

**Parasitical Political Pricing, Economic Calculation, and the Size of  
Government:**

**Variations on a Theme by Maffeo Pantaleoni**

Richard E. Wagner

Center for Study of Public Choice  
George Mason University  
Fairfax, VA 22030

Phone: 703-993-2324  
Fax: 703-993-2323  
e-mail: [rwagner@vms1.gmu.edu](mailto:rwagner@vms1.gmu.edu)

## **Parasitical Political Pricing, Economic Calculation, and the Size of Government:**

### **Variations on a Theme by Maffeo Pantaleoni<sup>1</sup>**

Whether economists speak of regulation or taxation, they seem to construe the state as acting within the market economy to modify the terms of market exchange. Regardless of the extent to which the state might modify market prices, resources are allocated through a system of market pricing in any case. Hence, there is but a single, unitary price system that governs the allocation of resources in society. My central theme, which was articulated initially by Maffeo Pantaleoni (1911), is that there exists two distinct price systems in contemporary mixed economies. One is a system of market pricing, the other is a system of political pricing. Each has its own mode and principle of operation. Moreover, the political price system cannot exist on its own, but can only exist parasitically upon the market price system. This is so regardless of whether government provides public goods or brokers wealth transfers between winning and losing coalitions. A point can be reached where a continued expansion in the extent of political pricing weakens the power of the assistance to economic calculation that market prices offer to economic participants. Not only would this increase the volume of economic wreckage in a society, but also it raises the possibility that these calculational difficulties may impose a kind of 'natural' limit on the extent of political pricing in a society.

## **1. The Dual Price Systems of the Mixed Economy**

A sampling of textbooks on the principles of economics yields many references to agricultural markets as providing a good illustration of a competitive market, with the discussion referring to such things as homogeneous products and large numbers of producers. What is left out of this effort to associate agriculture with the textbook model of perfect competition is any recognition that there are few agricultural markets where people are free to employ their capital to produce agricultural products and to offer them for sale on terms mutually agreeable to them and their customers. Entry into most agricultural markets typically is closed and requires government permission, and the marketing of products must flow through government offices or channels at approved prices. Someone who bought some land and cows and then sold milk could end up in jail and surely would collect no money.

Economic organization in such instances is most surely not governed by a system of open competition and market pricing. It is governed by a system of closed competition and political pricing. To be sure, there is a logic of economic relationships that is independent of the substantive forms those relationships might take. This logic is as applicable to contemporary mixed economies, regardless of the particular mix of collective and private participation in resource allocation, as it is to a pure market economy. The economic logic of markets can be used to analyze the market for milk when milk is produced under the closed market arrangements created by the contemporary system of marketing orders in the United States. It can likewise be used to analyze the production of milk

under conditions of open competition.

The logic of pricing and allocation, however, is distinct from the historical or institutional conditions under which pricing and allocation occur. Only if milk were produced under open competition would it be substantively correct to say that the production of milk was organized through a system of market pricing. When production takes place under marketing orders, resources are allocated through a system of political pricing. Market pricing and political pricing are two distinct, though connected, sets of rules that guide the allocation of resources, and the boundary between the two systems of pricing forms a combat zone in society, as Webber and Wildavsky (1986) explain, particularly in their distinction between individualist-market and egalitarian-sectarian regimes.

A system of market pricing rests upon the principles or institutions of private property, freedom of contract, and personal liability (Eucken 1952). These principles comprise the framework of rules that constitute a market economy. Within the rules of a market economy, choices regarding the allocation of resources reflect a consensus among market participants.<sup>2</sup> Is Tulsa, Oklahoma a suitable place to create a seaport? Within a regime of market pricing, the participants themselves effectively reach an agreement about the answer to such a question. There is no need for any resort to conflict and domination to reach a resolution over whether the conversion of Tulsa into a seaport would represent a more highly valued use of resources than any other use of those resources. People who think it would be a good idea to turn Tulsa into a seaport can compete openly with those who think the required resources

would be more valuable in other uses. Those who are unwilling to bid sufficiently to assemble the required resources are, within the framework of rules provided by property, contract, and liability, agreeing with those who secure command of resources, that the projects of the successful bidders are the most valuable anticipated uses of those resources.

