FINAL REPORT
2008-2009 Improving Teacher Quality State Grants
Title II, Part A, No Child Left Behind (NCLB) Act

A.C.T. NOW: ALGEBRAIC CONNECTIONS AND TECHNOLOGY IN MIDDLE GRADES MATHEMATICS

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FINAL PROGRESS REPORT GUIDELINES

2008-2009 Improving Teacher Quality State Grants
Title II, Part A, No Child Left Behind (NCLB) Act

Part A. **Project and Participant Information**

- **Name of project:**

A.C.T. NOW: ALGEBRAIC CONNECTIONS AND TECHNOLOGY IN MIDDLE GRADES

MATHEMATICS

- **Name of Institute**:

George Mason University
Academic department College of Education and Human Development
4400 University Drive, MS 4C2
Fairfax VA 22030

- **Names of individuals responsible for implementing the project.**

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Names of collaborating partners funded to implement the project. Describe support provided.
Fairfax County Public School (FCPS)

A report of the number of people participated in the project's activities.

39 teachers from grades 3-8

Contents

ATTACHED: A report of the number of people who participated in the project's activities (attach a copy of your attendance sheets, SCHEV's participant responses and project director summary of participant's forms).

Part B. Goals and Evaluation

B. 1: Restatement of the local needs addressed (from the proposal) to enhance content knowledge and improve pedagogical skills of teachers and/or leadership of principals and administrators. Provide summary of strategies and needs assessment instruments used.

B.2: List specific goals outlined in the proposal and provide details of how the project met the criteria and purposes. Provide a description of the activities, events, and/or programs that were implemented to address the problem or concern identified. Were the goals achievable as stated in the proposal? If not, explain why goals were not achieved.

B. 3: Provide a description of follow-up activities and dissemination of information including copies of flyers, posters, announcements, programs, news articles, evaluation or feedback forms, photographs, etc. that were developed or produced as part of the project.

Part C. Assessment

Assessment

An explanation of outcome(s) and assessment/evaluation methodologies. Describe and provide known scientific evidence that your project fostered high quality professional development and increased student achievement in the core academic areas. What strategies were used to provide greater access to diverse populations and lasting effects on classroom instruction so that all students could achieve the state's content and student performance standards? External evaluator's report should be included.

Research Report 1: Impact on Instructional Practices

Transforming Teachers' beliefs and practice by Developing Algebraic Connections Through Problem solving

Research Report 2: Impact on Student Learning

Building rules to represent linear functions through problem solving and technology

Part D. Cost effectiveness and adequacy of resources

Provide detailed budget narrative explaining the expenditure of funds to program objectives. (Total expenditures should be reported on the Final Budget Summary provided.)

Appendices

Table 1. Needs assessment based on SOL report card
Table 2. Survey to Pre & Post-assessment Preparedness for Math Instructional Practices
Table 3. Evidence that project fostered high quality professional development
Table 4. Impact on Teachers' Instructional Practices and Students Learning

Part B. **Goals and Evaluation**

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B.1-RESTATEMENT OF LOCAL NEEDS

School Division Demographics and Need

All students have one need in common, and that is the need for high quality mathematics instruction. In 2007, Fairfax County Public Schools failed to meet adequate standards, Adequate Yearly Progress (AYP), under the federal government's No Child Left Behind law the first time since the federal law passed in 2001. Sixty-eight of the 191 Fairfax County schools that were required to administer the tests failed to make AYP in 2006-2007, which was a 36 percent failure rate. Fairfax County Public Schools had 28 middle and elementary schools that failed AYP for the last two years or more. Schools receiving federal funds for low-income students under Title I of the federal Elementary and Secondary Education Act faced sanctions and had to take corrective actions when they failed to meet AYP for consecutive years. The federal law required that specific percentages of students in groups defined as ethnic minorities and students with disabilities, limited English skills and economic disadvantages earn passing scores. Fairfax County Public School (FCPS) has a total education enrollment of approximately 164,843 students for whom reaching full academic potential in mathematics is a high priority. The student population of FCPS is ethnically diverse, approximately 50% of the population is minorities. The demographics for the district includes: 10.8% African American, 0.3% American Indian, 17.4% Asian American, 16.0% Hispanic, 4.9% Multiracial, and 50.2% White. For the 2007-2008 academic year, there are 21,771 English Speakers of Other Languages (ESOL) , 24,000 students receiving special education services and about 33,000 (20%) students receiving free and reduced meals.

B.2 List specific goals outlined in the proposal and provide details of how the project is meeting the criteria and purposes.

The goal of A.C.T.: ALGEBRAIC CONNECTIONS AND TECHNOLOGY IN MIDDLE GRADES MATHEMATICS was to support teachers in Fairfax County Public Schools (FCPS) to address two strategic planning goals in the district: increasing algebra enrollment in the eighth grade and supporting teachers' integration of technology in instruction. In order to prepare students for success in the 21st century, Fairfax County Public Schools' strategic plan in mathematics was focused on increasing algebra enrollment in Grade 8, implementation of common problem solving strategies and encouraging students to pursue mathematics beyond Algebra II. Fairfax County Public Schools also had identified professional development in improving mathematics instruction for underperforming students as a key focus in the system's long range goals. To meet these goals, teachers needed to be better trained to develop algebraic thinking beginning in elementary grades and throughout middle school. This strategic goal aligned with the national initiative focused on the need for developing algebraic thinking in the elementary and middle grades.

The **A.C.T. Now: Algebraic Connections and Technology in Middle Grades Mathematics** addressed these local needs:

Teacher Knowledge and Practice	
• Improve teachers' algebra content-specific knowledge in mathematics.	
• Develop teachers' skills in the use of technology tools for mathematics.	
• Improve teachers' knowledge of state and national standards in mathematics.	
District Support and Development	
• Target participants in high-need schools as well as teachers of high-need populations (e.g., ESOL, special education)	
• Enhance the resources available to teachers for using mathematics tools, computers, and other technology	
• Disseminate strategies that develop mathematics content and technology-based instructional practices beyond teachers in the project	
• Promote connections and cooperation among grade levels 3-8.	
• Collaborate on an effort towards FCPS Strategic Goal focused on increasing algebra enrollment by Grade 8	

DESCRIPTION OF THE PROJECT**. A.C.T. Now: Algebraic Connections and Technology in Middle Grades Mathematics** targeted teachers in Fairfax County Public Schools including 40 teachers drawn from two grade-level bands (3-5th and 6th-8th). The project theme was “Algebraic Connections and Technology” – teaching rigorous mathematics content and developing connections among grade levels. The project addressed the need to develop teachers’ content knowledge, skills in the use of algebra tools and computer technology to plan and implement standards-based mathematics lessons. The professional development included two parts. Part I was a summer institute held in early August 2008. Part II was follow-up sessions throughout the school year. In Part I, teachers participated in interactive experiences with mathematics specialists and mathematicians to design lessons that support algebra outcomes at their grade level with an emphasis on technology use. Part II used a successful teacher-led professional development model called Lesson Study to specifically target teachers’ needs in the classroom as they implement materials and strategies during the school year. Under the direction of the FCPS Mathematics Supervisor, teachers designed dissemination plans that support teachers and schools in the system with the greatest need. By improving teachers’ content-specific knowledge in mathematics and their ability to use technology, the project increased the potential for improving the mathematics achievement of all students in Fairfax County Public Schools. The project used research-based instructional practices and mathematics technology tools to promote high quality, standards-based mathematics instruction. **A.C.T. Now: Algebraic Connections and Technology in Middle Grades Mathematics** offered a 40 hour summer institute and 15+ contact hours (per teacher) during the academic year, through follow-up lesson study meetings, online collaboration and communication. Teachers received a resource kit including course texts, resource notebook, an algebra manipulative kit, a CD of teacher-developed lesson plans, 3 university credits, and $125 in support to design personal dissemination plans (+ 5 hours) and attended a conference and presented to colleagues at their school sites.

B.3- DESCRIPTION OF PROJECT ACTIVITIES AND STRATEGIES

Outcome objectives –

During the project, teachers participated in many research based & teacher practice-based activities.

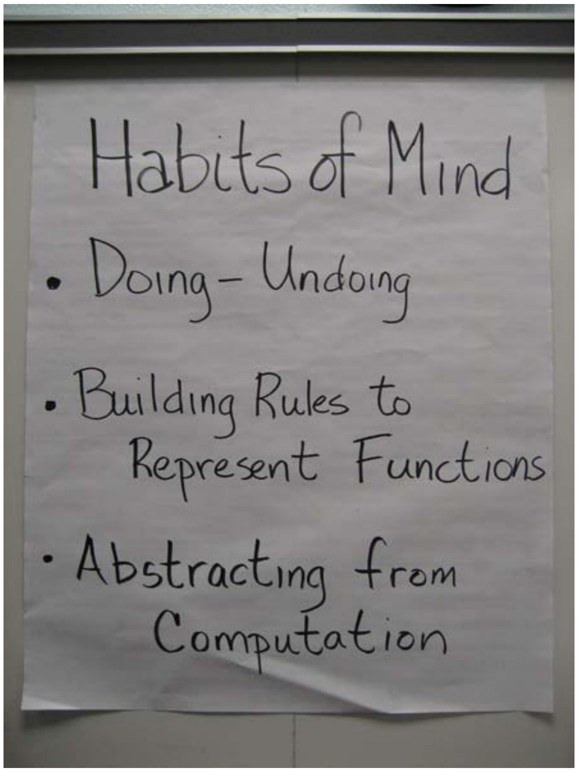
- Design grade-level specific algebra lessons with rigorous content and connections that bridge the gap between students' current, previous, and future mathematics benchmarks.
- Design lessons targeting populations who have demonstrated low-performance levels on past SOL testing by focusing on common error patterns.
- Use computers, virtual and physical manipulatives, calculators and other algebra tools to implement interactive mathematics experiences that support student learning.
- Teach mathematics to diverse learners by understanding strategies that reach females, ethnic minorities, individuals with disabilities, LEP individuals, and the economically disadvantaged. Design mathematics lessons aligned with the Virginia Standards of Learning and NCTM's Principles and Standards for School Mathematics (2000).
- Design and implement personal dissemination plans for sharing instructional strategies with others.

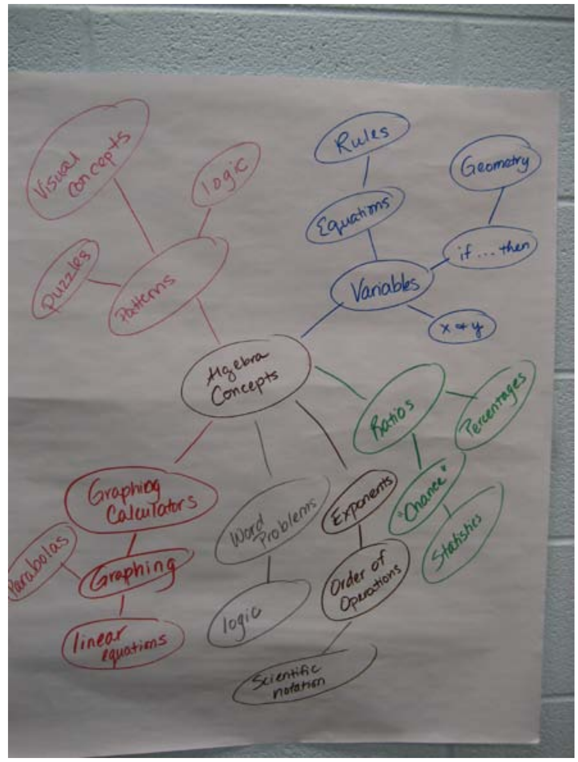
Key Project Activities

PART I: Summer Institute.

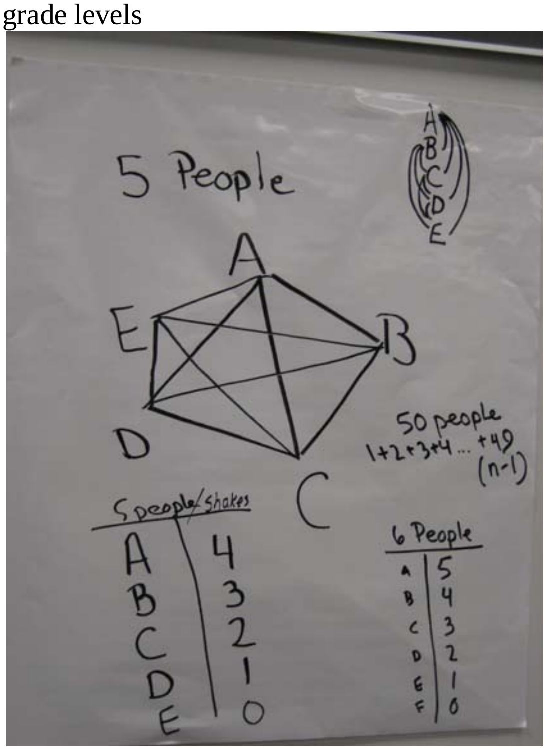
Forty elementary and middle grades teachers (from grades 3^{rd}- 5^{th} and 6^{th} -8^{th}) met in during a 2-week summer institutes, first two weeks of August, Monday through Thursday, from 9:00AM to 3:00 PM (40 hours). The separate sessions ran concurrently for the two grade level groups of 25 teachers with one Lead Instructor, one University Mathematician, and one Instructor's Assistant per group. Daily activities included research-based practices and model lessons using a variety of mathematics tools and technology. Participants engaged in mathematically rich activity that connects algebraic content with pedagogical strategies. They developed lessons and assessments based on grade level expectations and state and national standards.

