

HW1 Solutions.

(1)

1) $a(n+1) = 2a(n)$ $a(2) = 3.$

Solution $a(n) = c \cdot 2^n$

$$a(n) = c(2)^n$$

Find c $a(2) = 3 = c(2)^2 = 4c$
 $c = 3/4$

$$a(n) = 3/4 (2)^n$$

Equilibrium $a(n+1) = a(n) = A$

$A = 2A \Rightarrow$ True only if $A = 0$
 0 is the equilibrium point.

2) $a(n+1) = -2a(n)$, $a(2) = 3.$

Solution $a(n) = c \cdot 2^n = c(-2)^n$

Find c $a(2) = 3 = c(-2)^2 = 4c$
 $c = 3/4.$

$$a(n) = 3/4 (-2)^n$$

Equilibrium $a(n+1) = a(n) = A$

$A = -2A \Rightarrow$ True if $A = 0$
 0 is the equilibrium point.

3) $a(n+1) = 0.2a(n) + 5$ $a(0) = 3$. (2)

solution $a(n) = c r^n + \frac{b}{1-r}$ $r \neq 1$

$$a(n) = c (0.2)^n + \frac{5}{1-(0.2)} = c (0.2)^n + \frac{5}{0.8}$$

find c

$$a(0) = 3 = c (0.2)^0 + \frac{5}{0.8} = c + \frac{5}{0.8}$$

$$3 = c + 6.25$$

$$c = -3.25$$

$$a(n) = -3.25 (0.2)^n + 6.25$$

$$\text{Equilibrium} = \frac{b}{1-r} = 6.25$$

4) $a(n+1) = a(n) + 2$ $a(0) = -3$.

$$r = 1$$

solution $a(n) = bn + c = 2n + c$

$$a(0) = 2 \times 0 + c = c = -3$$

$$a(n) = 2n - 3$$

Equilibrium $A = A + 2 \Rightarrow$ Can never be true for any value of t

$$a(n+1) = a(n) = A$$

No equilibrium

5

$$a(0) = 1000$$

a)

5% growth

$$b = -25$$

$$\begin{aligned}
 a(n+1) &= 1000 a(n) + \frac{5}{100} \times 1000 a(n) - 25 \\
 &= 1.05 a(n) - 25
 \end{aligned}$$

Here we assume that 25 die after the 5% growth

Solution
$$a(n) = C(1.05)^n + \frac{(-25)}{1-1.05}$$

Find C

$$a(0) = 1000 = C(1.05)^0 + 500$$

$$C = 500$$

$$a(n) = 500(1.05)^n + 500$$

$$\begin{aligned}
 a(100) &= 500(1.05)^{100} + 500 \\
 &= 66250
 \end{aligned}$$

b)

$$a(n+1) = 1.05a(n) - 60$$

Solution
$$a(n) = C(1.05)^n + \frac{(-60)}{1-1.05}$$

$$a(0) = 1000 = C + 1200$$

$$C = -200$$

$$a(n) = -200(1.05)^n + 1200$$

$$a(100) = -200(1.05)^{100} + 1200 = \text{negative number.}$$

No one is alive in 100 yrs in the town

3