Tulsa is a seaport today, and has been for around 40 years. The resources required to convert Tulsa into a seaport were not, however, assembled through a system of market pricing. Those resources were instead assembled through a different price system, a political price system whose constitutive rules are not the principles of property, contract, and liability. They were assembled through the same type of alternative institutional framework that governs the allocation of resources throughout considerable portions of the social democracies of today.

In place of alienable and several ownership, a political price system invokes common ownership and management by committee. With private property the permission of no particular resource owner is required. Resources must be assembled with the agreements of owners, but there is no particular owner whose agreement is necessary. With common or state property, the permission of particular people is required, as with a zoning board or an Environmental Protection Agency. In place of freedom of contract, a political price system invokes duress, coercion, and side constraints. Command over resources can be secured through force and intimidation, and not just through agreement. In place of personal liability, a political price system invokes

common liability. Losses that result because enterprises do not operate as initially anticipated or projected are not borne by the sponsors of those enterprises but are spread throughout the polity.

Governmental decisions concerning resource allocation, unlike those of organizations that participate within the market economy, are not constrained by the consensual requirements that a market economy imposes. Where market pricing reflects contractual relationships, political pricing reflects rapacious relationships. Government is thus able to generate a system of pricing that is different from what would emerge through market pricing. Indeed, it is the creation of such alternative prices that surely provides the reason why people seek to use government in the first place. If it were not for its ability to replace market prices with political prices, there would be no support for a system of political pricing. As Pantaleoni observed, in the final analysis there can be no social reform without a change in prices.

Political pricing represents a different framework for governing resource allocation. It is an alternative price system whose operation is constituted by different rules than those that constrain the operation of a market price system. How different depends, perhaps, on constitutional and institutional matters. The Wicksellian approach or tradition in public finance seeks to have political pricing correspond to the same consensual principle that undergirds market pricing.<sup>3</sup> Other institutional formats diverge in various ways from the consensual principle.

I have no argument against the use of economic reasoning to examine the operation of markets under various governmental constraints. An economic logic

is as capable of illuminating how it happened that Tulsa's becoming a seaport was an outcome of a system of political pricing, as it is of explaining why it would not have arisen within a system of market pricing. It is a mistake, however, to go from a general logic of pricing and allocation to characterize actual pricing and allocation as occurring under a system of market pricing, regardless of the rules governing human relationships and resource allocation. Our knowledge of pricing and allocation is inadequate so long as we fail to recognize that we inhabitants of mixed economies live in the presence of two distinct, though connected, regimes of pricing and allocation.

## **2. The Parasitical Character of Political Pricing**

To note that market prices are self-generating and self-sustaining is not to assert that orderly and peaceful anarchy is a viable historical option.<sup>4</sup> Rather it is to assert only that if people were to limit their activities to those represented by the principles of property and contract, an organized network of economic relationships would arise and these relationships would sustain themselves. We have authentic and coherent knowledge about the self-organizing features of a market economy. If people were to conduct themselves according to the principles that constitute a system of market pricing, an orderly and sustainable network of economic relationships would arise.

No such claim can be made about a collective economy. In the absence of property, contract, and liability, market pricing will not arise. Resource allocation would require planning and allocation without markets. This is

impossible, as recognized even by those faint-hearted proponents of communism who advocated the fiction of market socialism. It is conceivable that there could exist a market economy without a polity and political prices; it is inconceivable that there could exist a collective economy without a market and market prices <sup>5</sup>

Political prices are neither self-generating nor self-sustaining. The presence of a market economy and market pricing is necessary for a collective economy and political pricing to arise. Political pricing can arise in two primary forms. One form is the direct pricing attached to specific enterprises, as illustrated by self-financed public enterprises. The other form is indirect pricing through taxation. I shall consider each in turn.

