers together to plan, implement and observe, and debrief lessons. The debriefing, or post-lesson discussion, is arguably the most important part of the process, as well as the most neglected and least written about part of the process in America (Tolle, 2010). The debriefing is also the most distinctive component, providing a means for reflection on both instructional theories and students’ learning while also laying the foundation for the next lesson study cycle (Groth, 2011). Still beyond the immediate classroom impact, lesson study affords collaboration and reflection that often confront larger social or cultural problems within the school (McMahon & Hines, 2008).

Loughran acknowledges that reflection in general is regarded as something useful and informative, but makes a point that *reflection* is different from *effective reflective practice*. He purports that effective reflective practice is a genuine lens into the world of practice, given that it questions taken-for-granted assumptions and encourages one to see his or her practice through others’ eyes. Further, he contends that reflection can become effective reflective practice by developing and enhancing it, as long as the learning is situated in a real context. Loughran infers that professional knowledge is a product of effective reflective practice when he quotes, “What is learned as a result of reflection is, to me, at least equally as valuable as reflection itself.” Loughran believes that effective reflective practice may provide the professional knowledge necessary in challenging the gap between theory and practice, in that the practitioner is continually developing professional knowledge to ultimately become truly responsive to the needs, issues, and concerns important in shaping practice. (Loughran, 2002)

Just as Loughran deems that practitioners can develop effective reflective practice through experience in real contexts, Lieberman deems that teachers can redevelop professional identities to include continual improvement by developing teacher learning communities through lesson study. Lieberman’s evidence is significant because it depicts lesson study as a vehicle that can facilitate change in the traditional norms that historically inhibited teachers from learning from one another. Through lesson study, teachers develop teacher learning communities that lead to teachers redeveloping professional identities to include continual improvement, where the norm is to innovate and collaborate to ultimately better serve students. Additionally, increasing teachers’ community results in agency. Lieberman defines teachers that have agency as teachers that have some control over what they do, where part of their job is to act, not just follow. (Lieberman, 2009)

This sense of agency stemming from collaboration is extremely important at a time in the USA when the pressure to increase student achievement is intense and teacher efficacy is being negatively impacted (Puchner & Taylor, 2006). Lesson study allows teachers to create a climate of collaboration and inquiry among themselves, with a relatively small amount of external intervention and minimal financial support, and deliver a tremendous three-fold impact: recognition of the benefits of collaboration, realization that they as teachers could significantly impact their students’ learning, and the belief that there may be changes in the way that math is taught that could impact student learning (Puchner & Taylor, 2006).

 Lesson study may be appealing to researchers and educators in the United States because it is situated in the classroom connecting theory and practice, because the students are at the heart of the learning, because it is teacher-led, because it has been shown to improve teaching and learning in Japan, because it is collaborative, continuous, and concrete, and/or various other favorable outcomes. Best yet is that “beginning lesson study and embarking on the road to improving teaching is within the reach of any teacher or group of teachers with enthusiasm and commitment to the profession (Takahashi & Yoshida, 2004).”

# **Research Questions**

1. How did the teachers perceive the cyclical model of lesson study as a framework for professional learning?
2. How did the teachers change following debriefing sessions (if at all)?

# **Methods**

**Background**

In June 2011, I was asked to co-teach a class to 11 elementary and middle school teachers from in and around Petersburg, VA. The class was funded by the State Council of Higher Education in Virginia and was a part of a program at George Mason University focused on creating a collaborative network for knowledge development in mathematics teaching and learning. The program is named I.M.P.A.C.T., which stands for Improving Mathematical Practices via Algebraic Connections and Technology. The goals of the course included providing a platform for teachers to collaboratively plan lessons, exchange best instructional practices and effective uses of “tech-knowledgy” tools to design instructional tasks that promote algebraic conceptual thinking.