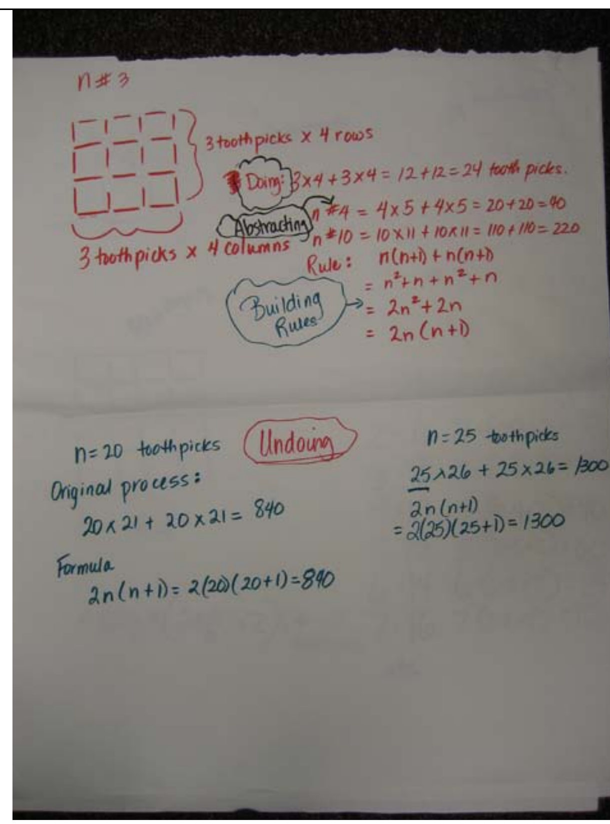
Developing Conceptual Understanding- Teachers worked on developing the conceptual underpinnings of algebraic understanding by unpacking the concept and discussing the importance of algebraic thinking through problem solving in earlier grades

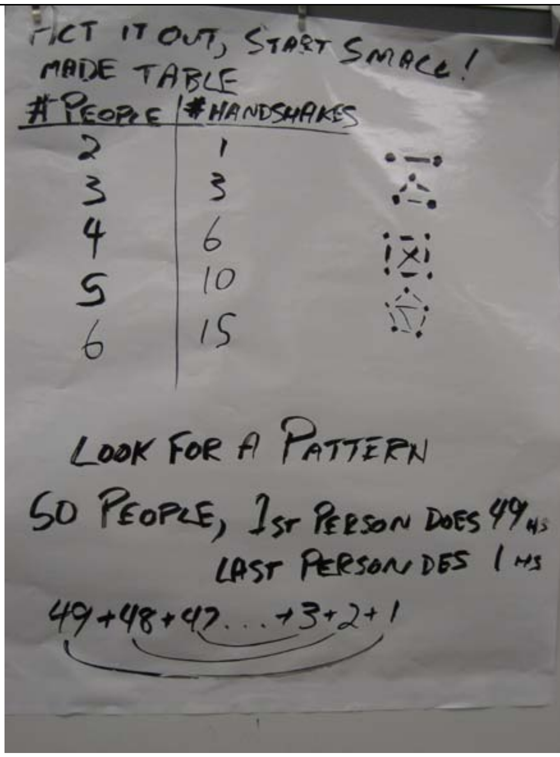




Teachers grappled with many types of algebra problems and discussed how to develop algebraic connections at their grade levels







Working with a Mathematician for George Mason University to gain deep integrated algebra content

LESSON STUDY COLLABORATION

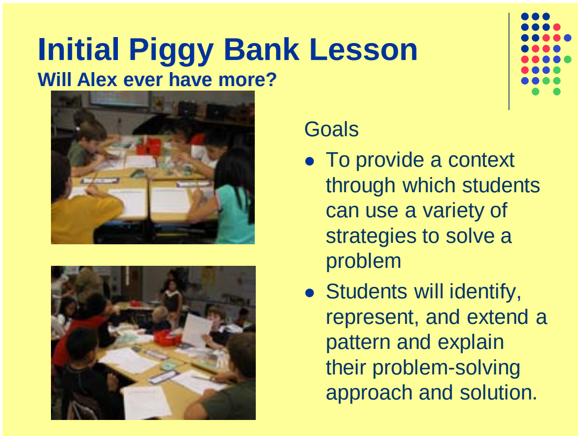
Teachers began the Lesson Study process in small groups that would take place during the academic year. The team taught mathematics to diverse learners by understanding strategies that reach females, ethnic minorities, individuals with disabilities, LEP individuals, and the economically disadvantaged. They designed mathematics lessons aligned with the Virginia Standards of Learning and NCTM's Principles and Standards for School Mathematics (2000) targeting populations who have demonstrated low-performance levels on past SOL testing by focusing on common error patterns.

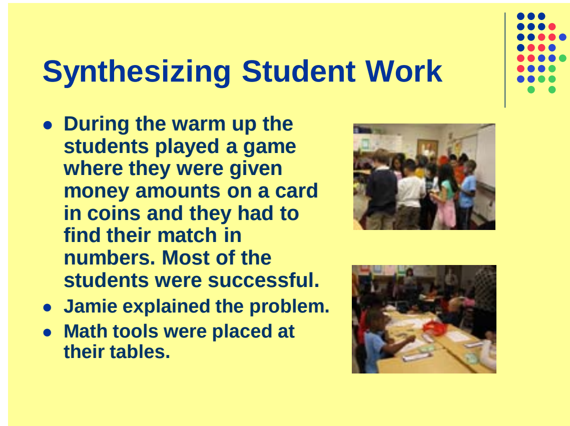
We used computers, virtual and physical manipulatives, calculators and other algebra tools to implement interactive mathematics experiences that support student learning.

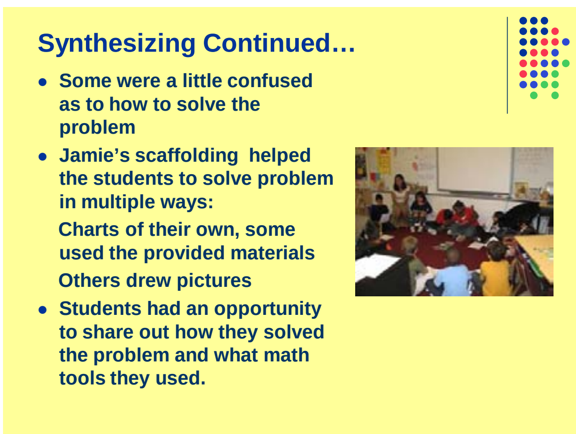
We designed grade-level specific algebra lessons with rigorous content and connections that bridge the gap between students' current, previous, and future mathematics benchmarks.

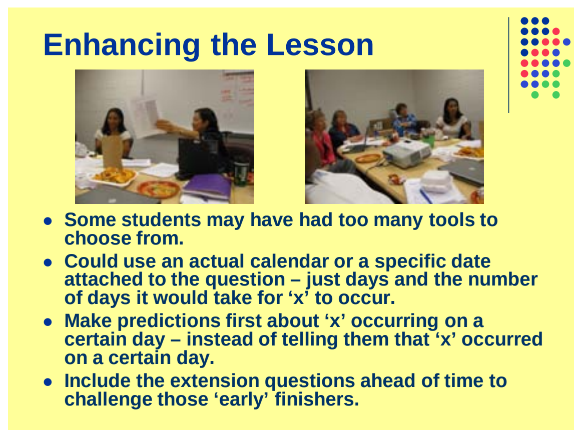
PART II: Follow-up Sessions. Teachers met two times during the academic year with the course instructors to continue their professional learning through a teacher-led professional development model called Lesson Study (10 hours/each group). The goal of these follow-up sessions was to provide teachers with continuing support in implementing content, materials, strategies, opportunities to share ideas across grade levels and analyze student learning. During these meetings, teachers participated in collaborative planning of lessons focused on algebraic connections and use of technology in groups using the Lesson Study model. Teachers observed one of the teachers in their group teach the lesson on which they collaborated and debriefed after the lesson by reflecting on the instructional strategies and students' learning and revised their lessons based on the outcomes. Communications among teachers and project staff was facilitated through the planning meeting, the debriefing and electronic discussions that addressed participants' challenges as they implement new models of instruction. During follow-up activities, the project's goal was to make connections between the strategies learned during the summer institute and classroom practice. These developed and implemented lesson plans was compiled on a CD by the project team for all participants and distributed at a spring follow-up session.

Example of one of the Lesson Study









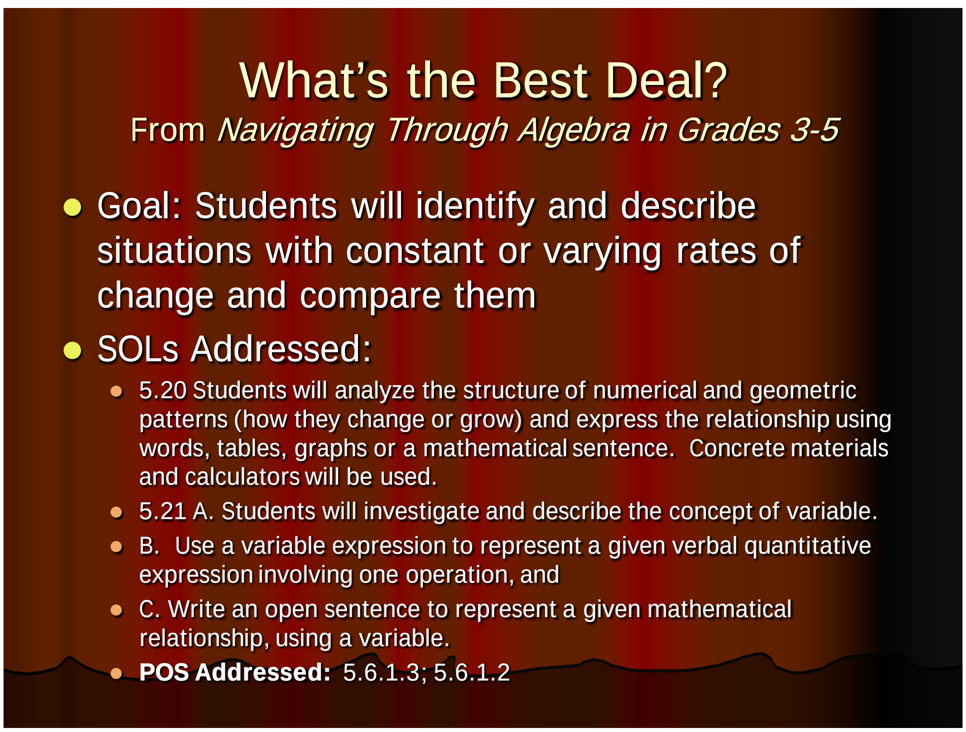
Key Elements in the Design of Project Activities

Four key elements of the design of the summer and follow-up experiences included (1) algebra content standards, (2) mathematics tools and technology, (3) research-based instructional strategies, and (4) targeting low-performing student populations.

<u>Mathematics Content Standards</u>.

Grades 3 through 5 placed emphasis on the algebraic connections to arithmetic focused on addition, subtraction, multiplication, division with whole numbers and rational numbers and solving problems involving fractions and decimals. Students also refined their estimation skills for computation and measurement and investigate geometric relationships involving area and perimeter, classifying triangles, plotting points in the coordinate plane, and variables. Students used ratios to compare data sets, make conversions within a given measurement system, make geometric constructions, classify threedimensional figures, and solve linear equations in one variable. In addition, five NCTM's mathematical processes such as problem solving, connections, communication, representations and reasoning and proof were interwoven into the activities.

Grades 6 through 8 placed emphasis on solving problems solving linear equations and inequalities, and use data analysis techniques to make inferences and predictions. One of the instructional module involved *a road trip application by car* and our objective was to introduce the teachers to a variety of algebraic concepts (involving slope, distance formula, Pythagoras theorem, linear equations, modeling variation, ratio and proportion, mean values, modeling rate of change, distance-time-speed relationship, quadratic equations, factoring polynomials, rectilinear motion, absolute value, functions, domain and range) through this single application in this module. We considered other similar modules with other real-world consumer applications as well that will illustrate various algebraic connections. Also, we used technology to model the kinds of hands-on explorations teachers would use during classroom instruction.



<u>Mathematics Tools and Technology</u>. Participants in this projec engaged in hands-on interactive sessions using a variety of mathematics tools and technology for instruction. Teachers learned grade-level appropriate uses for a variety of technology such as graphing calculator in conjunction with the Calculator-Based Ranger (CBR), Geometer's Sketchpad, databases, spreadsheets and virtual manipulatives. Various mathematics tools were used to model the kinds of hands-on explorations teachers would use during classroom instruction. In addition, the Project Director from the National Council of Teachers of Mathematics (NCTM) Illumination website, Patrick Vennebush, presented virtual manipulatives and investigative lessons related to algebra standards on one day of the summer institute.

Technology Guru Guest Speaker-Patrick Vennebush, (photo)
Project director of NCTM's Illuminations came to our class to present all the wealth of technology resources available for teachers to promote algebraic connections and technology.

