Chapter 3

Basic Physics

Rigid Bodies

Rotation

$x \rightarrow$ Displacement (Linear)

Theta $\theta \rightarrow$ angular displacement

omega $\omega = \dot{\theta} \rightarrow$ angular velocity

$v = \dot{x} \rightarrow$ linear velocity

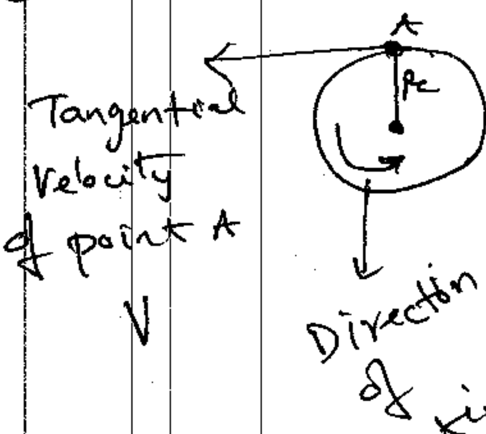
$\ddot{x} \rightarrow$ linear acceleration

$\ddot{\theta} \rightarrow$ angular acceleration

$\ddot{x} = a$

$\ddot{\theta} = \alpha$ → alpha

$r \rightarrow$ radius.



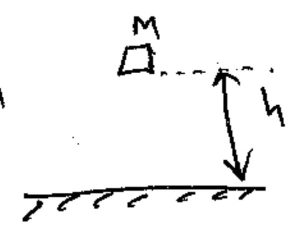
angular velocity $= \omega$

$$\boxed{V = r\omega}$$

$$\dot{V} = r\dot{\omega}$$

$$\boxed{a = r\alpha}$$

Potential Energy $PE = mgh$



Linear Kinetic Energy $KE = \frac{1}{2}mv^2$

Rotational $KE = \frac{1}{2} I \omega^2$

I → moment of Inertia

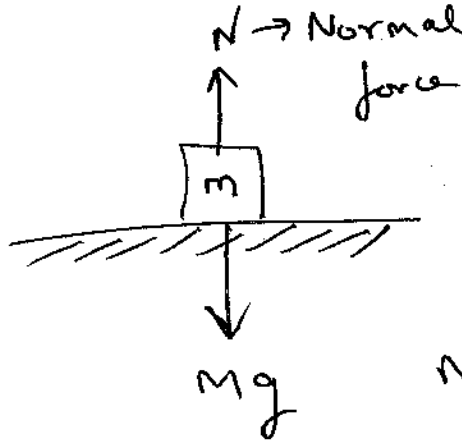
Torque = $I\alpha$ → Rotating (motion)

Force = ma → Linear / Translation (motion)

Solve in Terms of Torque or force depending on the type of motion.

Sometimes it is both

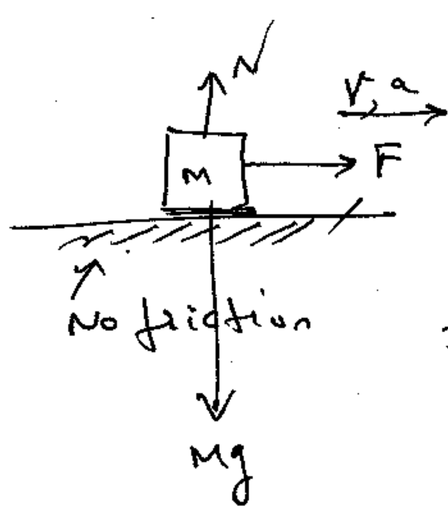
Static



What are the forces on M

$N = mg$

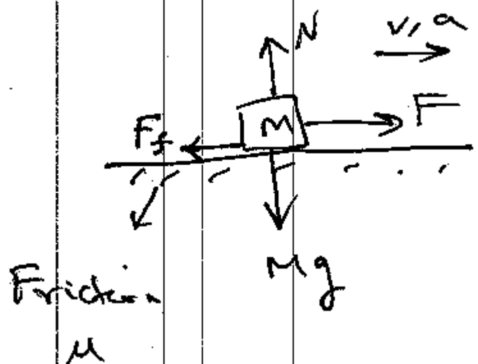
Dynamic



$N = mg$

$F = ma$

Force contributes toward M's acceleration (Newton's 2nd law) e



$F_f \rightarrow$ Frictional force due to coefficient friction μ

$\downarrow$

(Opposes motion)

$N = Mg$

$F > F_f$ (F has to overcome frictional force F_f)

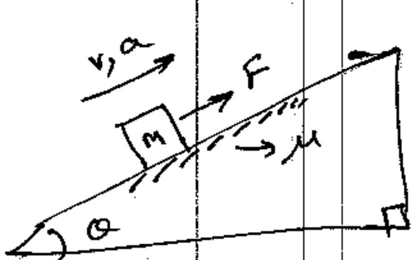
$$\underbrace{F - F_f}_{\text{Resultant force}} = Ma$$

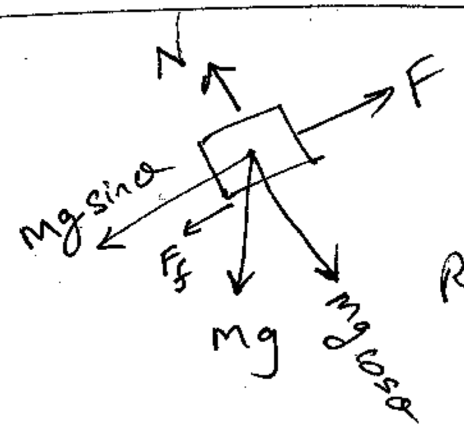
If $F \leqslant F_f$

No movement at all

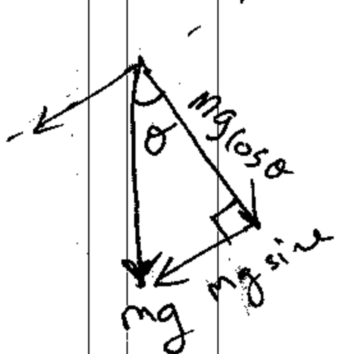
$V = 0$

$a = 0$





Resolve Mg along the plane & perpendicular to the plane.