In either case, the constituency for political pricing and enterprises arises from the ability of those enterprises to provide advantageous pricing for at least some set of people. On the product side of the market, political prices must be lower than market prices for at least some buyers. On the factor side, political prices must be higher than market prices for at least some suppliers. For expositional simplicity, I shall consider only the product side of the market, though a complete treatment of political pricing would include factor markets as well.

It is common in economic discourse to model a firm as producing a single product and selling it at a uniform price. Such price uniformity, if not impossible for collective enterprises, is at least a rare possibility. How could a collective enterprise that sells at a single price compete with a market enterprise? Impossibility cannot be claimed on conceptual grounds because of theories of



natural monopoly and public goods. All that can be claimed is that such a prospect is a remote one. In the absence of some such source of competitive superiority for collective enterprises, those enterprises will have to offer higher prices on average than private enterprises. To be sure, it is sometimes claimed that political enterprises do not have to earn returns for equity holders, and thus have a lower cost of capital that can be passed along through lower prices. This claim is mistaken because there is a cost of capital to collective enterprises. Governments can generally borrow more cheaply than private enterprises not because they involve less risk and, hence, a lower cost of capital, but because they can shift risk from bondholders to taxpayers. Hence, what appears to be a lower cost of capital is really a by-product of their being two prices at the factor supply level: one is a price to bondholders and the other is a tacit or implicit price to taxpayers. This latter component of cost is left out of the claim that the nonprofit status of government gives it a cost advantage over private firms.<sup>6</sup>

For the remainder of this paper I will operate with the assumption that a public enterprise cannot sell at a single price that is lower than the market price that market enterprises would offer. A political enterprise may offer prices that are lower to some buyers than what market enterprises offer, but only because they also charge higher prices to other buyers--and have the ability to force consumption at those higher prices. If a market enterprise sells at a single price, the creation of a political enterprise will thus involve at least two prices, one below the market price and one above the market price. The higher price to some buyers is necessary to make possible the lower price to other, politically-

avored buyers.

As a concrete example, consider the supply of bus service by a municipality. To start, the municipal bus enterprise will be parasitical upon the market relationships that have been established regarding such things as routes of travel and prices. Moreover, it will not be able simply to charge less for travel than existing market enterprises. It can charge less for some riders only by charging more to others. This raises the problem of how the political enterprise can secure customers. To those for whom it offers lower prices, the supporters of the enterprise, it is apparent why they would support the enterprise.

But why would those to whom prices would be raised not turn to market alternatives, where they could buy at lower prices? Should they do so, the ability of the collective enterprise to serve its clients would be weakened, provided only that the collective enterprise is not genuinely more efficient than alternative private enterprises. In the absence of such competitive superiority, political pricing will be unable to survive in direct competition with market pricing. For the collective enterprise with its political pricing to survive competition with private enterprises, the collective enterprise must extinguish, or at least impair, market pricing in its vicinity. With respect to municipal bus enterprises, ordinances against jitney service are one illustration (Eckert and Hilton 1972). By extinguishing or impairing market pricing in its vicinity, the political enterprise replaces the single market price with at least two political prices: a lower price to the supporting clientele and a higher price to everyone else.<sup>7</sup>

As the number of political enterprises expands, an increasing number of

market prices are suffocated or impaired, and in each case are replaced by at least two political prices--prices, however, that are administrative and not grounded in preferences and technology, as reflected through market prices. While it was market pricing that provided the "reformist" opportunities for political pricing in the first place, the political prices cannot compete against the market prices, so must impair or extinguish them. Political prices cannot sustain themselves, but can be sustained only through force or intimidation.