Research-Based Instructional Strategies. Project staff used a variety of instructional strategies for teaching and learning mathematics. Participating teachers engaged in cooperative learning, integration, scaffolding, examples of metacognition, and modeling with a focus on teaching mathematics for conceptual understanding. Teachers participated in hands-on practice sessions using manipulatives and technology in cooperative groups. Effective teaching and learning principles allowed opportunities for reflection on best practice instructional strategies, alignment with mathematics Standards, alternative assessment techniques, and setting high expectations for all learners. By integrating state (Standards of Learning, Commonwealth of Virginia, 1995) and national standards (Principles and Standards for School Mathematics, NCTM 2000) participants had the opportunity to see what the Standards meant and what they looked like in practice.

Dissemination Plans

Teachers met with the members of the project team to design personal dissemination plans for sharing project content and pedagogical strategies with other professionals. These dissemination plans supported the school system's goals and teachers submitted these plans as proposals for presentations at their district's mathematics conferences. Because some teachers were reluctant to participate in mathematics conferences, project staff assisted teachers in preparing proposals for presentation at these meetings (i.e., VCTM state conference, NCTM conference). Project staff supported teachers in designing and providing staff development aligned with the goals of the school system and the mathematics supervisor. The dissemination plan support and conference fee from the grant helped teacher participants disseminate their professional learning to other FCPS and Virginia teachers throughout the system and encourage them to become active in mathematics education issues.

Part C. **Assessment**

C.1- Outcomes(s) and assessment/evaluation methodologies up to now.
C.2- Evidence that your project fostered high quality professional development
C.3- Evidence of increased student achievement in the core academic areas.
C.4-What strategies are being used to provide greater access to diverse populations and lasting effects on classroom instruction so that all students could achieve the state's content and student performance standards?

C.1- Outcomes(s) and assessment/evaluation methodologies

Evaluating project for high quality professional development

To address questions such as: how sustainable the design is after the PD institute, systemic variables were collected: the degree to which it is sustained throughout the year, spread of use to other teachers and students. We used surveys and structured interviews with teachers. We also developed a questionnaire that addressed the advantages and difficulties teachers encountered in adopting this innovation in their classroom.

Assessing Student Learning

Success and failure of an innovation cannot be simply be evaluated in terms of how much students learn on some criterion measure. Different kinds of evaluation are necessary for addressing questions such as: How sustainable the design is after the PD institute, how much the design emphasizes reasoning as opposed to rote learning, how the design affects the attitude of students and teachers. TO evaluate different variables, it was necessary to use a variety of evaluation techniques, and systematic scoring of observations of the classrooms. Both qualitative and quantitative evaluations are essential parts of this design-research method. For this project, we measured

1. Climate variables, such as a) the degree of engagement; b) the degree of cooperation; c) the degree of risk taking.

To evaluate the climate variables, we employed observational techniques, through field notes while intervention in practice and video recordings of the intervention and scored those records. The video was scored systematically by two raters with respect to the three dimensions using a five point scale for each 5 minutes interval in the lesson. Raters were trained using benchmark lessons for which scores have been calibrated.

2. Learning variables, such as a) content knowledge, b) skills, c) learning strategies and metacognitive strategies and d) dispositions. To measure the content knowledge, skills,

strategies and dispositions, we collected student work with short answers and explanations form problems, oral interviews.

GOALS based on LOCAL NEEDS

Teacher Knowledge and Practice	
• Improve teachers' algebra content-specific knowledge in mathematics.	met
• Develop teachers' skills in the use of technology tools for mathematics.	met
• Improve teachers' knowledge of state and national standards in mathematics.	met
District Support and Development	
• Target participants in high-need schools as well as teachers of high-need populations (e.g., ESOL, special education)	met
• Enhance the resources available to teachers for using mathematics tools, computers, and other technology	met
• Disseminate strategies that develop mathematics content and technology-based instructional practices beyond teachers in the project	met
• Promote connections and cooperation among grade levels 3-8.	met
• Collaborate on an effort towards FCPS Strategic Goal focused on increasing algebra enrollment by Grade 8 • Build Capacity and infrastructure for Professional development	On-going effort:

ALGEBRAIC CONNECTIONS AND TECHNOLOGY IN THE MIDDLE GRADES

Quantitative Outcome Goals Table

Quantitative Outcome Goals	**Benchmark**			**Project**
	Summer 2008	Fall 2008	Spring 2009	Total
Teachers recruited to participate in mathematics professional development	40 teachers recruited	39 teachers continue participation	39 teachers continue participation	39 teachers complete the course
Teachers complete required number of mathematics professional development hours	40 hours (Summer Institute)	15 hours (2 days-Lesson Study Meeting day)	5 hours dissemination project	(60 hours)
Number of elementary students directly affected by ALGEBRAIC CONNECTIONS participants (assuming every teacher has 1 class of 20 students/year)		400	-	400
Number of middle school students directly affected by ALGEBRAIC CONNECTIONS participants (assuming every teacher has 4 classes/25students each)		2000	-	2000
Number of hours ALGEBRAIC CONNECTIONS participants spend in dissemination of professional development to other teachers (at least 5 teachers)			On going	Anticipating 150 teachers
Number of ALGEBRAIC CONNECTIONS participants attending national conferences	-	-	Teachers 30	Teachers 30
Number of lessons prepared by ALGEBRAIC CONNECTIONS participants and shared on CD (lessons/ teachers)	40 technology evaluation and lesson ideas	40 individual Lessons and teaching strategies	6 Group lesson study units	86 Individual and Group lesson study units

C.3- Evidence of increased student achievement in the core academic areas.

Based on many of the surveys (see appendix XX) and reflective comments collected from teachers, some recurrent themes emerged about student learning in their classroom as a result of this project.

1) Thinking like a Mathematicians

The Problem Solving approach encouraged students to try multiple approaches, abandon ineffective strategies, use multiple representations for communicating math, and compare and contrast strategies for efficiency. Students became pattern builders and pattern seekers and started making generalization and abstracting from computations. They started to think and talk like mathematicians as they built conceptual understanding and used precise mathematical vocabulary.

2) Learning socio-mathematical norm practices

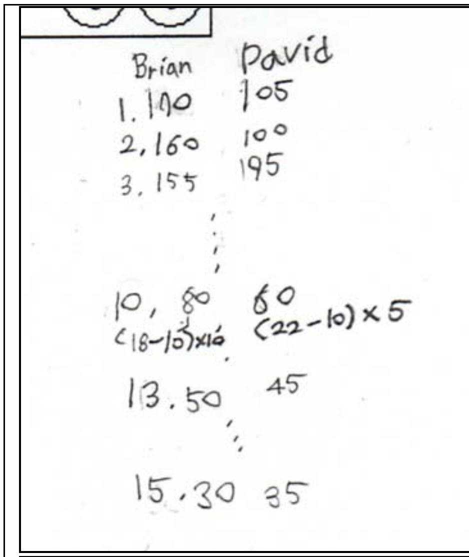
Through the problem based mathematics which emphasized communication and collaboration, students began to learn socio-mathematical norm practices that were new to them. They were taught how to justify and prove their thinking through mathematical discourse, how to argue about a mathematical problem. They improved gradually in their ability to describe the problem solving process, how to justify, share solutions, and actively listen to each other. In this way, they learned how different individual math thinking came together to build collective knowledge.

3) Productive disposition:

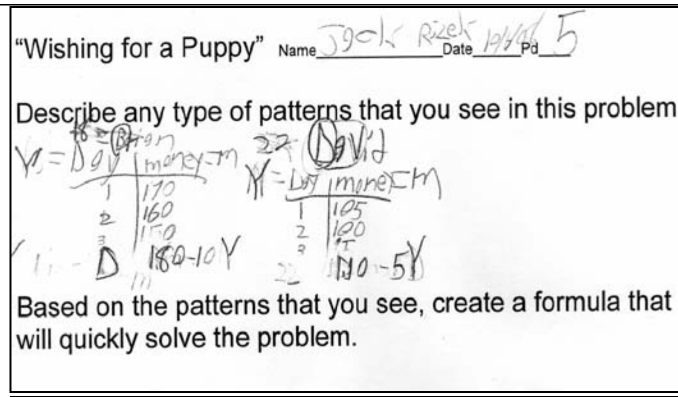
Students demonstrated engagement in algebraic problems and were willing to take risks and not afraid to attempt challenging problems. They demonstrated perserverence and determination as they became independent thinkers. They also began to accept grappling with a problem as a norm.

EXCEPT FROM TEACHERS OBSERVATION NOTES:
Evidence showing how the lesson contributed to student learning:

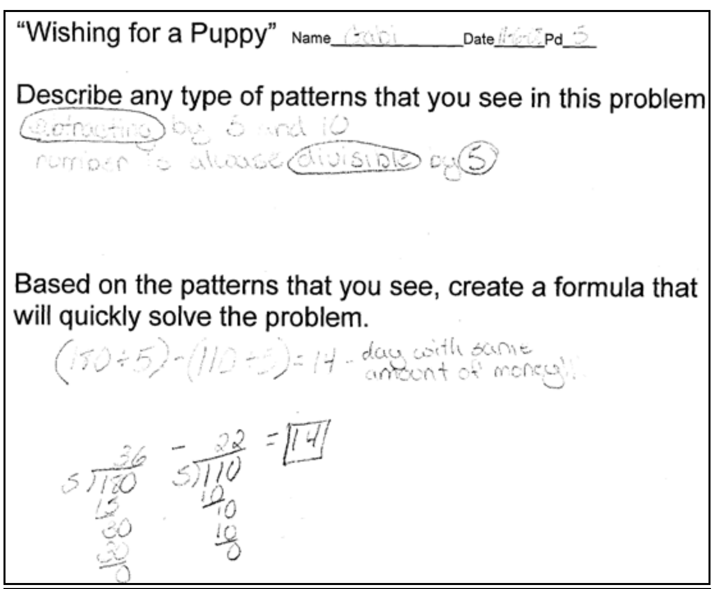
> **Doing and undoing-**One student's symbolic explanation led me to think that he was focusing in on the end result, when one brother's value was greater than the others. He knew that there were some repeated operations that were needed (rules-functions), but I also have a feeling he was finding how many days (some guess/check) had passed to get to that point.
>
> **Building Rules to Represent Functions-**Many students put data into a table, and began to show what each person's left-over value was on day 1, 2, etc. I was happy to see a student use the "…" to show a continuation of the pattern, and then get to the "important" days.
>
> **Abstracting from Computation-**One student focused on the difference between the two brothers' left-over values. She knew that the day that their difference was zero would be a day to focus in on.



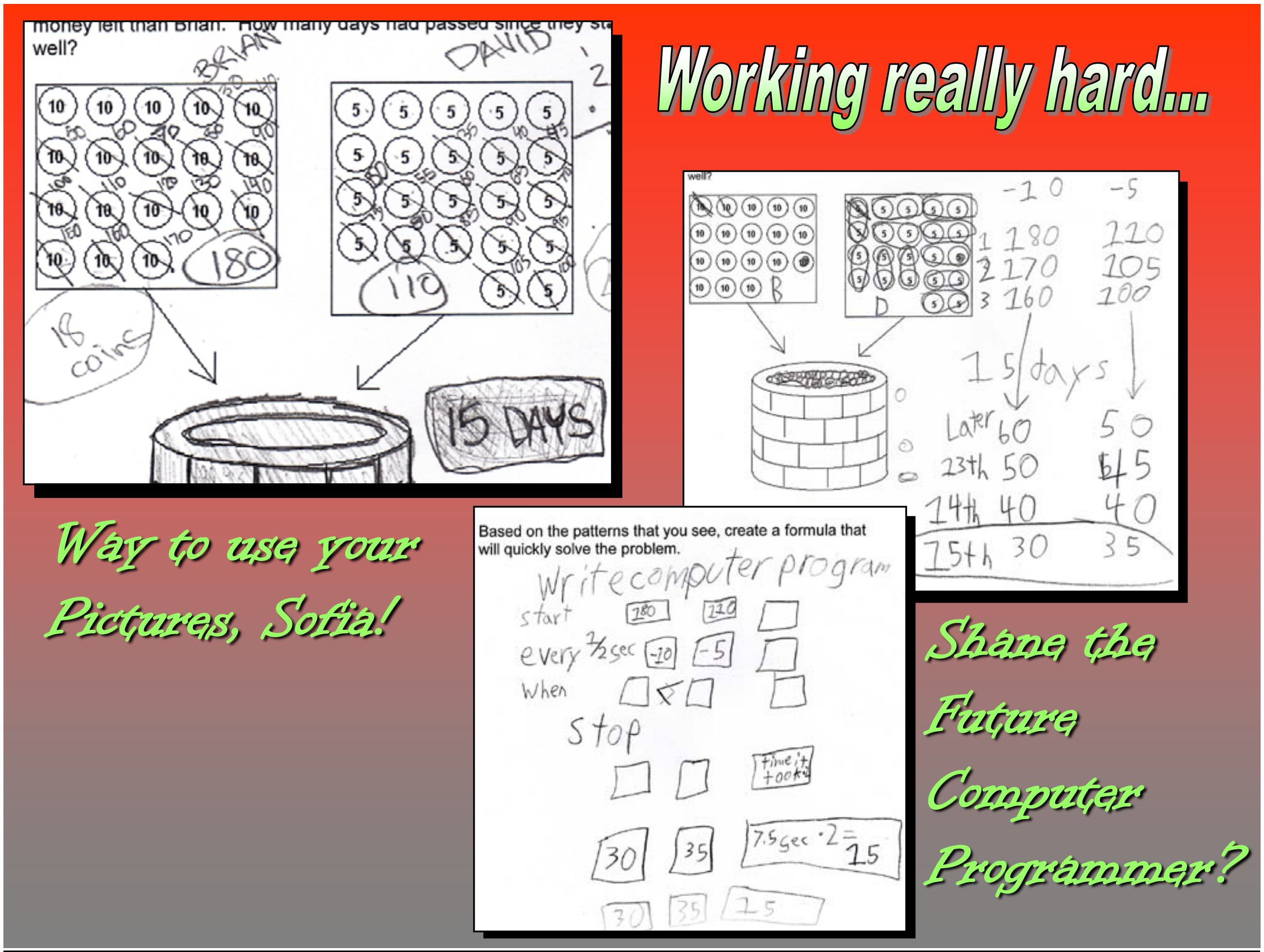
Kun Ho's formulas:
(18 – 10) x 10
(22 - 10) x 5



