1 Inventory control problem

In this section, we illustrate how dynamic programming can be used to solve inventory problem with the following characteristics:

1. Time is broken up into periods, the present period being 1, the next period 2, and the final period T. At the beginning of period 1, the demand during each period is known.
2. At the beginning of each period, the firm must determine how many units should be produced. Production capacity during each period is limited.
3. Each period's demand must be met on time from inventory or current production. During any period in which production takes place, a fixed cost of production as well as a variable per-unit cost is incurred.
4. The firm has limited storage capacity. This is reflected by a limit on end-of-period inventory. A per-unit holding cost is incurred on each period's ending inventory.
5. The firm's goal is to minimize the total cost of meeting on time the demands for periods 1, 2, $\cdots$, T.

In this model, the firm's inventory position is reviewed at the end of each period (say, at the end of each month), and then the production decision is made. Such a model is called a periodic review model. This model is in contrast to the continuous review model in which the firm knows its inventory position at all times and may place an order or begin production at any time. If we exclude the setup cost for producing any units, the inventory problem just described is similar to the Sailco inventory problem that we solved by linear programming. Here, we illustrate how dynamic programming can be used to determine a production schedule that minimizes the total cost incurred in an inventory problem that meets the preceding description.

1.0.1 Example 1: Inventory control problem from [?] page 969

A company knows that the demand from its product during each of the next four months will be as follows: month 1, 1 unit; month 2, 3 units; month 3, 2 units; month 4, 4 units. At the beginning of each month, the company must determine how many units should be produced during the current month. During a month in which any units are produced, a setup cost of \$3 is incurred. In addition, there is a variable cost of \$1 for every unit produced. At the end of each month, a holding cost of 50 cents per unit on hand is incurred. Capacity limitations allow a maximum of 5 units to be produced during each month. The size of the company's warehouse restricts the ending inventory for each month to 4 units at most. The company wants to determine a production schedule that will meet all demands on time and will minimize the sum of production and holding costs during the four months. Assume that 0 units are on hand at the beginning of the first month.

Solution:

Network is not given so you have to draw it. Often, you cannot draw it because its huge.

Define the stage, state, action, exogenous information, contribution and value function:

Define $f_t(i)$ as the minimum cost of meeting demand D at time $t, t+1,, 4$ if i units are **available** at the beginning of time t. Let $c(x)$ be the cost of producing x units, $c(0) = 0$ and $c(x) = 3 + x$.

Inventory at the beginning of a month = Inventory at the beginning of the previous month + production in the previous month-demand in the previous month. In general

$$f_t(i) = \min_x [c(x) + 0.5(i + x - D) + f_{t+1}(i + x - D)] \quad (1)$$

where D is the demand.

1.0.2 Example 2: Inventory control problem -Sailco example in DP

Sailco corporation must determine a production strategy for its sailboats for each day for the next 4 days. The demand for the next 4 days are 40, 60, 75, 25 sailboats. All demand must be met. Starting inventory is 10 boats on day 1. Sailco can produce upto 40 boats in a day with regular labor at a cost of \$400 per boat. However, Sailco can produce additional boats with overtime labor at a cost of \$450 per boat. At the end of the day the holding cost of a boat in the inventory is \$20 per boat. Formulate DP to minimize cost of production over the next 4 days.

Let x_t= number of boats made by regular labor during day t

y_t= number of boats made by overtime labor during day t

State i_t = inventory at the beginning of day t

Contribution function $c(i_t, x_t, y_t) = 400x_t + 450y_t + 20i_t$

Exogenous variable demand d_t

State transition function $i_{t+1} = i_t + x_t + y_t - d_t$

Action at stage t,

$$x_t = i_{t+1} - i_t + d_t, \quad y_t = 0, \quad if \quad x_t \leq 40, \tag{2}$$

$$y_t = i_{t+1} - i_t - 40 + d_t, \quad and \quad x_t = 40 \quad if \quad x_t > 40 \tag{3}$$

$$V(i_t) = \min_{(x_t,y_t)} [c(i_t, x_t, y_t) + V(i_{t+1})] \tag{4}$$

Set $V(i_5) = 0$ and solve backwards till $V(i_0) = 10$. See matlab code for solution.

1.0.3 Power plant problem as an Inventory control problem

An electric plant forecasts that r_t KWH will be need in year t. Each year it must decide how much generating capacity it must expand. It cost $c_t(x)$ dollars to increase capacity by x KWH during year t. At the end of each year 10% of the previous year's capacity is obsolete (except at the end of the first year of operation because previous year is 0 capacity). It costs the utility $m_t(i)$ dollars to maintain i units of capacity during year t. At the beginning of year 1, 100,000 KWH of capacity is available. Formulate DP to minimize cost of meeting power requirement

Hint: Define $f_t(i_t)$ to be the minimum cost incurred at t given that i_t capacity is available at the beginning of year t. Define x_t as the new capacity added at time t.