5.3 3rd edition, 4th edition.

First integrator block $1/s$

$4(F - 6X)\frac{1}{s}$ comes out of the block

Second integrator block $1/s$

$$\left[4(F-6X)\frac{1}{s} + (-G) - 8X\right]\frac{1}{s} = X$$

$$\left[\frac{4F}{s} - \frac{24X}{s} - G - 8X\right]\frac{1}{s} = X$$

$$sX + \frac{24X}{s} + 8X = \frac{4F}{s} - G$$

$$X\left[s + \frac{24}{s} + 8\right] = \frac{4}{s}F - G$$

$$\frac{X}{F} = \frac{4/s}{\left(s+\frac{24}{s}+8\right)} = \frac{\frac{4}{s}}{\frac{s^2+24+8s}{s}} = \frac{4}{s^2+8s+24}$$

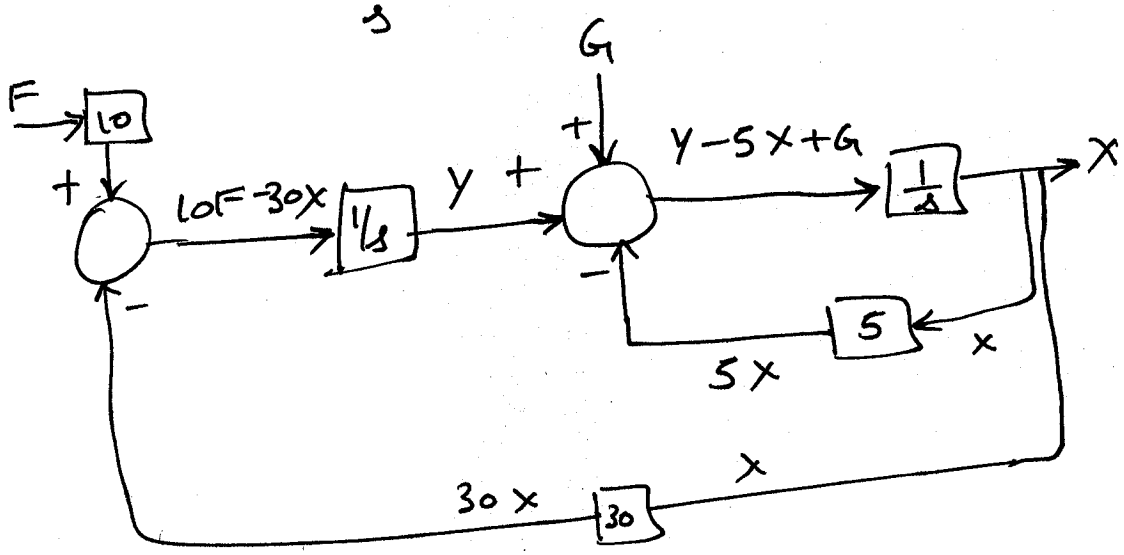
5.5 3rd edition, 4th edition

$\dot{x} = y - 5x + g(t)$ $\quad \dot{y} = 10f(t) - 30x$

Assume $x(0) = 0$ $\quad y(0) = 0$

$sX = Y - 5X + G$ $\quad sY = 10F - 30X$

$X = \frac{1}{s}[Y - 5X + G]$ $\quad Y = \frac{1}{s}[10F - 30X]$



5.17 3rd edition

$$2\ddot{x} + 5\dot{x} + 4x = 4y(t)$$

S-V form.

$$2s^2X + 5sX + 4X = 4Y$$

$$\frac{Y}{X} = \frac{2s^2+5s+4}{4}$$

$$\frac{\text{output}}{\text{input}} = \frac{X}{Y} = \frac{4}{2s^2+5s+4}$$

Method 1

$$\frac{X}{Y} = \frac{4/2s^2}{1+5s/2s^2+4/2s^2}$$

$$X\left[1 + \frac{5s}{2s^2} + \frac{4}{2s^2}\right] = \frac{4Y}{2s^2}$$

$$X = \frac{4Y}{2s^2} - \frac{5sX}{2s^2} - \frac{4X}{2s^2} = \frac{1}{2s^2}\left[4Y - 5sX - 4X\right]$$

$$X = \frac{1}{s^2}\left[2Y - \frac{5}{2}sX - 2X\right]$$

$$X = \frac{1}{s}\underbrace{\left\{\frac{1}{s}\left[2Y - \frac{5sX}{2} - 2X\right]\right\}}_{Z_1}$$

$$X = \underbrace{\frac{1}{s}Z_1}_{Z_2} \qquad X = Z_2$$

$$\boxed{x = z_2}$$

$$Z_2 = \frac{1}{s} Z_1$$

$$s Z_2 = Z_1 \qquad \boxed{\dot{z}_2 = z_1}$$

$$s Z_1 = 2y - \frac{5}{2} s x - 2x$$

$$s Z_1 = 2y - \frac{5}{2} s Z_2 - 2 Z_2$$

$$\dot{z}_1 = 2y - \frac{5}{2} \dot{z}_2 - 2 z_2$$

$$\boxed{\dot{z}_1 = 2y - \frac{5}{2} z_1 - 2 z_2}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -5/2 & -2 \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}_{B} y$$

$$x = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \underbrace{[0]}_{D} y$$