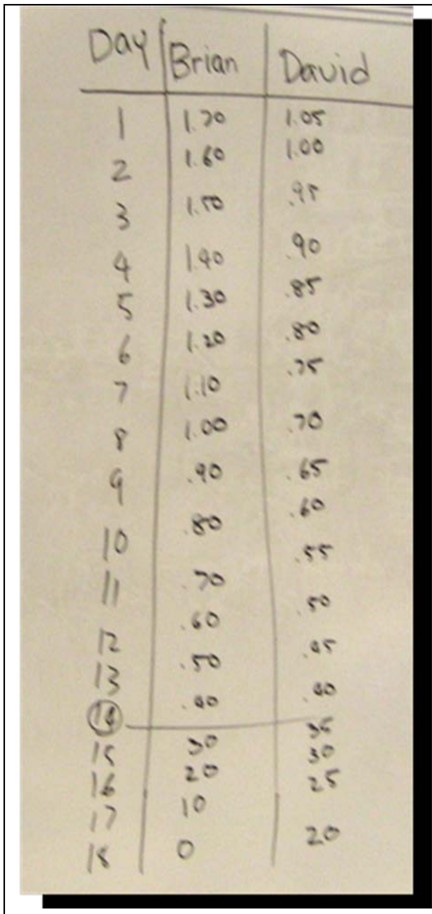
Jack's formulas: B = 1.80 – 10y
D = 1.10 – 5y



Gabi's formula: (180/5) – (110/5) = 14
The day they had the same amount of money.



Working really hard...
Way to use your Pictures, Sofia!
Based on the patterns that you see, create a formula that will quickly solve the problem.
Write computer program
15 DAYS
Shane the Future Computer Programmer?



Day	Brian	David
1	1.70	1.05
2	1.60	1.00
3	1.50	.95
4	1.40	.90
5	1.30	.85
6	1.20	.80
7	1.10	.75
8	1.00	.70
9	.90	.65
10	.80	.60
11	.70	.55
12	.60	.50
13	.50	.45
14	.40	.40
15	30	35
16	20	30
17	10	25
18	0	20

Analysis of Student Work:

Jasmine's Work:
Jasmine noticed a pattern between the values of the coins. She saw that two nickels equaled a dime, so she multiplied 7 (She determined the 7 by subtracting the amount David had from the total Brian had which left 7 dimes for Brian) by two and found that on the 14th day they had the same amount of money. She then knew that the final answer would be that David would have more money than Brian on the 15th day. This was a fantastic representation of algebraic thinking on her part by using patterns in the form of multiples of 5 and 10 to help solve the problem. Her work showed how she was able to go from generalizing her ideas to a formal representation of her discoveries.

Tiara's Work:
Tiara used both her visual representation and a list to determine the day that David had more money than Brian. She began by subtracting 10 cents every day from Brian's money and 5 cents from David's. From this she was able to determine that they had the same amount of money on the 14th day. She then checked her work by pairing up 2 nickels for every dime and found that Brian had 7 dimes left over. Since there were two people she multiplied the 7 dimes left by 2 to find again that they had the same amount of money left on the 14th day. I then encouraged her to try and draw a graphical representation of her work to see if she was correct. She found that the point of intersection of the two functions was the 14th day and therefore her final answer to the problem was that David had more money on the 15th day. Again, Tiara like Jasmine was able to generalize her thought patterns and then represent it in a formal way through graphs and tables.

Daniel's Work:
Daniel approached the problem by using a double sided table which showed the decrease in money for both of the brothers. He kept subtracting until he found that David had more money and then counted how many days it had been. Since he also finished quickly, I encouraged him to try and solve the problem in a different way to verify his work. I provided other materials that I let the students know about before they began, such as colored squares and graphing paper, to help them in their discovery. Daniel chose to try a graphical representation as Tiara did. Both these students were in different classes from each other, so it was really nice to see how they came up with the same type of graph. He, too, found the point of intersection was the day at which they had the same amount of money and that the 15th day David had more. Daniel was able formally show his algebraic thinking through both his table and his graph, however had some difficulty verbalizing his findings. But overall, he shows that he is developing skills to think in abstract ways.

C.4-What strategies are being used to provide greater access to diverse populations and lasting effects on classroom instruction so that all students could achieve the state's content and student performance standards?

Targeting Low-Performing Student Populations

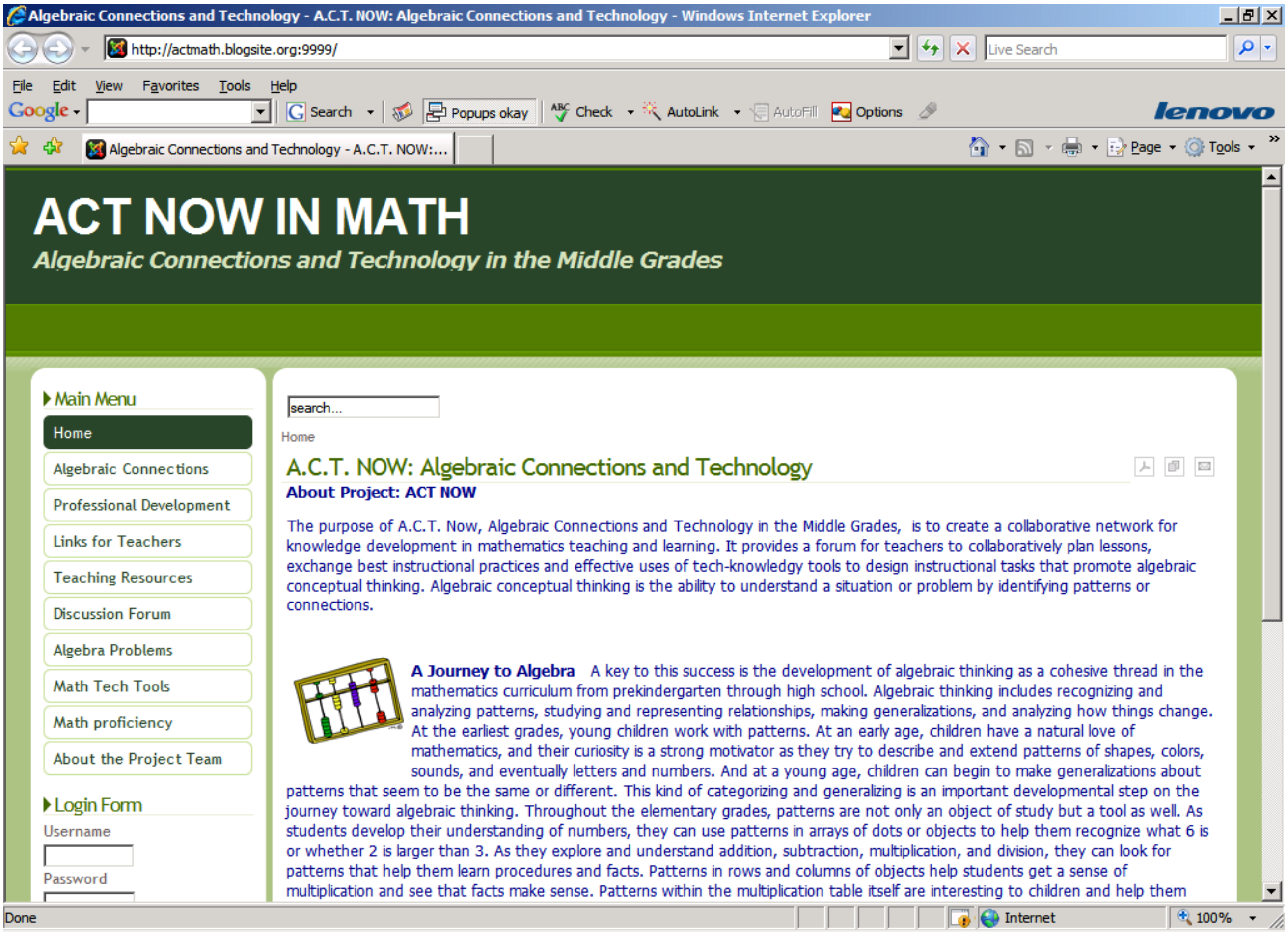
Focusing on the underserved population such as the at-risks and Title One schools addressed the need to increase access to rigorous mathematics and opportunity to develop conceptual understanding, procedural fluency and problem solving skills particularly among populations that have traditionally been underserved. Participants developed an understanding of effective instructional strategies for students who have difficulty learning mathematics, and for those with different learning styles such as females, ethnic minorities, individuals with disabilities, Limited English Proficient (LEP) individuals, and the economically disadvantaged. Instructors modeled the use of a variety of instructional strategies for diverse learners including the use of representations, connecting prior knowledge, explicit teacher modeling, scaffolding instruction, use of authentic contexts, teaching metacognitive strategies, providing structured language experiences, cooperative learning, and peer tutoring. These approaches emphasized a philosophy of success for *all* learners.

Development of Teachers and Special educators and English Language learners resource teachers

The project recruited 40 teachers from FCPS with a specific focus on recruiting new teachers and teachers in the at-risk and high-need target cluster schools (Title One Schools). In addition, recruitment efforts were focused on special education teachers as well as teachers of Limited English Proficient (LEP) students. 8th. By recruiting teachers at multiple levels, the project promoted dialogue and brought about continuity within the mathematics curriculum across grade levels.

Ensuring sustainability of the professional learning through Lesson Study and Technology Resources

The Lesson Study component was a critical piece to this project in ensuring sustainability of the professional learning. By exposing teachers and math leaders in the teacher-led professional development and the project created an infrastructure for continued development. In addition, the project team created a website as a resource for these teachers and their schools. This website has many important resources like links for teaching resources, more algebraic problems, a discussion forum, and links to video-based instructional resources on algebra. In addition, teachers have the lessons created by participants on a CD and on this website to share with their colleagues.



Research Report 1:

Documenting development of teacher' mathematical proficiency and productive dispositions towards teaching algebraic thinking through problem solving

This study documents teachers' development of knowledge in teaching algebraic thinking thorugh problem solving and the affective change that were elicited through the project activities. Forty elementary and middle school teachers in a summer institute called ACT NOW: Algebraic Connections and Technology with follow- up Lesson Study focused on developing algebraic connections and generalization strategies. The research examined the use of algebraic problem solving to develop teachers' integrated pedagogical content knowledge through analysis of teachers' reflections in problem solving tasks, the use of pedagogical strategies, tools and technology, and teachers' beliefs. Results revealed three critical areas necessary in transforming teachers' beliefs and practice: 1) building pedagogical content knowledge; 2) developing productive dispositions towards mathematics; 3) translating the knowledge into teaching contexts.

Underlying premise: to be able to teach algebraic reasoning through problem solving, teacher must know how to pose questions that elicit the algebraic connections. They must be able to extend students' thinking. But most importantly, they must relearn algebra through problem solving and develop a productive disposition towards understanding fundamental algebra necessary for elementary and middle grades.

Theoretic Framework

Research and initiatives emphasize the importance of fostering algebraic reasoning through problem solving and laying the critical foundations before students encounter formal algebra (Blanton, 2008; Driscoll, 1999; NCTM, 2000; NMAP, 2008). This leads to this important practical question: what experiences do elementary and middle grades teachers need, in order to teach through problem solving and make algebraic connections, especially if they have never learned in this way?

Algebraic Reasoning in the early grades

The problem is that before the 1980's "algebra was considered too abstract for younger students" (Greenes & Findell, p. 17). In the 1980s, the College Board (1985) recommended that foundations for algebra, in the form of arithmetic, be introduced to elementary students. Then in 2000, the National Council for Teachers of Mathematics (NCTM) published *Principles and Standards for School Mathematics* which identified algebra as one of the five strands of mathematics for elementary mathematics. In addition, more recently the *Curriculum Focal Points* (2006), outlining the most important math topics directly linked algebra to arithmetic in the numbers and operations and other content strands. In an earlier work, Kaput (1998) states that algebra "is a tool for generalizing and solving variety of problems" (p. 25). It entails making generalization which can identify mathematical structures, properties, and relationships through reasoning.