Rather than being financed through two levels of direct pricing, along with the attendant enforcement difficulties, the political enterprise could be financed at least partially through taxation. Direct pricing creates the problem of precluding resale and competition from market enterprises. When tax finance replaces direct pricing to disfavored buyers, this problem can be avoided. Taxation is typically described as modifying the terms of trade in the market economy, but it is really more than this. Taxation is one instrument by which the political price system expands relative to the market price system. To keep the tax side simple, suppose revenues are raised through a proportional tax on income. Tax finance creates different prices for different people, with the price varying in proportion to income. With tax finance the law of one price is replaced by differential pricing, with the number of price differences equal to the number of different incomes in society.

A collective enterprise that is financed wholly through taxation charges a unique price to each buyer, equal to that person's share in total income and paid regardless of whether the buyer makes use of the collective service.<sup>8</sup>

Regardless of income earned on factor markets, an expansion in the political product market would be accompanied by a growing equalization of standards of living. As collective provision and its accompanying equalization of real income proceeds, the supply of productive factors will decline. The limiting case of fully collective provision on the product market implies full equalization of real incomes on the factor market. In this instance, the political price system will have dissolved, leaving only three options: (1) an impossible attempt at genuine collectivist planning, (2) an effort to play at market through importing price-like information from outside, and (3) the replacement of political with market pricing.

### **3. Political Pricing and Calculational Degradation**

Although there is considerable controversy over the details of explanation, economists are agreed that a valuable feature of market pricing is the assistance it gives to economic calculation (Thomsen 1992). Market prices serve as navigational aids to economic calculation. Prices are not sufficient information for effective economic conduct, but they are necessary and powerful. Prices are navigational aids that improve the efficiency with which people get where they want to go. Prices cannot prevent people from wrecking or getting lost, because the realized value of any project will depend on future conditions that could not have been known at the time a project was undertaken. Market prices do, however, reduce the volume of economic wreckage.

Whatever the network of navigational aids that comprise a market economy might look like, the injection of political pricing disables some of those

aids. Sometimes it does so completely, as equivalent to destroying a buoy. At other times it does so only partially, as in giving the buoy a loose and variable anchorage. In any case, a political price cannot compete openly against a market price, provided only that the collective enterprise is not competitively superior to the displaced private enterprises. Therefore, the injection of political pricing into the economy must involve the degradation if not the destruction of market pricing in those areas most closely related to the price domain of the public enterprise.

For instance, a public enterprise of busses traveling fixed routes and staffed by full-time labor cannot compete with the jitney service provided by people who carry passengers to and from work as a by-product of their own journey to work and back. To create a place for the city bus enterprise to be successful, market pricing must be degraded. There are several ways this might be accomplished, on both the product and factor sides of the market. The prices of substitute services could be increased, as by placing a tax on jitney fees. Most likely, such taxes would be hard to collect, and an outright prohibition of jitney service might be more helpful to the public enterprise. As a result, genuine market signals concerning both the demand and supply sides of the market for rides are degraded, and are replaced by a source that is incapable of independent existence. And if restrictions on competitors were not sufficient for the success of the political enterprise, subsidization through taxation might be attempted. In this way, even the substitute signals are weakened.

Through the destruction or degradation of market prices, the growth of the

political price system reduces the quality of the navigational aids that inhabit the economy. Contrary to Littlechild (1978), however, I offer no argument for any “fallacy” of the mixed economy. Rather what I offer is a line of argument suggesting that political pricing becomes more damaging as its scope expands. Even in a world of full market pricing, economic wreckage would arise, and in many ways. There can be all kinds of ways in which enterprises and projects will have been undertaken that do not turn out as planned. In some cases the actual outcome will have been less satisfactory than what was planned or anticipated, and the promoter and those who relied on him will suffer various kinds of capital losses. The opposite outcome is possible as well, of course, in which case the promoter finds himself with too small an enterprise. In any case there will exist unanticipated capital gains and losses in a purely market economy because the navigational aids that market prices represent can never be fully sufficient instruments for judgment and choice. The absolute value of capital gains and losses is an indicator of the extent to which plans have not been fulfilled.