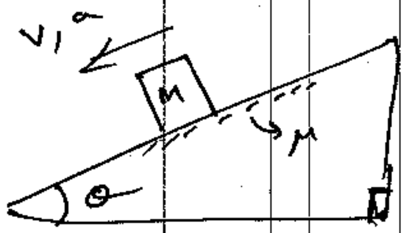
$$\boxed{N = Mg\cos\theta}$$

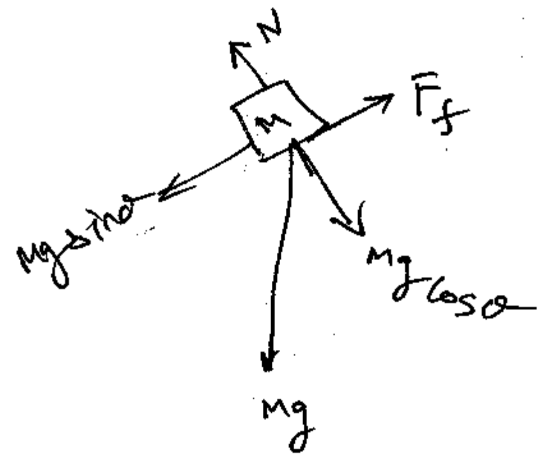
$$F > Mg\sin\theta + F_f$$

$$\boxed{F_f = \mu N}$$

$$\boxed{F - Mg\sin\theta - F_f = Ma}$$

M is sliding down the slope.



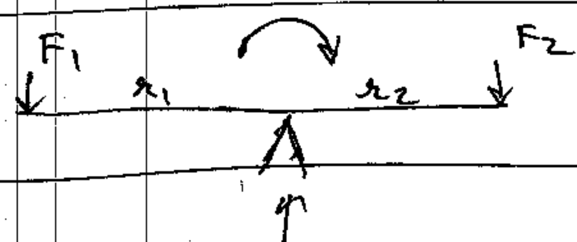


$$\boxed{\begin{aligned} N &= Mg\cos\theta \\ F_f &= \mu N \end{aligned}}$$

$Mg\sin\theta > F_f$

$$\boxed{Mg\sin\theta - F_f = Ma}$$

Rotation



Fulcrum.

Solve in terms of Torque and NOT IN terms of force

DO NOT SAY $F_2 > F_1$ INCORRECT

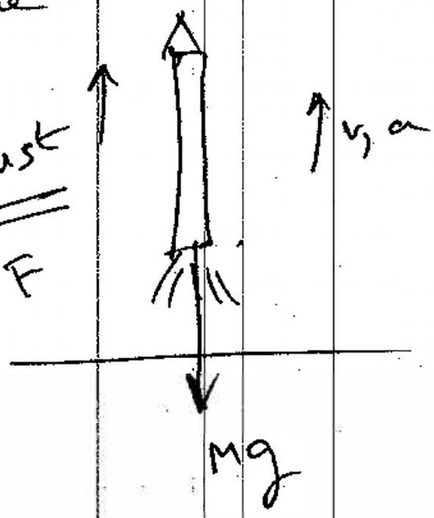
$$\underbrace{F_2 r_2}_{\text{Torque}_2} > \underbrace{F_1 r_1}_{\text{Torque}_1}$$

$$F_2 r_2 - F_1 r_1 = I\alpha$$

Type of force

Thrust

Force F



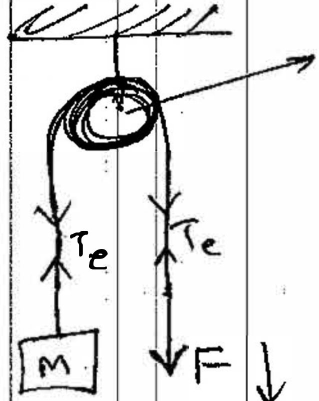
$F > Mg$

~~$F = Ma$~~

~~$F = M\ddot{x}$~~

$F - Mg = Ma$

$F - mg = M\ddot{x}$



v, a ↑

Pulley radius r

M_p Mass $= 0$ → mass of pulley

Write the equation of motion.

$M_P = 0$ means that there is no contribution of energy by the pulley. $I = 0$

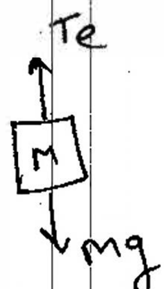
↑ moment of inertia.

Pulley adds zero torque to the system.