Several researchers have explored teaching early algebra for the elementary Grades and found that elementary students are capable of reasoning algebraic (Barstable & Schifler, 2007; Blanton & Kaput, 2003; Carpenter & Levi, 2000; Carraher & Blanton, 2007; Carraher, Schliemann, Brizuela, & Earnest, 2006; Falkner, Levi, & Carpenter, 1999; Dobrynia, 2001; Kaput, 1995; Moses, 1999). It takes a teacher who has a deep and **profound understanding of fundamental algebra** to provide opportunities for elementary and middle grades students to explore the foundation concepts for algebraic reasoning through patterning, relations, functions, and representations using algebraic symbols and utilizing mathematical models to represent relationships (NCTM 2000). Blanton and Kaput (2005) report teachers become better at teaching algebraic reasoning when the teacher's own mathematical knowledge and understanding is increased.

Blanton and Kaput (2008) conducted a 5 year professional developmentproject called, Generalizing to Extend Arithmetic to Algebraic Reasoning (GEARR), with the goal to embed algebraic thinking into instruction and to build teacher capacity to "algebrafy their classrooms" (p. 384). The evidence to support their findings was gathered from teachers' reflections and written work, students' reflections and written work, observations, and interviews. This professional development addressed both teacher content knowledge as well as teacher classroom practice. Their findings include the need for a professional community network, embedded professional development in the workday, the necessity of district leadership, and working within competing professional development agendas to build congruency.

Recent research looking at equality (Knuth et al., 2006) and symbolizing quantities (Blanton, et al., 2005), suggests teachers do not have the content knowledge to establish connections with algebraic thinking. Ma (1999) states "such understanding might be characterized as 'profound understanding of fundamental mathematics" (p. 120). Blanton and Kaput's (2005) earlier research on algebra and its many strands notes teachers must develop "eyes and ears" for algebraic reasoning while listening to students and looking at student work. However, teachers must have the content knowledge to know what to look for and what to listen for in the classroom. Driscoll (1999) suggested to foster the algebraic thinking, one must develop algebraic habits of minds which he outlined as seeing how algebra is about 1) doing and undoing; 2) building rules to represent a function; 3) abstracting from computation.

A summary of the research on early algebra (Mathematical Association of America, 2007) states young students are capable of the following activities:

(1) Describe, symbolize and justify arithmetic properties and relationships;
(2) develop an algebraic, relational view of equality;
(3) Use appropriate representational tools, as early as first Grade that will support the exploration of functional relationships in data;
(4) Identify and symbolize functional relationships;
(5) Progress from building empirical arguments to building justifications using problem contexts and learning to reason with generalizations to build general arguments; and
(6) Learn to compare abstract quantities of physical measures (e.g., length, area, volume), in order to develop general relationships (e.g., transitive property of equality) about these measures. (p. 9)

"Algebra is more than moving symbols around" (NCTM, p. 40) or application of a rule or formula. It involves variables, equality and the equal sign, describing and extending patterns, using models to make predictions, and understanding change (NCTM, 2000).The underlying concepts of algebra patterns, relations, and functions:

1) represent and analyze mathematical situations and structures using algebraic symbols;
2) use mathematical models to represent and understand quantitative relationships;
3) analyze change in various contests (NCTM, 2000, p. 39)

Mathematical knowledge for teaching

Recent reports from the National Mathematics Advisory Panel (2008) increased national attention to the need for improving mathematics teacher preparation and professional development with the goal of improvement of preK-12 student learning. Ma (1999) reports that many elementary teachers in the United States lack deep knowledge of mathematics content and pedagogy, and Fennell (2007) agrees that "the pre-service background and general teaching responsibilities of elementary teachers do not typically furnish the continuous development of specialized knowledge that is needed for teaching mathematics today" (p. 2). A critical component of this recommendation is that teachers be given ample opportunities to learn mathematics for teaching. That is, teachers must know in detail the mathematical content and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach. The report further acknowledges that content knowledge is not sufficient. For example, educators must have pedagogical content knowledge including the interconnectedness among conceptual understanding, procedural proficiency and problem solving (Shulman, 1986). According to Ball (2003), having mathematical knowledge for teaching means that you have practice-based knowledge such as being able to pose meaningful problems, represent ideas carefully with multiple representations, interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems, and insights (both predictable and unusual).

"Mathematical Knowledge for Teaching" is a deep understanding of mathematics that allows teachers to explain why common algorithms work, evaluate students' problem-solving strategies, anticipate students' misconceptions, and analyze students' errors. Teachers need to know the mathematical content and standards that should be taught and recognize the relationships among those mathematics topics. They must understand mathematical procedures in detail, but also have a clear understanding of why the procedures work. They must be able to represent mathematical ideas in multiple forms, choosing and using mathematical models skillfully. In addition, they must know their students and be able to adjust their teaching techniques according to the needs of their students. They should be able to interpret students' computational errors, evaluate students' alternative algorithms for usefulness, and understand students' mathematical thinking. Teachers with "mathematical knowledge for teaching" have an extensive and complex set of knowledge and skills that facilitates student learning.

Researchers have had some success in developing reliable measures of teachers' knowledge for teaching mathematics and these studies have suggested that professional development designed to improve teachers' mathematical knowledge for teaching could have a positive impact on student achievement. (Hill & Ball, 2004; Hill, Ball, & Schilling, 2004; Hill, Rowan, & Ball, 2005; Rowan, Schilling, Ball, & Miller, 2001).

Defining Teachers' Affect & Beliefs

Affect is defined as "a disposition or tendency or an emotion or feeling attached to an idea or object. Affect is comprised of emotions, attitude and beliefs.

Emotions- feelings or states of consciousness. Emotions change more rapidly and are felt more intensely than attitudes and beliefs. Emotion may be positive (e.g. the feeling of aha) or negative (e.g. the feeling of panic) emotions are less cognitive than attitudes.

Attitudes- manner of acting, feeling or thinking that show one's disposition or opinion. Attitude change more slowly than emotions but they change more quickly than beliefs. Attitude like emotions may involved positive or negative feelings and they are felt with less intensity than emotions

Beliefs psychologically held understandings about the world; harder to change than attitudes.Beliefss might be thought of as lenses that affect one's view of some aspect or as dispositions towards actio (Phillip, 2008, p. 261).

If beliefs are the lenses through which we humans view the world then the beliefs we hold filter what we see. Philip (2008) asks the questions, How do math educators change teachers' beliefs by providing practice based evidence if teachers cannot see what they do not already believe? He states that the essential ingredient for answering this question is reflection upon practice. "When practicing teachers have opportunity to reflect upon innovative reform oriented curricula they are using, upon their student thinking or upon other aspects of their practices, their beliefs and practice change."

Another important consideration for teacher educators working with teachers to develop MKT is to consider teachers affective nature towards mathematics. According to McLeod (1992) "all research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction" and he drew this conclusion form Mandler's theory that connects ones knowledge and beliefs to affective factors. For example, if a student solving a problem believed that all math problems should be solved in a couple of minutes but the student was unable to solve it in that period of time, the student might experience arousal that he or she would interpret as negative. This negative feeling develops into a negative attitude towards story problems and in many cases such negative attitude towards one aspect of math generalize to negative attitudes towards math in general or toward the students view of himself or herself as a learner. If however students believed that story problems can challenge even good problem solvers and require a longer period of time to solve, then arousal at an inability to quickly solve the problem might not be interpreted as negative. Research on prospective teachers indicate a fear and dislike for mathematics which is associated with a fragile understanding of mathematics. Many of these teachers indicated negative experiences in their middle and highschool mathematics classroom. In order to help teachers gain a new appreciation for mathematics, teachers must experience positive experiences while relearning the mathematics.

Effective professional development

There has been quite a lot of research on how to design effective professional development for teachers. Researchers have found that the best professional development is "intensive, ongoing, and connected to practice" (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009, p. 5). This type of professional development is called "practice-based professional development" and is "situated in practice" (Smith, 2001). It involves the work that teachers do every day and includes thoughtful inquiry and reflection about the lessons they plan, the tasks in which they engage children, the instructional strategies they use, and the ways in which they assess students. Ongoing practice-based professional development allows teachers to deepen their understanding of mathematics, examine their own instructional practices, and learn about their students' mathematical thinking (Smith, 2001; Weiss & Pasley, 2009). Ball and Cohen (19999) state "Much of what teachers would have to learn must be learned in and from practice rather than in preparing to practice" (p. 10).

The Design for the Professional development

Research about teacher professional development supports the design of learning communities focused on the fundamental mathematics concepts students need to learn. This professional learning should be embedded in their practice in order to have the greatest impact on students' learning. Effective job- embedded professional development models, where professional learning is directly related to the work of teaching include lesson study where teachers collaborative plan, observe, and debrief (Lewis, 2002a, 2002b; Lewis & Tsuchida, 1998; Wang-Iverson &Yoshida, 2005); co-teaching, mentoring, reflecting on actual lessons (Schifter & Fosnot, 1993); and group discussions surrounding selected artifacts from practice such as student work (Ball & Cohen, 1999). Research has shown that content-focused professional development leads to improvements in teacher content knowledge and needs to focus on student learning goals, highlighting the concepts being addressed, how they are developed over time, and how to monitor student understanding (Cohen & Hill, 2001; Desimone, Porter, et al, 2002; Garet et al., 2001; Hill, Rowan, & Ball, 2005). For the development of mathematics teacher leaders, the National Council of Mathematics Supervisors proposed a Leadership Framework incorporating four areas of leadership: equity, curriculum, assessment, and teaching/learning. Mathematics teacher leaders should both demonstrate leadership of self via their own professional learning (including significant reflection and analysis of their own teaching) and serve as a model for best practices. Teacher leaders should move beyond their own classrooms to sharing and collaborating with other mathematics teachers to influence mathematics teaching and learning at their school (National Council for Supervisors of Mathematics, 2008). This also means teachers need to implement and to develop culturally responsive pedagogy for their classrooms in order to achieve equity in student learning.

Effective job-embedded professional development models, where professional learning is directly related to the work of teaching include Lesson Study where teachers collaborative plan, observe, and debrief (Lewis, 2002a, 2002b; Lewis & Tsuchida, 1998; Wang-Iverson &Yoshida, 2005); co-teaching, mentoring, reflecting on actual lessons (Schifter & Fosnot, 1993); group discussions surrounding selected authentic artifacts from practice such as student work or instructional tasks (Ball & Cohen, 1999); curriculum materials (Ball & Cohen, 1996; Loucks-Horsley, Hewson, Love, & Stiles, 1998; Remillard, 2005). Research has shown that content-focused professional development lead to improvements in teacher content knowledge and needs to keep the focus on student learning goals, highlighting the concepts being addressed, how they are developed over time, difficulties students may encounter, and how to monitor student understanding (Hill & Ball, 2004; Garet et al., 2001; Cohen & Hill, 2001; Desimone, Porter, et al, 2002). For local systemic change to occur through teacher enhancement, (Heck, Banilower, Weiss & Rosenberg, 2008), there needs to be a shared mission (school-wide/district-wide/state-wide), a critical mass of teachers involved. In addition, principals need to know what teachers are learning and how they can best support them and state/district policies need to aligned with the same vision as the professional development. While few teachers in the United States have opportunities to participate in this type of sustained high-quality professional development, it makes a difference in student achievement (Darling-Hammond, et al., 2009). It can take the form of centralized workshops (Weiss & Pasley, 2009), but greater numbers of teachers are engaging in job-embedded professional development activities at their own school sites. Job-embedded professional development helps teachers "to implement instructional change in their classrooms" and to "take ownership of their own professional growth" (Weiss & Pasley, 2009, p. 39). It allows teachers to receive support for their own learning on a daily basis within their own classrooms. It can be provided in a number of ways including collaborative lesson planning, co-teaching with other teachers, working with a coach, and participating in lesson study. Lesson study (Smith, 2001) is a model where groups of teachers design and implement lessons together.