Upon successful completion of the course, teachers would receive 3 graduate credits, technology resources (including a subscription to a suite of interactive mathematics simulations), professional books for teaching algebraic reasoning (*Young Mathematicians at Work: Constructing Algebra* (Fosnot, 2010) and *Patterns, Functions, and Change* (Schifter, 2008) - as described in the related research section above), a stipend of $200, and an all-expenses paid opportunity to present at the Virginia Council of Teachers of Mathematics conference.

**Procedure**

Classes met on 6 occurrences from 9am-3pm over the course of a 6-week period in June and July and one additional date in September. The class had two components: a content component and a lesson study component. The earlier class sessions were focused on content - engaging in algebraic reasoning activities using multiple representations, examining and discussing vignettes of students’ thinking, and gaining exposure to a variety of technology tools for teaching such as videos, apps, online games, and interactive whiteboards. These class sessions provided common experiences that were foundational to the later component where the eleven teachers were to participate in two cycles of lesson study.

Although the teachers had never participated in lesson study before, my co-instructor and I decided to ‘teach it by doing it’ rather than introducing them formally to the framework first. The eleven teachers split up into four groups according to the grade-levels that they taught and were instructed to choose a lesson idea or teaching topic that they would like to explore further as a group, given the expectation to embed algebraic reasoning and encouragement to integrate technology. They worked over the next couple class sessions designing and refining a lesson with the following components:

* Learning Objectives
* Materials (including all Blackline Masters)
* Instructional Plan – directions so that *any* teacher would be able to conduct the lesson cohesively from start to finish
* Questions for Students (to lead classroom discussion)
* Assessment Options (\*that you must be able to archive somehow so we can reflect on student work and understanding)
* Extensions (particularly to differentiate - scale up/down the lesson)
* Guide for Teacher Reflection
* Standards that are targeted (list both the CCSS and the VA SOLs relevant)

Once the groups were satisfied with their lessons, they rehearsed their deliveries within their collaborative groups. The first cycle of lesson study was scheduled to take place during summer school, where each group had the opportunity to teach a lesson in the context of real classroom with real students – just not their own. Although those not presenting the lesson attended as observers, these lessons were also videotaped so that the presenters would have an opportunity to see themselves teaching and the observers would have the opportunity to add to their observations. The following class meeting was spent watching the videos and debriefing, allowing the presenters to reflect on their lessons before welcoming observers to share their ideas, as suggested by research (Lewis, 2002). The co-instructor and I facilitated the discussion and offered our own feedback, according to an observation guide that we shared with the class while they were still designing their lessons. The observation guide was presented as a guide, and teachers were told that it was possible that not all questions would be appropriate for every lesson. The guide follows:

**Academic Learning:**

What were the objectives of the lesson?

Were multiple representations used? Which ones?

Was it possible for students to use different problem-solving strategies? Give examples.

How did students of different learning groups (visual, gifted, ESL, special needs) react to the various instructional tasks? How was learning evident in these groups?

What evidence of reasoning and proof were students constructing?

How well were students able to communicate mathematical ideas?

Were connections made to other content? Which?

How was student understanding assessed? Was this form of assessment effective?

**Motivation and Engagement:**

Were students engaged during each of the parts of the lesson?

Did students attempt to solve the problem independently?

Did students share strategies and help each other during small group sharing?

Were all students engaged during large group sharing? How so?

Which parts of the lessons were most motivating and engaging to students? Was this true for all students or were some less motivated/engaged? Can you offer reasons why?

Were there opportunities for students of different levels of understanding to be challenged? (both scaled up and scaled down appropriately)

**Social Behavior:**

Did students stay focused and on task?

Did students share and discuss with peers appropriately?

What kinds of comments were made by students of various learning groups?

If small groups were used, how were they chosen? Did the groups demonstrate productive learning?

Did students need teacher guidance during sharing times or were the students able to share appropriately when asked? What was the reason for guidance, if applicable?

Did students demonstrate proper listening behavior during instruction and sharing times?

How did the teacher maintain classroom management?

How did he/she respond to problems?

**Instructional Features:**

What instructional tasks appeared to be most effective?

What questions did the teacher use to guide student learning and understanding-during each part of the lesson?