Tension T_e

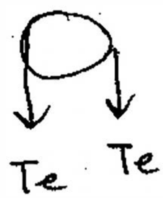
Force type on strings

v, a ↑



$T_e > mg$

$T_e - Mg = Ma$

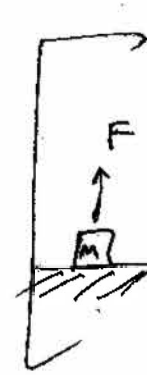


$T_e r = T_e r$

Torque

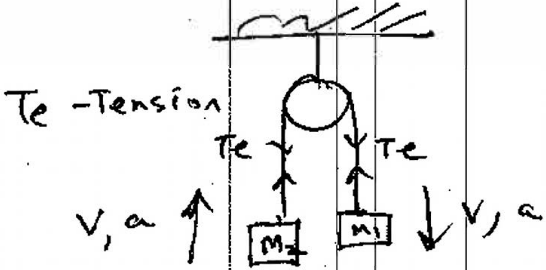
$F = T_e$

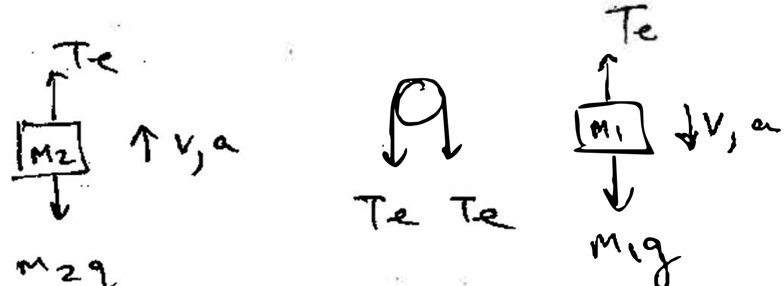
$$\boxed{F - Mg = Ma}$$



M_p - pulley mass = 0

$I = 0$





Te*r = Te*r
Torque

$T_e > M_2 g$

$M_1 g > T_e$

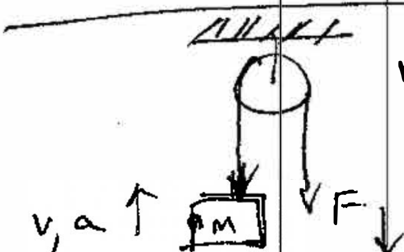
$T_e - M_2 g = M_2 a$

$M_1 g - T_e = M_1 a$

$T_e = M_2 a + M_2 g$

$$M_1 g - (M_2 a + M_2 g) = M_1 a$$

$$(M_1 - M_2) g = (M_1 + M_2) a$$



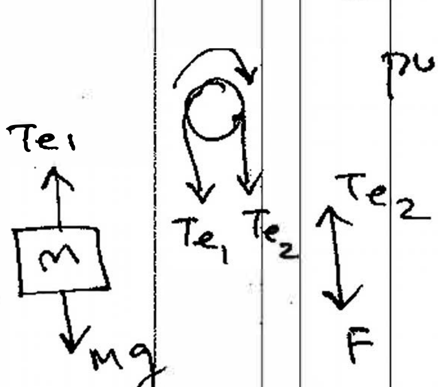
$M_p \neq 0$ pulley has mass m_p

$I \rightarrow$ moment of ~~...~~ inertia

$r \rightarrow$ radius of the pulley.

<u>Cannot ignore the pulley</u>

pulley contributes energy to the system.



$T_{e1} \neq T_{e2}$ Tensions are <u>not equal</u> on either side of the pulley

$$T_{e_1} > mg$$
$$T_{e_1} - mg = ma$$

$$\frac{Fr - I\alpha}{r} - mg = ma$$

$$F - \frac{I\alpha}{r} - mg = ma$$

$$v = r\omega$$
$$a = r\alpha$$

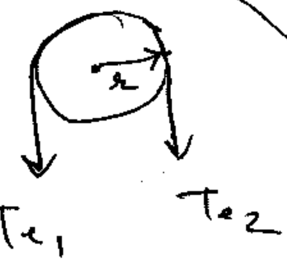
$$F - \frac{I}{r^2}a - mg = ma$$

$$F - mg = \left(m + \frac{I}{r^2}\right)a$$

$$T_{e_2} > T_{e_1} \text{ (Force)}$$

$$rT_{e_2} - T_{e_1} r = I\alpha \text{ (Torque)}$$

$$F = T_{e_2}$$



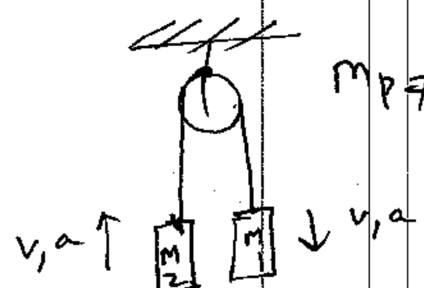
$$Fr - T_{e_1} r = I\alpha$$

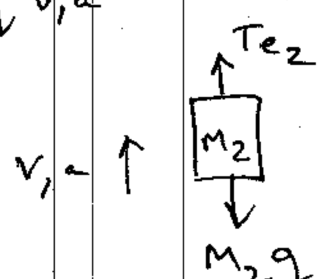
$$T_{e_1} r = Fr - I\alpha$$

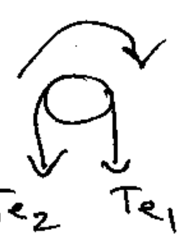
$$T_{e_1} = \frac{Fr - I\alpha}{r}$$

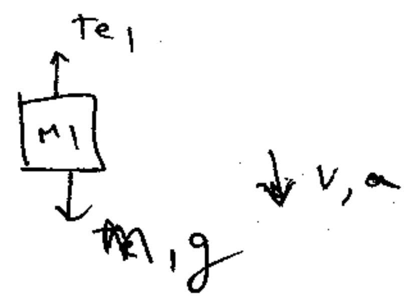
$m_p \neq 0$ pulley has mass m_p.

$I \neq 0$.









$$T_{e_2} \neq T_{e_1}$$

$$M_1 g > T_{e_1}$$
$$M_1 g - T_{e_1} = M_1 a$$

$$T_{e_2} > M_2 g$$
$$T_{e_2} - M_2 g = M_2 a$$

Force equation

$$T_{e_1} r > T_{e_2} r$$
$$T_{e_1} r - T_{e_2} r = I\alpha$$

Torque equation

Force Equation.