5.21 3rd edition.

$$\frac{Y}{F} = \frac{6s+7}{s+3}$$

Method 1

$$\text{output} = \frac{Y}{F} = \frac{6s/s + 7/s}{1 + 3/s}$$

$$\text{input}$$

$$Y[1+3/s) = F(6+7/s)$$

$$Y = 6F + \frac{7F}{s} - \frac{3Y}{s}$$

$$Y = 6F + \frac{1}{s}\underbrace{[7F-3Y]}_{X}$$

$$Y = 6F + X \qquad \boxed{y = x + 6f}$$

$$sX = 7F - 3Y \qquad \dot{x} = 7f - 3(x+6f)$$

$$\dot{x} = -3x + 7f - 18f$$

$$\boxed{\dot{x} = -3x - 11f}$$

$$A = -3 \quad B = -11$$

$$C = 1 \quad D = 6$$

Method 2 $\quad Y = (6s+7)\left(\frac{1}{s+3}\right)$

$$Y = 6sX + 7X \qquad X$$

$$X = \frac{F}{s+3}$$

$$sX + 3X = F$$

$$sX = -3X + F$$

$$\boxed{\dot{x} = -3x + f}$$

$$y = 6(-3X + F) + 7X$$

$$y = -18X + 6F + 7X$$

$$y = -11X + 6F$$

$$\boxed{y = -11x + 6f}$$

$A = -3 \quad B = 1$

$C = -11 \quad D = 6$

B & C got interchanged in Method 2 when compared with Method 1

5.22 3rd edition

$$\frac{Y}{F} = \frac{s+2}{s^2+4s+3}$$

method 1

$$\frac{Y}{F} = \frac{s/s^2 + 2/s^2}{1 + 4/s^2 + 3/s^2}$$

$$Y\left[1 + \frac{4s}{s^2} + \frac{3}{s^2}\right] = F\left[\frac{1}{s} + \frac{2}{s^2}\right]$$

$$Y = \frac{F}{s} + \frac{2F}{s^2} - \frac{4Y}{s} - \frac{3Y}{s^2}$$

$$Y = \frac{1}{s}\left[F - 4Y + \underbrace{\frac{1}{s}\left[2F - 3Y\right]}_{x_1}\right]$$

x_2

$Y = X_2$ $\boxed{y = x_2}$

$sX_1 = 2F - 3Y$ $\qquad \dot{x_1} = 2f - 3y$

$\boxed{\dot{x_1} = 2f - 3x_2}$

$sX_2 = F - 4Y + X_1 = F - 4X_2 + X_1$

$\boxed{\dot{x_2} = f - 4x_2 + x_1}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{[0]}_{D} u$$

Method 2

$$\frac{Y}{F} = \frac{s+2}{s^2+4s+3}$$

$$Y = (s+2)\left(\frac{F}{s^2+4s+3}\right)$$

$$Y = sX_1 + 2X_1$$

$$y = \dot{x}_1 + 2x_1$$

$$\boxed{y = x_2 + 2x_1}$$

↑ obtained on next page

$$X_1 = \frac{F}{s^2+4s+3}$$

$$s^2X_1 + 4X_1s + 3X_1 = F$$

$$X_1 = \frac{1}{s^2}\left[F - 4X_1s - 3X_1\right]$$

$$X_1 = \frac{1}{s}\left\{\frac{1}{s}\left[F - 4X_1s - 3X_1\right]\right\}$$

$$sX_1 = \underbrace{\frac{1}{s}\left[F - 4X_1s - 3X_1\right]}_{X_2}$$

$sX_1 = X_2$ $\quad \boxed{\dot{x}_1 = x_2}$

$sX_2 = F - 4X_1 s - 3X_1$

$\dot{x}_2 = f - 4\dot{x}_1 - 3x_1$

$\boxed{\dot{x}_2 = f - 4x_2 - 3x_1}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}}_{A} \begin{matrix} x_1 \\ x_2 \end{matrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} f$$

$$y = \underbrace{\begin{bmatrix} 2 & 1 \end{bmatrix}}_{C} \begin{matrix} x_1 \\ x_2 \end{matrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} f$$

A is Transposed

B & C got interchanged.