How did the teacher select students to share?

How did the teacher encourage students to participate? React to those that were not participating, if any?

How did the integration of technology enhance or inhibit teaching and learning? Can you offer reasons?

**SUMMARY:**

What are three components of the lesson that went well? What impact did those components have on the overall teaching and learning?
What are three suggestions you can offer that might improve this lesson? What components in the teaching and learning do these suggestion address?

Following the debriefing session, teachers were asked to individually write up informal reflections on the lessons they presented, and include the ways that they would revise the lesson. They met the following week to collaborate with their groups and talk through their individual reflections, making decisions and revising the lessons. They were to each modify the lesson to fit their own classrooms’ needs, as could be best predicted a month before school was actually back in session. They presented the lessons again in September after school was back in session, and the entire group gathered for a debriefing session directly following the lesson. Both the lessons and the debriefing session were videotaped, this time transcribed instead of used as a second opportunity to observe and reflect.

**Data Sources and Collection**

I used a collective case study methodology with the goal of collecting enough data about the lesson study groups in order to understand how the groups functioned (Berg, 2004; Bogdan & Biklen, 2003). Specifically, I was interested in how the teachers would perceive the cyclical model as a framework for professional learning and how they would change following debriefing sessions (if at all). I was also interested to see how they would embed algebraic reasoning and/or technologies, but I didn’t include this as a third research question because I felt like the themes would be emerge when analyzing that data for evidence of change.

Data was collected throughout the course. It included online surveys, blog posts, critiques of technology tools, responses to vignettes, lesson plans, written observations, and reflection papers. Video was also taken when the collaborative groups presented their lessons. Video from the first cycle of the lesson study was viewed as a full group and used to inform the observation and reflection, as part of the debriefing process. Video from the second cycle was transcribed for the purpose of archiving data that could be included in analysis.

**Data Analysis Techniques**

Here is where I switch voices and casually admit that I am happy that this is a pilot study at this point. ☺ This pilot was clearly a case-study, since I studied one particular group of teachers participating in one particular class in one particular location over one particular period of time under particular circumstances. Although I did not have intentions of being able to claim that any of my findings were generalizable, I did have intentions of being able to analyze the data both quantitatively and qualitatively. Folks may assume that case-studies are qualitative in nature, but there is no reason why cases cannot be measurement-based (PREST, 2004).

Mixed methods is a procedure for collecting, analyzing and “mixing” both quantitative and qualitative data at some stage of the research process within a single study, to understand a research problem more completely (Creswell, 2002). The most compatible mixed methods research questions are open-ended and non-directional in nature, and they both seek to discover, explore, or describe a particular participant(s), setting, context, location, event, incident, activity, experience, process, and/or document (Onwuegbuzie & Leech, 2006) - precisely as my two research questions were crafted and all components of the case were particular.

The qualitative component consisted of organizing a plethora of archived data, such as assignments from the content-portion of the class (critiques of technology, responses to case studies, presentations on constructing algebra), blog posts, field notes, written observations of their peers teaching lessons, reflections at different stages of the lesson study, open-ended responses to online survey questions, and videos of both presenting and debriefing lessons. Before analysis of the archived qualitative data could begin, the videos had to be transcribed.

Even after all of the videos were transcribed, I was overwhelmed by the sheer amount of data that I had for qualitative analysis. I decided that limited by both time and the scope of this pilot study to focus on just one of the four lesson study teams. I justified my selection of one particular group because I had the richest archived data for this group. For that particular three-person group, I had their full-lesson as well as the full group debriefing session on video and transcribed. It would have been problematic to narrow the scope of the qualitative data by narrowing my sample since I had not planned on limiting my quantitative data to a subset of my sample, but it turned out that the quantitative data could not be used with confidence anyway. I will return to this issue shortly.