$T_{e_2} = m_2 a + m_2 g$ $\qquad$ $T_{e_1} = m_1 g - m_1 a$

plug into the Torque Equation.

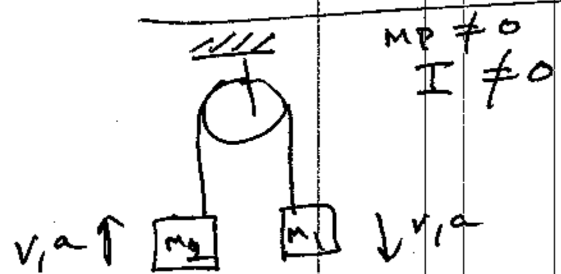
$$r[m_1 g - m_1 a] - r(m_2 a + m_2 g) = I\alpha$$

$$m_1 g - m_2 g = \frac{I\alpha}{r} + m_1 a + m_2 a$$

$\boxed{\alpha = \frac{a}{r}}$

$$(m_1 - m_2) g = \left(\frac{I}{r^2} + m_1 + m_2\right) a$$

Energy Equivalent Approach



$$m_1 g > m_2 g$$

$$m_1 g - m_2 g = M_e a$$

Mass equivalent

M_e

Belace the ~~energy~~ KE of the System.

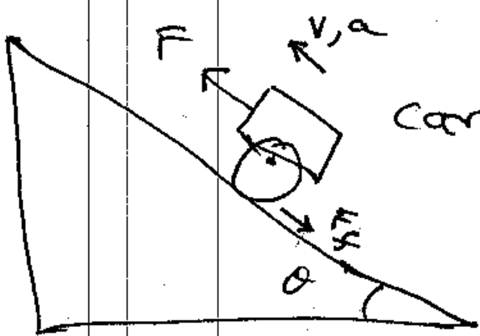
$$\frac{1}{2} M_e V^2 = \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2 + \frac{1}{2} I \omega^2 \qquad (v = r\omega)$$

KE of the whole system

$$= \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2 + \frac{1}{2} I \frac{v^2}{r^2}$$

$$\frac{1}{2} M_e v^2 = \frac{1}{2} \left[\underbrace{m_2 + m_1 + \frac{I}{r^2}}_{M_e} \right] v^2$$

$$\boxed{m_1 g - m_2 g = \left(m_1 + m_2 + \frac{I}{r^2}\right) a}$$



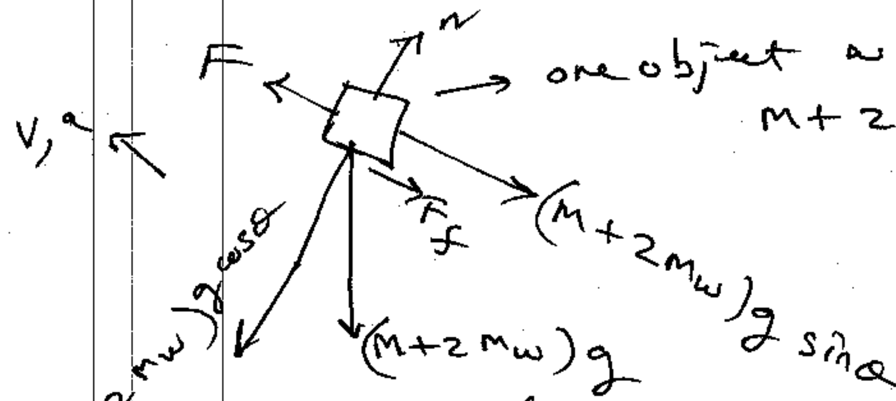
cart with 2 wheels.

mass of cart $= m$
Mass of wheel $= m_w$

Find an equation for acceleration a.

Use energy approch (much easier)

Treat the wheels + the cart as one object.



→ one object with $M+2M_w$ as mass.

$$F > F_f + (M+2m_w)g\sin\theta$$

$$F - F_f - (m+2m_w)g\sin\theta = m_e a$$

KE of whole system

$$\frac{1}{2} m_e v^2 = \underbrace{\frac{1}{2} m v^2}_{\text{cart}} + \underbrace{\overset{\text{2 wheels}}{2\times}\frac{1}{2} m_w v^2}_{\text{wheel linear}} + \underbrace{\overset{\text{2 wheels}}{2\times}\frac{1}{2} I \omega^2}_{\text{wheel rotation}}$$

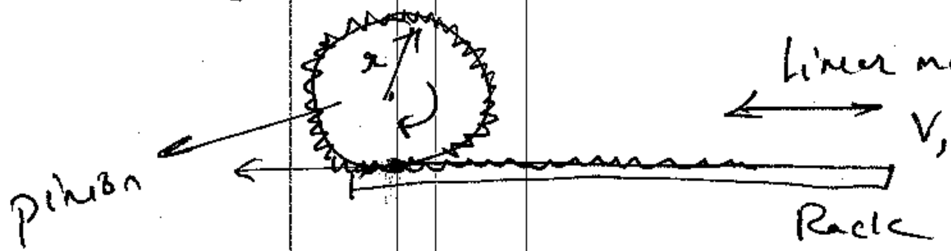
$v = r\omega$

$\omega = \dfrac{v}{r}$

$$\frac{1}{2} m_e v^2 = \frac{1}{2}\underbrace{\left[m + 2m_w + \frac{2I}{r^2}\right]}_{m_e} v^2$$

$$\boxed{F - F_f - (m+2m_w)g\sin\theta = \left(m + 2m_w + \frac{2I}{r^2}\right)a}$$

Rack & Pinion



Pinion is fixed in position. ω, m_p, r

Angular velocity (ω); mass of pinion (m_p)

Rack m_r, v, a

mass of rack (m_r); velocity of rack (v)

A Torque T is applied to the pinion. Find the equation for acceleration a of the Rack.