As political pricing grows and market pricing weakens, the volume of economic wreckage will surely increase. Political prices attach themselves to particular market prices, extinguishing the market prices in the process. Due to the connectedness among market prices, there is little problem for political prices to substitute for market prices when their numbers are small. But as political prices proliferate and an increasing number of market prices have been extinguished, the informational value of market prices declines and economic calculation becomes increasingly difficult and mistaken.

The economic wreckage that might be attributed to the degradation of the calculational aids represented by market prices are unlikely to be distributed uniformly throughout the economy. Their distribution would probably reflect the pattern of injection of political pricing into the economy. Economic wreckage would increase in those places where political prices had more fully degraded market prices.

This has a number of potential empirical implications, or so it would seem. For one thing, the growth of political pricing would increase the volume of unanticipated capital gains and losses in the economy, relative to various economic aggregates. Growth in an economy characterized by a robust political price system would be distributed differently than growth in an economy dominated by market pricing. Provided only that political prices are not distributed uniformly throughout the economy, the variability of growth rates across the economy would expand along with the expansion in political pricing. More rapid growth would occur in those areas where the navigational aids offered by market prices had been less degraded by political pricing.

On the other hand, in those areas where political pricing was relatively strong and the reliability of navigational aids relatively weak, economic wreckage would loom larger. This greater riskiness would be accompanied by larger costs of capital to market participants who sail those waters. With respect to the approach to the evaluation of commercial enterprises suggested by the capital asset pricing model, the stock market betas of such enterprises should be larger due to the expansion of political pricing. Conversely, enterprises operating in

areas where political pricing was relatively slight would have lower costs of capital and lower betas.<sup>9</sup>

#### **4. A Natural Limit to Political Pricing?**

It is obvious that there is some limit to the extent to which the political price system can operate in a society, due simply to the parasitic nature of political pricing. Without a system of market pricing, political pricing cannot exist. This purely formal point sheds no insight into just where short of 100 percent that limit might reside. Whether it is possible to advance some numerical limit is dubious, because the limit could well depend on the particular location at which political pricing replaced or degraded market prices.

Indeed, to posit the principle of a limit does not even imply that the equilibrium relationship implied by that limit will be attained. For instance, and by way of illustration, suppose the relationship between the extent of market pricing (M) and the extent of political pricing (P) can be represented by the following pair of equations:

$$dM/dt = (a_1 - b_1 P)M, \text{ and}$$

$$dP/dt = (-a_2 + b_2 M)P.$$

The positive sign on  $a_1$  reflects some “natural” rate of expansion in market pricing in the absence of impediments from political pricing. In contrast, the negative sign on  $a_2$  reflects the inability of political pricing to survive in the absence of market pricing. The negative sign on  $b_1$  conveys the claim that an expansion in political pricing undermines the vitality of market pricing, while the positive sign



on  $b_2$  conveys the claim that an expansion in the hosting system of market prices allows an expansion in the parasitical system of political prices.<sup>10</sup>

This pair of equations has the solution

$$a_2 \ln M - b_2 M + a_1 \ln P - b_1 P = \text{constant.}$$

Figure One provides a graphical representation of these equations, and this representation illustrates the points noted above. First of all, there is an equilibrium solution to this pair of equations. Taking parameter values of  $a_1 = 1.0$ ,  $b_1 = 0.1$ ,  $a_2 = 0.5$ , and  $b_2 = 0.02$  a point of stability results where  $M = 25$  and  $P = 10$ .<sup>11</sup> Different initial values of  $M$  and  $P$ , in conjunction with different values for the constant, generate the three curves portrayed in Figure 1. Each curve shows some endless, repetitive oscillation in light of the initial conditions, and thus represents a static representation through time. This representation yields an interval where the domain of market pricing expands while that of political pricing falls, as well as a subsequent transformation into a situation where both market pricing and political pricing expand. Eventually, the expansion in market pricing reverses itself and the domain of market pricing contracts, though the domain of political pricing continues to expand. This expansion in political pricing also ends, and is replaced by an interval where both market and political pricing contract. And then the process repeats itself.