The multiple forms of qualitative data were then read and read again with the purpose of uncovering themes. As themes became evident, they were categorized and color-coded for easy reference. Then, they were recorded as a matrix in a spreadsheet with the categories listed along the top and the participants by row. The matrix allowed for clear organization of the otherwise overwhelmingly boundless data. The matrix served as a working document as I iteratively deleted examples from the inside cells such that the richest examples of the most prominent themes remained.

As for the quantitative data, it was limited. The eleven students completed four separate online surveys over the course of the class and throughout phases in the lesson study cycles. The first survey had both Likert-scale type questions and free-response type questions. The free response questions were demographic in nature or asked them to type in their goals and priorities. The Likert-scale questions asked the teachers to rate themselves on a scale from 1-5 on their own teaching practices, where 1= novice and 5=exceptional. All 11 teachers completed this survey. The second, third, and fourth surveys were all administered at the end of the course in September. The second survey was all free-response and was a systematic way of having each student enter their observations and reflections after the second cycle of lesson study in the settings of their actual schools (unlike the first cycle when they presented in summer-school classes and posted their observations to the blog). All of the teachers filled this survey out, and some filled it out more than once. The third survey was the post-survey that matched the pre-survey, but unfortunately only 5 teachers filled this one out. The fourth survey had both demographic and Likert-scale type questions, but the responses were to be used strictly for a statewide report to the federal government and not used in evaluating the program. (This explains why there was not a companion survey earlier in the course…I think…)

The observations could be used as more qualitative data to be coded and added to the matrix, if applicable. Since the fourth survey was for a federal report and there was no pre-survey anyway, the only surveys that could be used for descriptive statistical analyses were the matched-pair responses to surveys one and three. Unfortunately, since only 5 completed the post-survey, more than half of the responses from the pre-survey had to be thrown out – negating any validity of quantitative analyses.

# **Results**

**Discussion of Major Findings**

Since I consider myself an optimist and since I will be teaching this class again, I will follow through with the quantitative analyses anyway. ☺ The independent variable was the status of before or after the lesson study took place. Since the same teachers took both surveys, this is considered a matched-pairs study. The several dependent variables (several because each question tested a different construct) afforded Likert-scale style responses, which are defined as ordinal variables (not necessarily interval variables). The correct statistical test for a matched-pairs design with ordinal dependent variables is the Wilcoxon signed ranks test.

After running the Wilcoxon signed ranks test for each of the 12 different constructs, only one significant difference was

The Wilcoxon Signed Ranks Test showed that of the 12 different dependent constructs related to teaching practices, the lesson-study course only elicited one statistically significant change between the pre-survey and post survey; the statistically significant change is the item that reads “have participated in a full-cycle of lesson study and understand its components” (Z = -2.121, P = 0.034).

At first this finding was comical to me because the quantitative analysis indicated that due to the lesson study course itself, the teachers participated in a full-cycle of lesson study. Of course they did! But then I realized the fact that it was statistically significant that the teachers perceived an *understanding* of the components of the lesson study could not be taken for granted. By participating in lesson study, the teachers understood it. Now, the question was still, “How did their participation in the lesson study course change them, if at all?”

And that is a perfect transition to discuss the emerging themes from the qualitative analyses. ---------- [ TO BE CONTINUED… ] ----------

**Conclusion** **and** **Reflection on the Pilot Study**

As I reflect now, just hours before I must present this pilot study, I realize that when I started writing, my intentions were to write about the lesson study “in their own words” – yeh, the words of the teachers, which I have not even begun to do yet! After all the transcribing, reviewing the literature, analyzing all of the artifacts to look for themes, and being disappointed with missing survey data to invalidate the descriptive statistical analyses, I offer these 20 pages and 24 sources and not the 4 pages and 5 sources as outlined on the syllabus. I will however, continue to investigate quantitative themes from the 2011 Petersburg data if upon your review, you advise me that the time and effort is warranted.

Nonetheless, all in all, if I do indeed teach another section of this course this summer, I will be able to collect exactly the data I need (and verify survey completion!!!), and better yet, I will have the first 15 pages of a manuscript already ready to submit for publication before the data is even collected. ☺

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