$\boxed{T = I_e \alpha}$ with the rack meshed with the pinion.

$\boxed{\text{without the rack} \quad T = I\alpha}$

$$\underline{\underline{v = r\omega}}$$
$$a = r\alpha$$

Rotational KE of the whole system

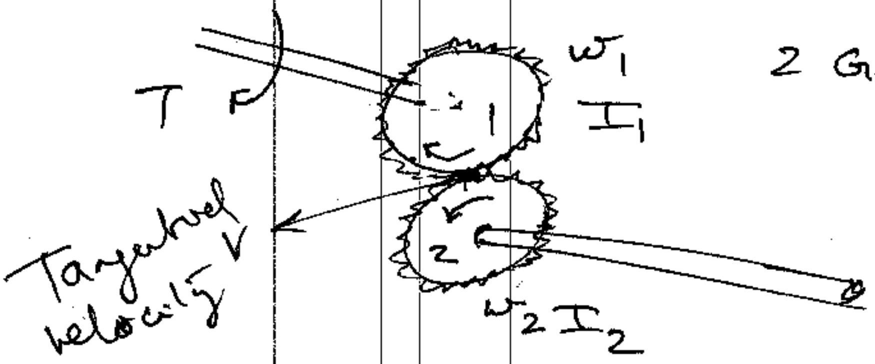
$$\frac{1}{2} I_e \omega^2 = \frac{1}{2} I \omega^2 + \frac{1}{2} m_r v^2$$

$$\frac{1}{2} I_e \omega^2 = \frac{1}{2} I \omega^2 + \frac{1}{2} m_r r^2 \omega^2$$

$$= \frac{1}{2} \underbrace{\left[I + m_r r^2 \right]}_{I_e} \omega^2$$

$$T = \left[I + m_r r^2 \right] \alpha$$

$$T = \left[I + m_r r^2 \right] \frac{a}{r}$$



2 Gears & they are meshed.

T is Torque supplied to Gear 1

Gear ratio $N = \dfrac{\omega_1}{\omega_2} = \dfrac{\text{\# of Teeth on gear 2}}{\text{\# of Teeth on gear}}$

larger r $\quad \dfrac{5}{10} = \dfrac{20}{40}$ → # of Teet[h] on smalle[r] / → # of Teet[h] on larger gear

$\uparrow$ smaller

$$\boxed{T = I_1 \alpha_1 \text{ if not meshed}}$$

$T = I_e \alpha_1$ if meshed.

$$KE \Rightarrow \frac{1}{2} I_e \omega_1^2 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2$$

$$= \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \frac{\omega_1^2 r_1^2}{r_2^2}$$

$$= \frac{1}{2} \underbrace{\left[I_1 + \frac{I_2 r_1^2}{r_2^2} \right]}_{I_e} \omega_1^2$$

(OR)

$$= \frac{1}{2} \left[I_1 + \frac{I_2}{N^2} \right] \omega_1^2$$

$V = r_1 \omega_1$

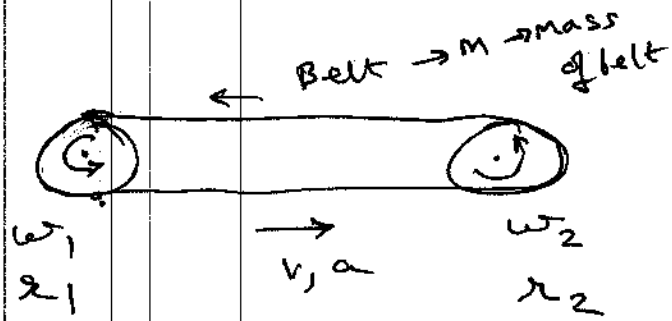
$V = r_2 \omega_2$

$\omega_1 r_1 = r_2 \omega_2$

$\omega_2 = \dfrac{\omega_1 r_1}{r_2}$

$\dfrac{1}{N^2} = \dfrac{r_1^2}{r_2^2}$

$N = \dfrac{r_2}{r_1} = \dfrac{\omega_1}{\omega_2}$



2 pulleys & a belt

pulley position is fixed.

T → Torque is applied.

$$T = I_e \alpha_1$$

$$KE \quad \frac{1}{2} I_e \omega_1^2 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} m v^2$$