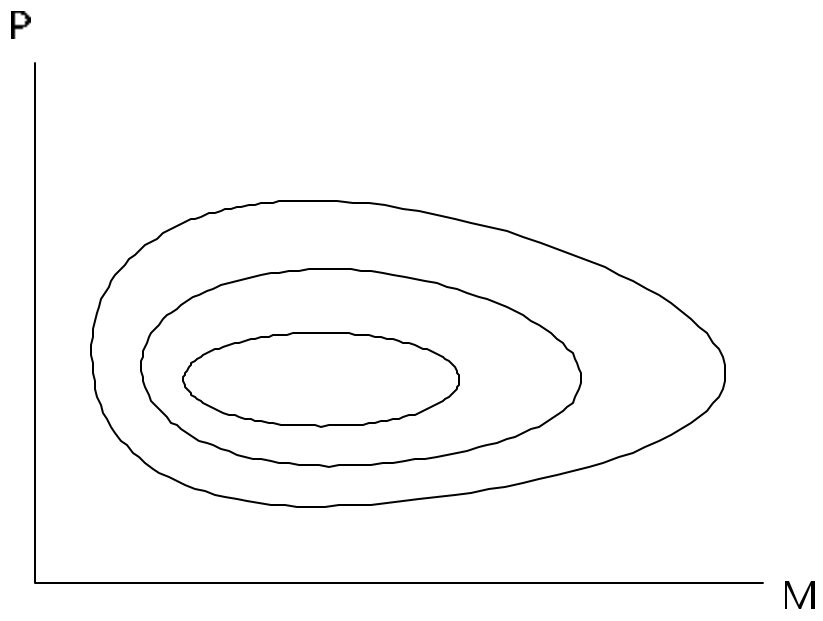
## 5. Final Remarks

I do not offer Figure 1 and its associated equations as any kind of realistic portrait of actual social processes. It is far too static for that. I offer it rather as a

simple indicator of some of the logical implications of the parasitical character of political pricing. A robust system of market pricing can accommodate some amount of political pricing without undergoing any significant impairment of the health of the market pricing system. But a point can be reached where the continued expansion in political pricing does impair the health of the system of market pricing, which in turn can eventually impair the viability of those enterprises organized within the framework of political pricing as well.

In any case, and in closing, contemporary mixed economies do *not* possess a single pricing system that is in various ways shaped and constrained through government. Rather they possess dual pricing systems, with political pricing bearing a parasitic relationship to market pricing. To be sure, some forms of parasitism are beneficial to both parties, and these are illustrated by genuinely public goods. Other forms of parasitism are beneficial to the parasite while detrimental to the host. This is illustrated by many of the state-sponsored activities in contemporary social democracies, where enterprises that could not survive within the openly competitive framework represented by the principles of private property and freedom of contract are sheltered from such competition through a wide-ranging variety of restrictions on property and contract, including, but not limited to, taxation. Recognition of the parasitical character of political pricing would surely represent a cognitive advance over the presumption of a single price system, regardless of where such recognition might lead.