Context of the project

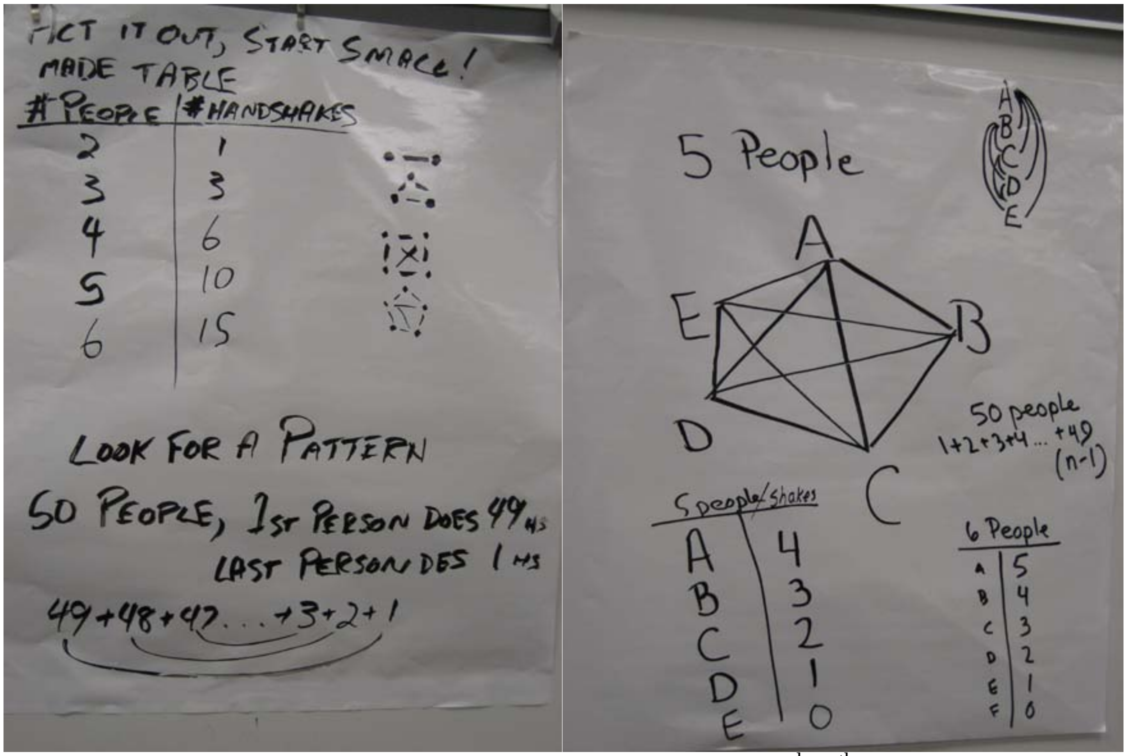
The designers and instructors of this project included a university mathematics educator, a mathematician, an elementary mathematics specialist and a middle school Algebra I teacher. We designed *ACT NOW in MATH: Algebraic Connections and Technology in Middle Grades Math* based on research and the current needs in mathematics education. Research on effective professional development in mathematics identified three key factors focused on: 1) content knowledge; 2) active learning where teachers become actively involved in both discussions and planning; and 3) coherence, directly related to their practices (Garet, Porter, Desimone, Birman & Yoon, 2001). In addition, professional development must "foster a culture of sharing and providing sustained support for teachers (i.e. knowledge networks)" (Barab et.al, 2001, p. 74); occur through collaborative planning and implementation, engage teachers in opportunities that promote continuous inquiry and improvement that is relevant and appropriate to local sites (NWREL, 1998); and "facilitate joint construction of knowledge through conversation and other forms of collaborative analysis and interpretation" (Cochran – Smith & Lytle, 2001, p.53).

Consequently, merely implementing these factors in professional development may not bring about change in teaching practices. Teacher change is much more complex due to the affective nature of one's beliefs, along with attitudes and emotions (McLeod, 1992). The affective domain is most simply described as feelings – how an individual feels about, in this case algebra. Research has revealed that the formation of teachers' beliefs about mathematics teaching and learning come from their own experiences as a learner of mathematics (Fosnot, 1989). Thus, the goal for all professional development should be to develop teachers' productive disposition towards mathematics, which has been described by the National Research Council (2002) as is "the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy"(p. 116).

The goal of *ACT NOW in MATH: Algebraic Connections and Technology in Middle Grades Math* was to support teachers to transform their practices by bridging more algebraic connections to the existing mathematics curriculum through problem solving and technology. One goal was to help teachers to reconceptualize algebra in the elementary and middle grades by re-examining the existing curriculum and bringing out the algebraic reasoning that was embedded in the arithmetic already being taught. This strategic goal aligned with the district's mathematics initiative which focused on the need for developing algebraic thinking in the elementary and middle grades.

Design of the Professional Learning
Content focus: The project focused on developing teachers' mathematics content knowledge, especially building their algebraic habits of mind (Driscoll, 1999): 1) Doing and Undoing, 2) Building rules to represent functions and 3) abstracting from computations. In addition, the teachers discussed research on how students learn specific mathematics concepts and strategies and the learning progressions in the middle grades students (Grades 3-8).
Active learning-Teachers as learners: During the summer institute teachers grappled with rich problems and uncovered all the mathematics connections and algebraic thinking. Over the academic year, a small Lesson Study (Lewis, 2002) team collaborated on a research lesson. This job- embedded professional development models was directly related to the work of teaching where teachers collaborative plan, observe, and debrief
Coherence: The content of the professional learning centered on the curricular objectives for the grade bands and districts' initiatives were incorporated in the application of best practices (i.e. promoting math discourse and effective use of technology).
Duration: The professional learning would begin in the summer through the institute but it will be sustained throughout the year through face to face meetings for Lesson Study in the fall and follow up blended seminars in the spring that were online and onsite.
Collective participation School teams were encouraged to participate so that there would be at least 3 teachers from a site. This would allow for a support network for the professional learning to continue beyond the project. Administrators and other special resource teachers (ELL and Special educators who co teach math) were also encouraged to join as part of the school teams.

This project focused on developing teachers' algebraic connections and generalization strategies through the use of problem solving and technology. The professional development summer institute and the follow up Lesson Study throughout the academic year focused on (1) engaging teachers in algebraic problem solving tasks, (2) exploring pedagogical strategies, mathematics tools and technology, and (3) promoting algebraic connections in elementary and middle school curricula.



Forty elementary and middle grades teachers from grades 3rd -8th met for a 2-week summer institute. Daily activities included research-based practices and model lessons using a variety of mathematics tools and technology. Participants engaged in mathematically rich activities that connected algebraic content with pedagogical strategies through problem solving (see Figure 1). In addition, teachers met during the academic year in small groups of 6-7 teachers with the instructors to continue their professional learning through a teacher-led professional development model called Lesson Study. The goal of these follow-up sessions was to provide teachers with continued support in learning and implementing algebraic content, materials, strategies, opportunities to share ideas across grade levels and to analyze student learning. The Lesson Study component was a critical piece to this project in ensuring sustainability of the professional learning. By exposing teachers and math leaders in the teacher-led professional development and the project created an infrastructure for continued development. In addition, the project team created a website as a resource for these teachers and their schools. This website has many important resources like links for teaching resources, more algebraic problems, a discussion forum, and links to video-based instructional resources on algebra. In addition, teachers have the lessons created by participants on a CD and on this website to share with their colleagues.

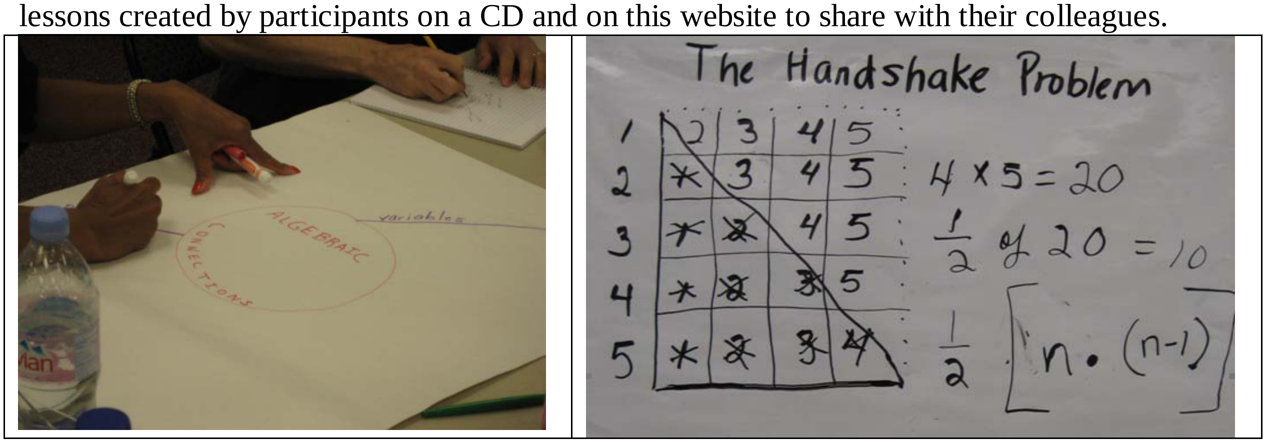


Figure 1 a & b. Teachers at work: “Relearning the math”

During the course, the participants were immersed in a problem solving environment. That is, problems were used as a way to introduce concepts in mathematics, mathematics teaching, and mathematics learning. There were problems that were assigned to be worked on in class, as homework, and as a project. Participants kept a mathematician's reflective journal in which they responded to assigned problems. The data sources included videotapes of the class sessions and teacher reflections that included (1) teachers' algebra content knowledge and feeling about the problems (2) teachers' use of mathematics tools and technology and pedagogical strategies and (3) teachers' beliefs.

The Lesson study reflection included teachers' evaluation of instructional strategies used to promote algebraic thinking through problem solving, teachers' analysis of student thinking and what they learned from the process of collaborative planning, teaching, observing and debriefing with colleagues. The Lesson Study process included developing and refining a research lesson, assessments items, and analysis of their students' learning.

Research Questions

1) How did teachers in the study develop algebraic connections and deeper mathematical knowledge of middle grades mathematics?
    a) What designed activities and professional learning opportunities elicited the development of the algebraic connections for teachers?
    b) What pedagogical content knowledge and beliefs are revealed in their problem solving reflections as they take on the role of teachers as learners?
2) How do teachers translate what they learned into the teaching context?

Methods

This project used a mixed method of survey analysis and qualitative analysis. Using a Grounded Theory approach (strass and corbin, 1994), the researcher used a constant comparative method. In a grounded theory study, the researchers intend to generate a theory that is "grounded in data systematically gathered and analyzed". Grounded theories are not generated before a study begins but are formed inductively from the data that are collected during the study itself. The researchers start with the data they have collected and they then develop generalizations after they look at the data. Researchers used the constant comparative method. This method allowed for continual interplay between the researchers, the data and the theory that is being developed. Potential categories for grouping items of data are created, tried out, discarded until a "fit" between theory and data is achieved. The method used to reach a grounded theory is termed the ****constant comparative method. During data collection, data usually are analyzed concurrently. As the data are analyzed, the researcher searches for a ****core variable, which will serve as the foundation for theory generation. The core variable usually has some of the following characteristics:

* recurs frequently,

* links various data

The analysis of the data began with the reading of the reflections from the summer institute during which patterns in the participants responses were identified. The ideas which emerged from the reflections were categorized into themes and cross checked with teachers' comments during the taped video sessions and researchers' notes so that a set of common themes began to emerge.

Data Collection

Survey from the beginning of the summer institute and at the end of the Professional development course in the spring. Problem solving Reflections were collected during the summer institute. In addition, researchers kept a researcher memo based on open-ended interviews and observation from the summer institute and the research lessons during lesson study. These different data sources were triangulated. Observational data were used for the

purpose of providing a rich description of settings, activities, people, and the understandings of what is observed from the perspective of the participants.
Moreover, case study field researchers rely most heavily on the use of field notes, which are running descriptions of settings, people, activities, and sounds. Since it is difficult to write extensive field notes during an observation, Lofland and Lofland (1984) recommend jotting down notes that will serve as a memory aid when full field notes are constructed. This should happen as soon after observation as possible, preferably the same day. In addition to field notes, researchers may use photographs, videotapes, and audio tapes as means of accurately capturing a setting. Audio recordings and written field notes of the sessions were employed and used in this case study. Through these lesson study activities, the researcher team had access to participants who explained their experiences, ideas, and allowed for additional conversations and data collection to occur. Through the summer institute and lesson study meetings, teachers built trust with the research team during the lesson study discussions. The researchers also had also visited the schools, met teachers, and attended meetings at the participating elementary and middle school prior to embarking on the case study research.

During the lesson study planning cycle, there were at least three opportunities to interview participants and observe the participants' face-to-face, in small group training and planning/reflection sessions. Also, field notes of observations of the classroom planning, implementation of the research lesson that was developed from these training and planning sessions also provided additional information and data. Educators' documents were collected and analyzed. "Physical artifacts have less potential relevance in the most typical kind of case study. However, when relevant, artifacts can be an important component in the overall case" (Yin, 2003, p. 96). Document review conducted by studying documents "follows the same line of thinking as observing or interviewing" (Stake, 1993, p. 68). Teacher learning artifacts that were collected from lesson study professional development included training and planning agendas, lesson plans, student work samples and/or student data, reflections, schedules, and evaluations contributed to building the picture of teacher learning experiences.

Data Analysis

The data analysis was aimed at answering the research questions and identifying themes, categories, or types. To begin the data analysis the researcher went through three processes: data reduction, data display, and conclusions and verification. These flows are present in parallel during and after the collection of data (Miles & Huberman, 1994). Data reduction refers to the process of selecting, focusing, simplifying, abstracting, and transforming the collected data. Data displays are intended to organize the collected data in such a way that it permits conclusion drawing (Berg, 2001; Miles & Huberman, 1994). The third component of the data analysis process is conclusion drawing and verification. During the collection of data, preliminary conclusions were drawn and verified during the process (Miles & Huberman, 1994). The strategy that was for data analysis in this case study was the technique of pattern matching. To develop internal validity and external validity, specific analytical technique of pattern matching was followed (Yin, 2003). When all collected data are available in textual format, data was methodologically and systematically analyzed using Nvivo. In pattern matching, a pattern is compared with a predicted, consistent or proposed one. Pattern matching or coding has four important functions (Miles & Huberman, 1994). First, it reduces large amounts of data into a smaller number of units to be examined. Second, it gets the researcher into analysis during data collection, so that later fieldwork can be more focused. Third, it helps the researcher understanding participants' interactions. Fourth, it lays the groundwork for cross-case analysis by promoting common themes. Transcribed and coded interviews and field notes from the teachers' conversations, reflections, and discussions is critical to analyzing the data. In analyzing additional data collected from six teachers' problem solving reflections, observations, documents, and other artifacts, breaking down data using codes provided descriptive information linked to practice and concepts that were critical to the reporting of the results. The coding also reflected patterns focused on people, events, issues, etc. The data were reorganized into themes followed by the interpretation of the themes and drawing and verifying conclusions and finally, the writing of the research findings (Miles & Huberman, 1994). All evidence, data, and documentation were carefully and methodically analyzed as they were vital to creating the picture of teachers' understandings for teaching through problem solving to make algebraic connections and the lesson study professional development process.