$v = r_1 \omega_1$

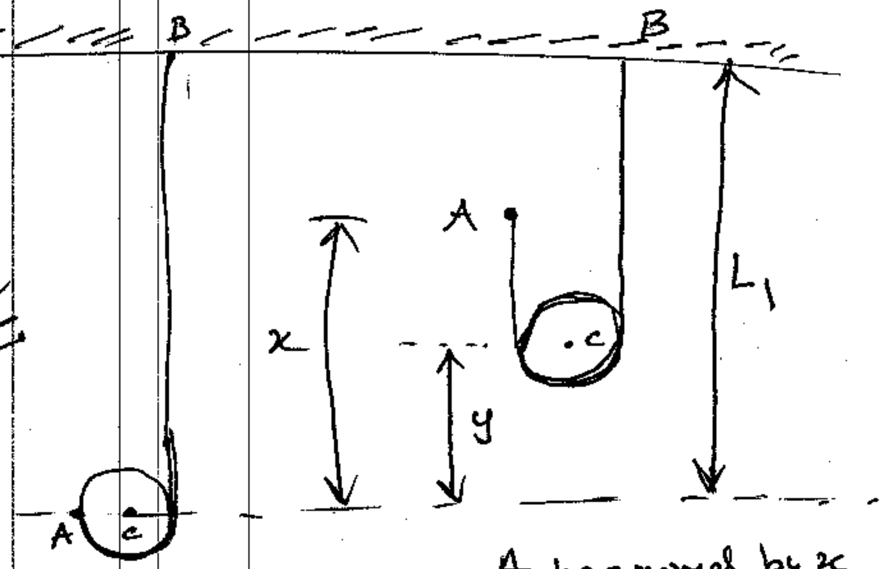
$v = r_2 \omega_2$

$r_1 \omega_1 = r_2 \omega_2$

$\omega_2 = \frac{r_1 \omega_1}{r_2}$

$$= \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \frac{r_1^2 \omega_1^2}{r_2^2} + \frac{1}{2} m r_1^2 \omega_1^2$$

$$= \frac{1}{2} \underbrace{\left[I_1 + \frac{I_2 r_1^2}{r_2^2} + m r_1^2 \right]}_{I_e} \omega_1^2$$



pulley radius $= r$

A-B is a string.
c → pulley.

pulley is NOT FIXED

A has moved by x
pulley c has moved by y

String wraps around half the perimeter of the pulley

L = Length of string

A to B is L

What is the relation between x & y?

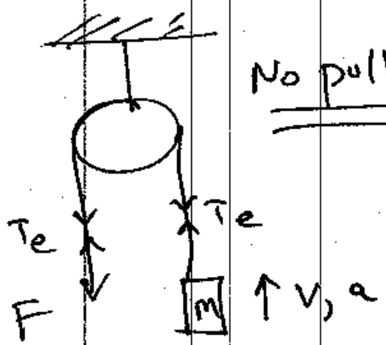
Length of string $L = L_1 + \pi r$

Length of string $L = (x - y) + \pi r + (L_1 - y)$

$$\cancel{L_1} + \cancel{\pi r} = (x - y) + \cancel{\pi r} + (\cancel{L_1} - y)$$

$$x = 2y \qquad \boxed{y = x/2}$$

No pulley Mass



$$F = T_e$$

$$T_e - mg = ma$$

$$\boxed{F - mg = ma}$$

Example.

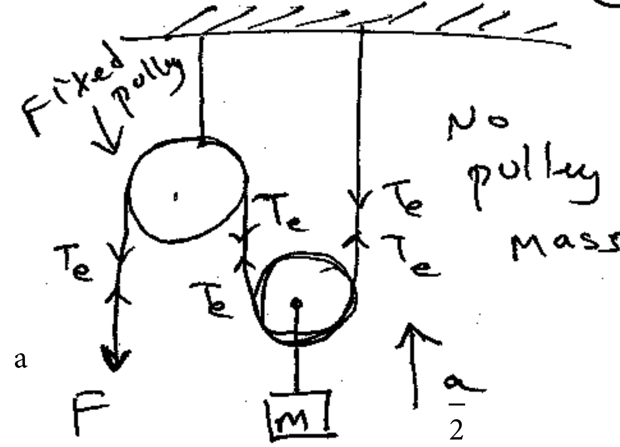
$m = 1\ kg$

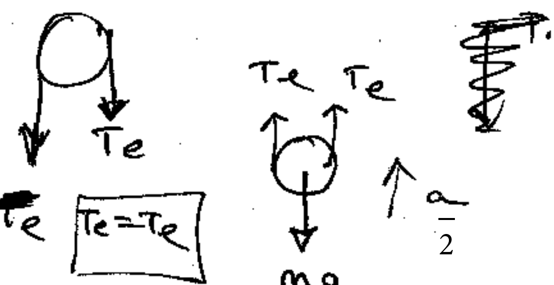
$g = 9.8\ m/s^2 \approx 10\ m/s^2$

$F = 20\ N$

$20 - (1 \times 10) = 1 \times a$

$a = 10\ m/s^2$





$$\boxed{T_e = T_e}$$

$T_e \updownarrow F$

$$\boxed{F = T_e}$$

$$2T_e > mg$$

$$2T_e - mg = m\frac{a}{2}$$

$$T_e = \frac{ma/2 + mg}{2}$$

$$F = \frac{ma/2 + mg}{2}$$

$$\boxed{F - \frac{mg}{2} = \frac{ma/2}{2}}$$

In the same amount of time to get acc a= 10m/s2 for m you must pull twice as fast at the point of pull with a = 20m/s2 but effort put in is only 10 N. You are pulling twice the length of rope with less effort to get the same effect for m as the diagram on the left top

$a = 20\ m/s^2$

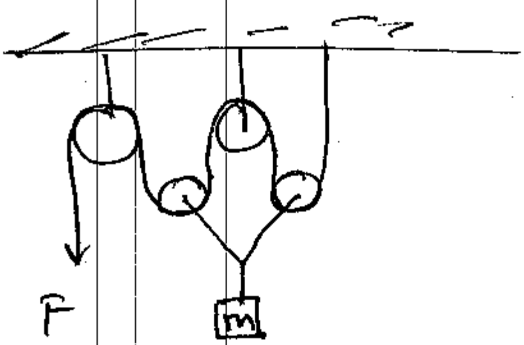
$m = 1\ kg$

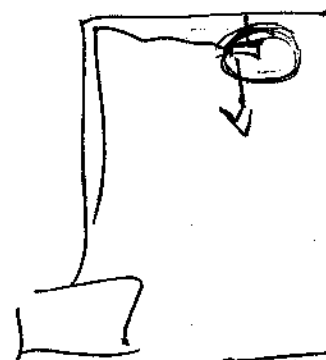
$g = 10\ m/s^2$

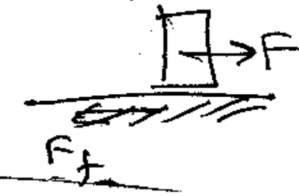
$F - \left(1 \times \frac{10}{2}\right) = \frac{1 \times 20/2}{2}$

$F = 5 + 5 = 10\ N$

To reduce to 1/4 F.