Figure 1



## REFERENCES

- Buchanan, J. M., *The Limits of Liberty*, Chicago: University of Chicago Press, 1975.
- Bush, W. C., "Individual Welfare in Anarchy," in G. Tullock (ed.) *Explorations in the Theory of Anarchy*, Blacksburg, VA: Center for Study of Public Choice, 1972, pp. 5-18.
- Cowen, T. (ed.), *The Theory of Market Failure*, Fairfax, VA: George Mason University Press, 1988.
- De Angelo, H., "Competition and Unanimity," *American Economic Review* 71, 1981, pp. 18-27.
- De Jasay, A., *The State*, Oxford: Basil Blackwell, 1985.
- Eckert, R. D. and Hilton, G. W., "The Jitneys," *Journal of Law and Economics* 15, 1972, pp. 293-325.
- Eucken, W., *Grundsätze der Wirtschaftspolitik*, Tübingen: J. C. B. Mohr, 1952.
- Hayek, F. (ed.), *Collectivist Economic Planning*, London: George Routledge and Sons, 1935.
- Hennipman, P., "Wicksell and Pareto: Their Relationship in the Theory of Public Finance," *History of Political Economy* 14, 1982, pp. 37-64.
- Littlechild, S., *The Fallacy of the Mixed Economy*, London: Institute for Economic Affairs, 1978.
- Lott, John R., Jr., "Predation by Public Enterprises," *Journal of Public Economics* 43, 1990, pp. 237-51.
- Makowski, L., "Competition and Unanimity Revisited," *American Economic*

- Review* 73, 1983, pp. 329-39.
- Mises, L. von., "Economic Calculation in the Socialist Commonwealth," 1920, reprinted in Hayek, (ed.) (1935), pp. 87-130.
- Pantaleoni, M., "Considerazioni sulle proprietà di un sistema di prezzi politici," *Giornale degli Economisti* 42 , 1911, pp. 9-29 and 114-33.
- Roberts, P. C., *Alienation and the Soviet Economy*, Albuquerque: University of New Mexico Press, 1971.
- Thomsen, E. F., *Prices and Knowledge: A Market-Process Perspective*, London: Routledge, 1992.
- Wagner, R. E., "The Calculus of Consent: A Wicksellian Retrospective," *Public Choice* 56, 1988, pp. 153-66.
- Webber, C. and Wildavsky, A., *A History of Taxation and Expenditure in the Western World*, New York: Simon and Schuster, 1986.
- Wicksell, K., *Finanztheoretische Untersuchungen nebst Darstellung und Kritik des Steuerwesens Schwedens*, Jena: Gustav Fischer, 1896.

---

<sup>1</sup> \*I am grateful to Domenico Da Empoli for valuable suggestions regarding an earlier version of this paper.

<sup>2</sup> See, for instance, De Angelo (1981) and Makowski (1983).

<sup>3</sup> Besides Wicksell (1896), see Hennipman (1982), and Wagner (1988). For a cogently skeptical line of argument, see De Jasay (1985).

<sup>4</sup> See Bush (1972) and Buchanan (1975) for skeptical arguments regarding such historical viability.

<sup>5</sup> See, for instance, the papers collected in Hayek, ed (1935). In a related vein,

---

Roberts (1971) shows that the Soviet Union did not replace market allocation with collective planning, but rather created a form of political price system that sought some anchorage in some semblance of market pricing. Pantaleoni's argument about the impossibility of a complete system of political pricing predated the widely-cited argument of Mises (1920), reprinted in Hayek (1935), on the impossibility of genuine socialism.

<sup>6</sup> Even Lott's (1990) model of predatory pricing by public enterprises derives some of its features by virtue of the public enterprise's being able to operate without regard to returns to stockholders-taxpayers, and hence to operate with a second source of financing outside that raised directly from prices.

<sup>7</sup> What is called "cream skimming" is the activity of market participants in otherwise eroding the foundations of the political enterprise.

<sup>8</sup> It is even possible to conceptualize collective enterprises that varied prices according to buyer's incomes, as proportional taxation accomplishes.

<sup>9</sup> To be sure, the presence of political pricing is not the only reason for betas to vary among enterprises. I am not advancing any semblance of a theory of beta here, but am trying only to illustrate some of the implications of political pricing and economic calculation.

<sup>10</sup> It should be noted that cases of genuinely public goods provide an exception, for such goods cannot be provided efficiently through alternative private arrangements. While this caveat must be made on conceptual grounds, the domain over which genuinely collective goods might operate in contemporary societies might be quite small, as suggested, for instance, by the essays collected in Cowen (1988).

<sup>11</sup> This is where  $M = a_2/b_2$  and  $P = a_1/b_1$ .