# CS 682 - Homework 4

Mihail Sharov

## 1 Optical Flow (a)

Instead of $W(x)$, I will denote the neighborhood by $W(z)$ in order to avoid confusing it with the subscript $x$ in $I_x$ and $u_x$.

$$E(u) = \sum_{W(z)} (I_x u_x + I_y u_y + I_t)^2$$

The partial derivatives with respect to the two components of $u$ are set to 0:

$$\frac{\partial E}{\partial u_x} = 2 \sum_{W(z)} \left(I_x^2 u_x + I_x I_y u_y + I_x I_t\right) = 0$$
$$\frac{\partial E}{\partial u_y} = 2 \sum_{W(z)} \left(I_x I_y u_x + I_y^2 u_y + I_y I_t\right) = 0$$

The 2 can be ignored. The sums can be split up, with the constants $u_x$ and $u_y$ factored out:

$$u_x \sum_{W(z)} I_x^2 + u_y \sum_{W(z)} I_x I_y + \sum_{W(z)} I_x I_t = 0$$
$$u_x \sum_{W(z)} I_x I_y + u_y \sum_{W(z)} I_y^2 + \sum_{W(z)} I_y I_t = 0$$

Combining into a single matrix equation:

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} + \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} = 0$$

This can be expressed as

$$G\mathbf{u} + \mathbf{b} = 0$$
$$G\mathbf{u} = -\mathbf{b}$$

## 2 Optical Flow (b)

The solution to the above linear system may be undetermined. Since $G$ is a $2 \times 2$ matrix, the only possible scenarios in which $G$ is singular are:

$Rank(G) = 1:$ The neighborhood contains only an edge (1D shape) and therefore a single point from it can be matched with any other point on the edge.

$Rank(G) = 0:$ The neighborhood contains no texture and therefore a single point can be matched with any other point.