The findings are summarized in the following sections.

Results and Conclusions

The teachers wrote reflections on the problems that they worked on in class. Analysis of teachers' reflections from the algebraic problem solving tasks revealed their algebraic content knowledge, their use of mathematics tools and technology, their feelings about the problems and how they could adapt the problems to make algebraic connections in their respective grades. The following reflection illustrates the "relearning" that took place for the participant as she tackled the problem individually than worked collaboratively with her colleagues to build collective knowledge by sharing their different strategies. In addition, it shows how the use of a chart (math tool) helped the teachers see the patterns and ultimately see the connection to the Pascal triangle (see Figure 2). The following problem was a combination problem which asked to find the total number of possible pizzas with a variety of toppings.

> *"Taking the chart that I made to compare toppings to pizzas, I decided to add some columns showing the number of combinations for each amount of toppings... I also noticed the symmetrical feeling the pattern in the table had, where the first and last numbers are the same, and the second and second to the last numbers were the same. I shared my discovery with my group, and Lucy noticed a triangular number pattern in something she was doing as well. When Jamie looked at the table, she quickly stated that it was Pascal's triangle. WOW… I totally missed that! The really big WOW was still to come for this problem. When we were working on binomials, Dr. S said something that really blew my mind. After already multiplying out $(x + y)^2$, he said to look at the coefficients when you multiply out $(x + y)^3$. So, I did it all out and got $x^3 + 3x^2y + 3xy^2 + y^3$, the coefficients were 1, 3, 3, 1. While I have held a belief that math makes sense and is full of patterns, this reaffirmed that belief and added on another level."*

# toppings	# pizzas	0 topping	1 topping	2 topping	3 topping	4 topping	5 topping
2	4	1	2	1			
3	8	1	3	3	1		
4	16	1	4	6	4	1	
5	32	1	5	10	10	5	1
6	64	1	6	15	20	15	6
n	2^n	1	n	[n(n-1)]/2			

Figure 2. Teacher reflective entry shows "relearning" through problem solving

Analysis of the problem solving reflections and researchers' notes on the learning process for the teachers during their engagement with the algebraic problems revealed two levels of learning. The first level was the learning that took place for teachers as learners as indicated by the first row in the diagram below (see figure 3). The teachers learn to grapple and solve algebraic problems, use math tools such as graphs, tables, formulas, pictures and technology and finally evaluate multiple solutions with colleagues. On the second level, teachers gained pedagogical knowledge while engaged in this process as indicated in second row. Each process that teachers participated in as learners, allowed them to consider the pedagogical implications to making algebraic connections such as, understanding the importance of designing rich problems that elicit algebraic reasoning and understanding the metacognitive processes and mathematical concepts important within these problems. As teachers used a variety of math tools (graphs, tables, equations, diagrams, technology) and shared multiple strategies, they recognized that different tools and representations are better and more efficient for different classes of problems.

Developing Algebraic Connections through Problem solving	**Immerse in rich problem with algebraic connections**	**Solve the problem independently**	**Discuss strategies, tools and build collective knowledge**	**Reflect on problem solving strategies and make explicit connections**
Opportunities for Teachers as learners	Grapple with the problems & experience disequilibrium Use and make connections to fundamental algebra	Self-monitor one's problem solving process & making sense of mathematics Rediscovery of the algebra that teachers learned procedurally through a conceptual approach Be in the "shoes of students"	Share strategies with colleagues Build on ideas or repair understanding Make new connections Present via multiple representations Communicate ideas Social learning	Probe thinking to make deeper connections Make explicit connections to fundamental algebra Related math concepts and problems
Linking teacher professional learning with elicited teaching practices				
Teachers engaged in rethinking their teaching practices	How to pose rich problem to students How to set up a task How to engage students How to prepare tools for thinking	How to breakdown the essential math learning How to identify common student misconceptions How to scaffold and differentiate for diverse learners	How to navigate math discourse How to ask higher level questions How to distinguish and highlight strategies for collective inquiry How to respond to student questions	How to make connections for deeper understanding How to assess students responses for efficiency and depth of understanding

Figure 3. Parallel learning path documented from problem solving reflections

Developing productive dispositions towards mathematics

As teachers grappled with algebraic problems in class and as homework, they kept a mathematician's reflective journal in which they responded to assigned problems and wrote about the feelings they encountered as they solved the problems. The data from teachers' reflections revealed not only teachers' learning the content but also their feelings about the learning process. The instructors collected teachers' work samples on algebraic problems and teachers' reflections of their problem solving process for analysis. The emerging themes from the reflective narratives revealed that: a) teachers felt frustrated and afraid to tackle the algebra problems at first. But as they continued on with the course and persevered through the problem tasks, teachers gained confidence, satisfaction and a renewed appreciation for algebraic thinking which they hoped to instill in their students; b) teachers needed to "relearn" the algebraic concepts through a problem solving approach because they were not taught in that fashion which required them to shift their traditional mode of algebraic thinking; c) teachers valued the collaborative work and rich mathematics discourse which allowed them to make generalizations and build collective knowledge.

In one class session, a teacher shared how she struggled in mathematics when she arrived to the United States in middle school. As a second language learner, she tried her best to follow along with the instruction in her math class. She recalled how she would take the problems and translate it in her language to make sense of it and then solve it. However, before she could finish the problem or have a chance to articulate it in some way to make sense of her learning, another student would give an answer and the teacher would go onto the next problem. This was the first time, as an adult, that she was able to work on a mathematics problem and get a chance to share her solution strategy with her colleagues. She shared with the class how this experience was significant in changing her attitude towards mathematics and giving her confidence in a subject that she never liked as a student. This testimony from the teacher also helped move our discussion on how to engage students from diverse populations (ethnic, linguistic, and diverse abilities) to engage in rich mathematics and classroom discourse.

At the end of the course, participants reflected on "How have your ideas changed through your participation in this course? The most common themes were that they "relearned" the math by being in the "shoes of the student" and that having to solve challenging problems while breaking down the important mathematics helped them see the early building blocks for algebraic connections. These experiences also built their confidence in mathematics and a productive disposition towards mathematics. The following quotes reveal teachers *transforming their practice and beliefs about bridging algebraic connections in earlier grades.*

> "I used to be uncomfortable with kids struggling with problems that I think I may have "guided" their thinking which also "robbed" the critical thinking process. Through this experience, I realized that the struggling part is part of the problem solving process that mathematicians need to go through to make sense of the mathematics."

> "I really need to change how I teach. I need to do more of these problems so that they can make those algebra connections."

> "I will use more problems like these to teach the students to think and to make them more independent and confident learners. I use a lot of direct instruction which does help the Special Ed students but more "thinking" problems will be a great balance for my class."

> "As a student in the class I am learning the power and benefit of struggling through a concept as a student; instead of simply receiving an equation or an answer. When I teach this year I want my students to feel this disequilibrium and then have satisfaction through understanding more through time as their mathematics learning continues."

> "Unlike the algebra courses that I have taken before, this course is fulfilling my expectation of challenging me to "think algebraically."

> "It really has been good to put me back into thinking like my students. I feel the frustration that I feel, and the great questioning and example setting gives me great insight as to how I can teach in my own classroom."

Translating knowledge into practice in a classroom context

Teacher reflections from the follow-up Lesson Study revealed several themes of importance to mathematics teaching and learning. First and foremost, teachers commented on the value of collaboration, the opportunity to observe a teacher in action, the time to collectively reflect on a shared experience, and establishing the trust, support and expertise of colleagues. Lesson Study provided teachers the opportunities to see how a teacher effectively (a) uses technology to make algebraic connections; b) helps diverse learners access the algebraic concepts; c) guides discussion to make connections and generalizations explicit; d) builds collective knowledge through shared activities.

> "The fact that I taught the lesson twice that day provided for us a nice opportunity to debrief after the first time, and modify the lesson for the second time. My analogy was that of a football coaching staff making modifications to the game plan at halftime. Once again, this was a very balanced and constructive process, with everyone's input considered and valued." (teacher who lead the Lesson Study with 8th graders)
>
> "The debriefing / enhancing discussion was done in a very supportive environment. The term "enhancing" a lesson immediately helped me feel like any suggestions were put towards the lesson, and not as a critique of me. I hope that my students feel as safe as I felt taking this class!" (teacher who lead a lesson in a 6th grade class)
> "This lesson study format allowed me to challenge even the lowest of my students. All children can learn through this method and having colleagues to bounce ideas off of made it so much more valuable." (a 4th grade teacher observer and participant in Lesson Study)
>
> "Lesson study is a very powerful tool to gain insight into student learning and understanding of a specific topic. I learned more about my students during this one hour lesson than I have any other day in math this year. By engaging in the cooperative lesson study cycle I felt stronger as a teacher and more knowledgeable about where my students are and where I need to push them. Lesson study is not something that can be done alone and requires a unique blend of people who are willing to take risks and work collaboratively." (a 5th grade teacher who lead a lesson)

Before the project began, we assessed teachers' beliefs about the most important practice based skills necessary to make algebraic connections in the early grades. The top four practice -based skills were: posing good mathematical questions and problems that are productive for students' algebraic thinking (95%); responding productively to students' mathematical questions and curiosities (94%); assessing students' mathematical learning and taking the next steps (89%); and giving access to algebraic thinking to all members of a diverse population (89%). This survey indicated the areas that teachers felt the greatest need in their professional learning.

The pre and post surveys entitled " Measure of importance and preparation for teaching for mathematics proficiency with emphasis on making algebraic connections". Teachers took the survey at the beginning of the summer institute and then again after the winter conference in December. Using a paired sample t-test, teachers self-report of preparedness was compared for each participant that completed a pre and a post survey (n=21). Some participants had completed a presurvey but did not complete the post survey so the researchers only included the matching surveys. The analysis showed significant change reported by teachers on several categories ($p < .05$): 1.Developing students' algorithmic thinking through algebraic connections; 3. Building rules to represent functions; 4. Abstracting from computation; 5. Assessing students' mathematical learning and taking the next steps; 8. Responding productively to students' mathematical questions and curiosities; and 10. Using technology with students to make algebraic connections.

Table X.
Results from the Paired Samples Test

	Paired Differences					t	df	P value
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
1.Developing students' algorithmic thinking through algebraic connections	-.66667	.91287	.19920	-1.08220	-.25113	-3.347	20	.003
2. Using doing and undoing to promote algebraic thinking	-.33333	.91287	.19920	-.74887	.08220	-1.673	20	.110
3. Building rules to represent functions	-.47619	.92839	.20259	-.89879	-.05359	-2.351	20	.029
4. Abstracting from computation	-.47619	.81358	.17754	-.84653	-.10586	-2.682	20	.014
5. Assessing students' mathematical learning and taking the next steps.	-.57143	.81064	.17690	-.94043	-.20243	-3.230	20	.004
6. Posing good mathematical questions and problems that are productive for students' learning.	-.28571	.90238	.19691	-.69647	.12504	-1.451	20	.162
7. Making judgments about the mathematical quality of instructional materials and modify as necessary.	-.61905	.86465	.18868	-1.01263	-.22546	-3.281	20	.004
8. Responding productively to students' mathematical questions and curiosities.	-.42857	.81064	.17690	-.79757	-.05957	-2.423	20	.025
9. Using mathematically appropriate and comprehensible definitions with students.	-.57143	.81064	.17690	-.94043	-.20243	-3.230	20	.004
10. Using technology with students to make algebraic connections	-.66667	.85635	.18687	-1.05647	-.27686	-3.568	20	.002
11. Giving access for mathematical learning to all members of a diverse population.	-.28571	.95618	.20866	-.72096	.14953	-1.369	20	.186

12. Identifying and making algebraic connections among various mathematical topics.	-.61905	.86465	.18868	-1.01263	-.22546	-3.281	20	.004
13. Representing mathematical ideas and concepts carefully in multiple ways.	-.52381	.81358	.17754	-.89414	-.15347	-2.950	20	.008
14. Making connections between physical, graphical models and symbolic notation.	-.80952	.74960	.16358	-1.15074	-.46831	-4.949	20	.000

Discussion

According to our work, changing teachers' beliefs and transforming their practices requires construction of new pedagogical content knowledge, productive disposition towards mathematics, and translating this new knowledge in their teaching contexts (See figure 4).