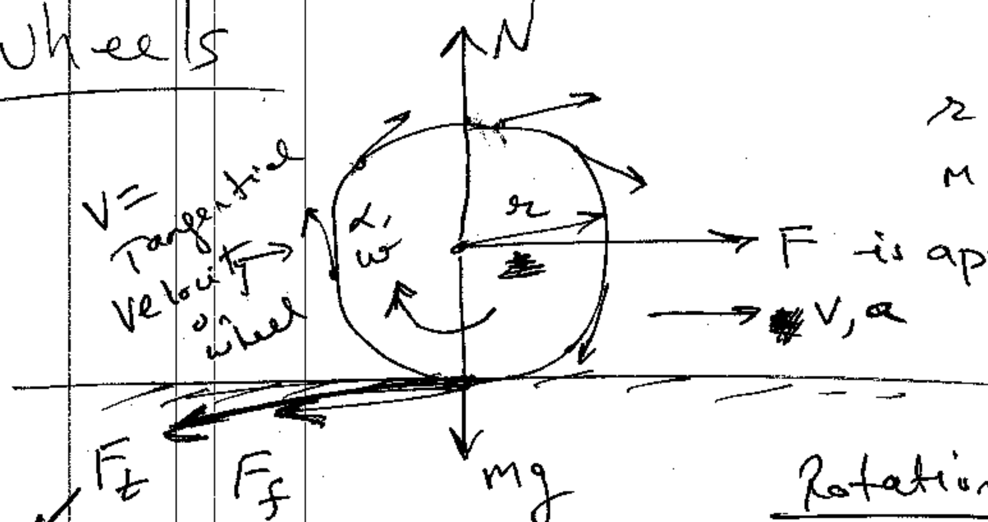






Wheels

r – radius
M – mass of wheel



Tangential Force (Exists only for wheels)

Frictional Force.

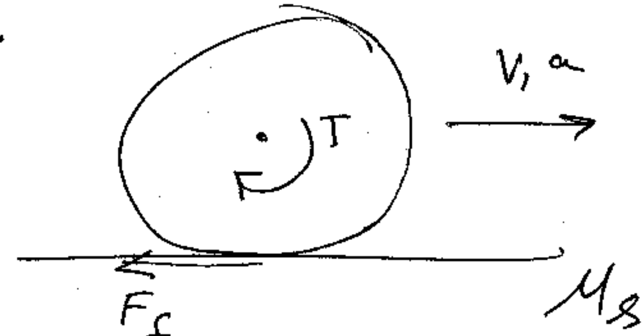
Only one of them will dominate

Rotation

F_f dominates over F_t

$V = r\omega$

Torque $T = F_f r$



μ_s

μ_d

$F_f = \mu_s N \rightarrow$ static

$F_f = \mu_d N \rightarrow$ dynamic

$\mu_d <<< \mu_s$

Rotation	Sliding	Slipping (Some rotation & sliding)
$v = r\omega$	$\omega = 0$	$\omega \neq 0$
$F_f > F_t$ ≠	$v \neq 0$	$v \neq 0$
dominates	(on ice) wheels are locked	but $v \neq r\omega$.
	$F_f = F_t$	$F_f < F_t$ dominates

$$F - F_f = ma$$

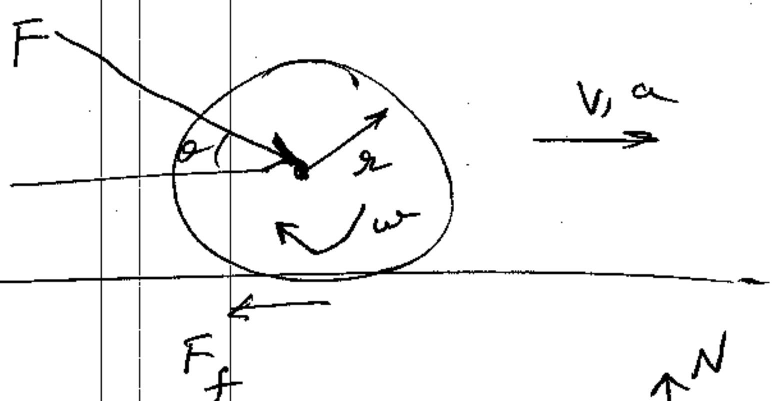
$$N = mg$$

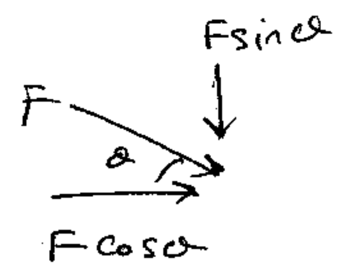
$$F_f = \mu_s N = \mu_s mg$$

coefficient of static Friction

$$T = \text{Torque} = I\alpha$$

$$T = F_f r$$





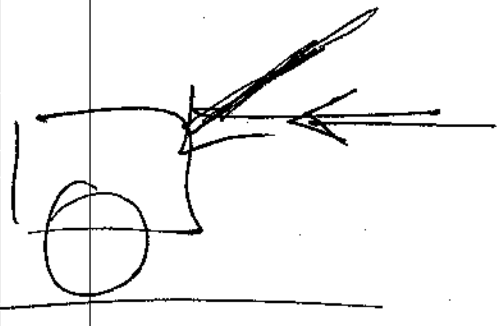
$$F\cos\theta > F_f$$

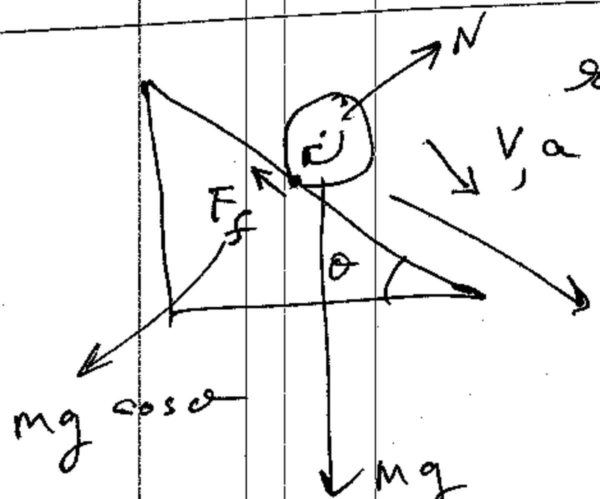
$$F\cos\theta - F_f = ma \qquad N = mg + F\sin\theta.$$

$$F\cos\theta - \mu_s N = ma \qquad F_f = \mu_s N$$

$$F\cos\theta - \mu_s(mg + F\sin\theta) = ma$$

$$\text{Torque} = I\alpha = F_f r$$





r = radius.
m = mass of wheel.

$I = \frac{1}{2} m(r_1^2 + r$

$mg\sin\theta - F_f = ma$

$N = mg\cos\theta$

$F_f = \mu_s N$

$T = I\alpha = F_f r$

$T = I\frac{a}{r} = F_f r$

$\frac{1}{2} m\cancel{r^2}\frac{a}{\cancel{r^2}} = F_f$

$F_f = \frac{1}{2} ma$

$mg\sin\theta - \frac{1}{2}ma = ma$

$mg\sin\theta = \frac{3}{2}ma$

$V = r\omega$

$a = r\alpha$

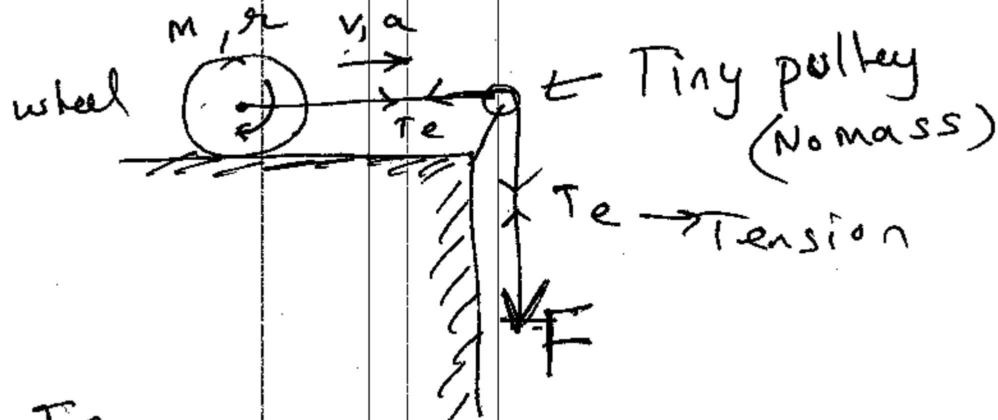
$\boxed{I = \frac{1}{2}mr^2}$

For a solid wheel

Fact

$g\sin\theta = \frac{3}{2}a$

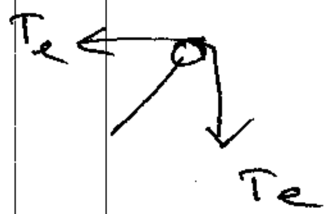
$\boxed{a = 2/_3\, g\sin\theta}$



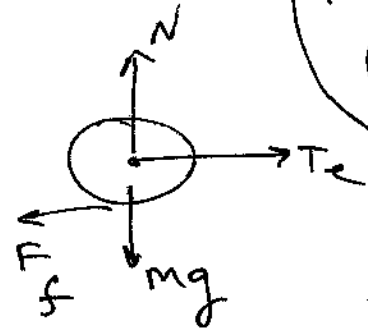
Te

$\updownarrow$ $F = T_e$

F



$T_e = T_e$



Rotation ($F_f > F_t$)

$$T_e - F_f = ma$$

$$N = mg$$

$$F_f = \mu_s N = \mu_s mg$$

$$F - F_f = ma$$

$$\text{Torque} = I\alpha$$

$$I\alpha = F_f r$$

$$F_f = \frac{I\alpha}{r}$$

$$\alpha = a/r$$

$$F_f = \frac{I a}{r^2}$$

$$F - \frac{I a}{r^2} = ma$$

$$F = \left(m + \frac{I}{r^2}\right) a$$

Energy method

$F = m_e a$ (In the vertical direction)

↑ mass equivalent.

$v = r\omega$, $\omega = \frac{v}{r}$

KE

$$\frac{1}{2} m_e v^2 = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v^2 + \frac{1}{2} \frac{I v^2}{r^2} = \frac{1}{2}\left[m + \frac{I}{r^2}\right] v^2$$

$$M_e = m + \frac{I}{r^2}$$

$$F = M_e a \qquad F = \left[m + \frac{I}{r^2}\right] a$$