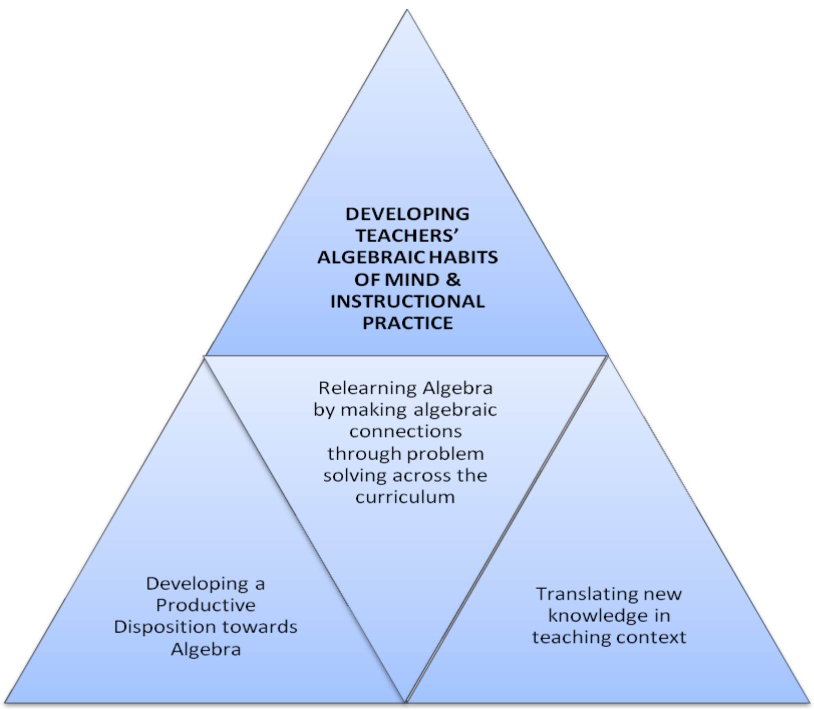


Figure 4. Critical areas of development necessary in transforming teachers' beliefs and practices

We share the voices of teachers of how they learned from this project in hopes that it will have a broader impact in teacher education and professional development. Through their own experiences with grappling with the algebraic problems, teachers came to see, and to believe, in the value of teaching mathematics through problem solving, making connections, communicating and justifying their solutions. Many were never taught this way in their own schooling and needed to experience this as "students" themselves before they could teach this way. In addition to having the *opportunity to become "students"* during the summer institute, we believed it was important to provide teachers the opportunity to *translate their learning into practice* as they collaboratively planned a lesson, observed and debriefed through Lesson Study. ACT NOW gave teachers these critical experiences that provided them with the *teaching and learning schema* for transforming their practice and teaching through reform oriented practices that promoted algebraic connections. One teacher commented," As I worked in the cooperative group setting on Tuesday as we worked on building our Lesson Study, I realized just what a great asset such a concept would be for me as a teacher. You could feel the energy and excitement as ideas were being bantered about. This is what good teaching is about. I do not believe that teaching was meant to be done in solo, but rather, the educational experience is made more valuable when teachers work together toward an ultimate destination, an educated child."

The results of this project contribute to the growing body of knowledge on what teachers need in professional development: the time to "relearn the mathematics" for teaching, the positive experiences that help develop a productive disposition towards mathematics, and an opportunity for teachers to work together to translate their learning into practice. This research impacts teacher education and professional development in developing algebraic teaching practices in the earlier grades. The results of this study contribute to the growing body of research on what elementary and middle school teachers need in the form of professional development to become effective in bringing out the algebraic connections in what they already teach.

Research Report 2: Impact on Student Learning

Building rules to represent linear functions through problem solving and technology

This anecdote shares evidence of student learning during a lesson study experience in planning and teaching an algebraic lesson on linear functions with a real world application. Participants included eight seventh and eighth grade teachers and lesson study facilitators, including a university mathematics educator, a mathematician and a school mathematics specialist. The overarching goal of the lesson study was to developing students' algebraic connections, communication, problem solving and algebraic habits of mind. One algebraic habit of mind that is critical in middle school is building rules to represent functions. To elicit this type of thinking, teachers may pose questions like: Is there a rule or relationship here? How does the rule work and how is it helpful? How are things changing? Does my rule work for all cases?

The lesson we describe is called the MP3 purchase plan and the purpose of this activity was to allow students to use tables, graphs to build rules to represent linear functions with a real world application and determine which mp3 plan was most beneficial to them as consumers. Some of the research goals set out by teachers was to develop students to become persistent and flexible problem solvers and to communicate their mathematical ideas clearly and respectfully. Through the design of the lesson, teachers wanted students to be able to recognize patterns and create and analyze functional relationships.

The Lesson Study on Linear functions

The Lesson Study took place in a middle school with diverse learners. The lesson was taught in two classes: Algebra I Honors: 1st period, 47 minute class with a total of 18 students, 12 girls and 6 boys who tend to work well independently and Math 8: 5th period, 1 hour class with a total of 26 students,13 girls and 13 boys who tend to need more teacher prompting/questioning. The teacher group included five seventh and eighth grade teachers teaching a range of middle school mathematics and two special education teachers.

The lesson study process involved three phases which began during a summer professional development institute and continued through the follow-up lesson study: 1) collaborative planning phase, where the teacher group defined the overarching goal, the important mathematics, and planned the lesson; 2) teaching and observation phase, where one teacher taught the focus lesson and the others observed using a predetermined observation form;

and 3) debriefing phase, where teachers reflected on the lesson design, representations, student engagement, evidence of learning and discussed future steps. Some of the essential elements crucial to the planning, teaching, learning, observing and reflecting processes were posed as the following questions: What is the important mathematical understanding that students need to learn? How do we pose this problem in an engaging and meaningful way? What different forms of representations will give students access to this concept? What conceptual supports and instructional strategies can best address our students' learning? How do we assess evidence of their learning? How do we modify the lesson and use our discoveries to improve the teaching and learning of algebra?

In the following sections, we will address these six questions as revealed through the Lesson Study process as essential elements in improving mathematics teaching and learning.

Essential Element # 1: Identifying the important mathematical understanding

To begin the Lesson Study planning process, teachers gathered their curricular materials and identified important mathematical concepts for the mathematics they taught at different levels, (7^{th} grade, 8^{th} grade math, Algebra Honors, and Special Education and English Language Learners). This process allowed for teachers to have vertical articulation between and among grade levels and discuss different developmental learning issues. A common goal teachers identified as one of the important mathematical understanding for students was for middle school students to a) describe and represent relations and functions, using tables, graphs, and rules; and b) relate and compare tables, graphs, and rules as different forms of representation for relationships; c) solve multistep linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable, and apply these skills to solve practical problems.

Essential Element #2: Posing meaningful problem targeting the important mathematics

Based on the identified goals, the Lesson Study Team decided on posing a meaningful problem that would help build rules to represent linear functions. They chose to modify an existing lesson on their county's problem solving resource called the MP3 purchase plan. In order to complete this activity, students would need background knowledge of how to write linear equations, complete a function table, graph on the coordinate plane, and analyze data. The problem was as stated,

> You have decided to use your allowance to buy an mp3 purchase plan. Your friend Alex is a member of i-sound and pays $1 for each download. Another one of your friends, Taylor, belongs to Rhaps and pays $13 a month for an unlimited number of downloads. A third friend, Chris, belongs to e-musical and pays a $4 monthly membership fee and $0.40 a month per download. Each friend is trying to convince you to join their membership plan. Under what circumstances would you choose each of these plans and why?

Essential Element #3: Planning for different representations to access students' thinking

In teaching and learning, representations can play a dual role, as instructional tools and learning tools. As Lamon (2001) states, representations can be "both presentational models (used by adults in instruction) and representational models (produced by students in learning) which can play significant roles in instruction and its outcomes" (p.146). Another way to think about representations is that they allow for construction of knowledge from "models of thinking to models for thinking" (Gravemeijer, 1999). By focusing on this element, we heightened teachers' awareness of the importance of multiple representations and how teachers need to thoughtfully and critically select models that would facilitate the teaching and learning of a mathematics concept.

Essential Element #4: Design features (Conceptual supports and instructional strategies) addressed diverse learning needs

In both, the algebra honors and Math 8 classes, the teacher began with a class discussion regarding mp3 players and downloads. Then he presented the problem as a hand-out. Teacher led class discussion on translating problem into verbal expression. First, students worked individually to solve the problem. The teacher gave students plenty of time to work out the problem on their own. The lead teacher and the Lesson Study teachers observed the students' reactions closely. As the teacher observed students having difficulties, he passed out a hint card. A chart with the five different representations (verbal, concrete or pictorial, graph, and table) was on the board for students to see. Then, students worked in small groups to discuss and compare their solutions. Teacher moved from group to group offering input as needed. Class discussion focused on the solutions that students reached. Finally, the teacher called on groups to share their methods/solutions with a focus on the multiple representations.

Design modification for the task sheet for the two classes (see figure 2)

The algebra class quickly came up with linear formulas for the three different types of MP3 plans then they immediately filled the table using the formula they had developed. However, for the Math 8 class who had not yet been formally introduced to linear formulas, the Lesson Study Team decided to reverse the order and have students, first fill in the table for each of the plans. Then use the table to find patterns that might aid them in verbalizing the rule.

Data analysis interpretation questions

For both classes, the data analysis/interpretation questions on the last page dealing with which plan was best under various circumstances brought meaning to the day's activity. They realized that this type of thinking and analysis allows them to become better thinkers and smarter shoppers.

1. If you buy less than 5 mp3s a month on average, which plan would you choose? Explain your answer.
2. If you buy between 10 and 22 mp3s per month on average, which plan would you choose? Explain your answer.
3. If you buy more than 22 mp3s per month which plan would you choose? Explain your answer.
4. Which company has the best plan for you? Explain your reasoning.
5. Which representation — the advertisements, the table, or the graph — helped you most in deciding which plan is best? Explain your reasoning.
6. What other real life purchase plans could be analyzed this way?

Essential Element #5: Assessing students' ability to build rules to represent functions

The students, both algebra and Math 8, could predict early on in the lesson that Rhaps was the ideal choice for someone downloading a very large number of songs each month. Both classes saw the application of the concepts of this lesson toward other relevant areas of their lives, such as cell phone plans. In this lesson, students were asked to verbally translate a rule to represent each mp3 purchase plan. They used words to help translate each plan into language they were more comfortable with. In the Algebra class, students were asked to algebraically represent the rules using two variables, while in Math 8, they used the table and graph to verbalize the pattern of change. It is of importance to note that the students came very close to making this connection from the verbal description to generating a rule.

**Algebra Honor Students: Evidence of the development of students' algebraic thinking*

For Algebra, discussions came up when establishing the equations and whether or not the $4 monthly service fee needed to be added each time or if that was a one time fee. Also students were heard making generalizations about the graphs and were heard asking each other about how many points they needed to create a line graph as well as whether or not they needed to graph each individual point. Discussions were also had regarding which was easier to read, the table or the graph which helped to develop the idea that everyone has different learning styles, but they can all do the same work in different ways and learn the same overarching concepts.

The lead teacher commented "In moving about from group to group, I was able to elicit from the students what the abstract notations of domain and range meant in the context of this mp3 problem. The students were able to explain why the domain was the whole numbers (one cannot download negative or fractional amounts of songs) and why the range was the rational numbers ≥ 0 (total cost will never be a negative value); and thus why we were only concerned with Quadrant I."

**Math 8 Students: Evidence of the development of students' algebraic thinking*

The lead teacher noted that exposure to this problem allowed students to demonstrate their understanding in several advanced concepts. Namely, students drew and analyzed the graphs of the three download plans. They understood that for a given number of downloads (i.e. input value) the line which was "lowest" represented the best deal. They could also interpret what the points of intersection of the lines meant. They understood that total cost depended on the number of downloads, and not the other way around. The "aha" moment was witness by an observer, when a student made a realization of plotting several points not all points. This made graphing easier for the Math 8 students. The Math 8 students also, in a looser less formal way than the algebra students, could gather why we were only concerned with Quadrant I. The Math 8 students needed more guidance since they did not have all the background information regarding functions and equations. The chart really helped the students visualize the pattern and compare the plans. This was something they seemed comfortable creating and something they could easily use to compare the plans. Students were heard making connections between the graph and a process of cost analysis based on the graphs. Additionally the students were observing and employing patterns to help them construct the table and create the line graphs. In fact, one student was even overheard saying "up one, over one" about plotting the points for one of the mp3 purchase plans. Hearing a student discuss slope without knowing what slope is, was really a great way to see how teachers can use a students' innate sense of recognizing patterns to help foster algebraic thinking.

The activities that were more effective varied between the two classes. For the Math 8 class, the most effective activity was establishing the table and transferring the table into a visual representation. In doing this, many students began to visualize patterns and make connections between the algebraic table and the visual graph. It was very effective to have the class analyze the graph as a group. Some students took a little longer to make the connection between the idea that the line that is lower on the graph was the cheapest, but we think that the discussion helped to make sure the majority of students made the connection between the math and cost analysis in a real life application. For the Algebra classes, the most effective activity had to do with writing the equations and connecting them to the graph. While the Algebra students did not rely on the patterns as much to help them construct the graph, they were able to connect the table to the visual representation. They were also able to take the visual representation to help them to conduct a cost analysis of the three plans. It was effective and worthwhile for the students to think through the domain of the graph within the confines of the word problem. This spurred some interesting conversation about whether or not we could buy parts of mp3s or not, and whether the domain is all real numbers, integers, or whole numbers.

Essential Element #6 Making modification after teaching and reflecting on the lesson

The beauty of Lesson Study is that it is an on-going iterative process of lesson refinement and professional learning. The fact that the teacher taught the lesson twice that day provided for the lesson study team a nice opportunity to debrief after the first time, and modify the lesson for the second time. The analogy was that of a football coaching staff making modifications to the game plan at halftime. The collegial atmosphere was very professional and the discussion was very balanced and constructive, with everyone's input considered and valued.

Some of students' common misconceptions were revealed during the observation of the lesson and the analysis of their work. Many were related to graphing such as: confusion with zero where does it go on the graph or whether to use a line graph vs bar graph, Use of arrows on the end of lines, How do we label the y-axis? The algebra students had a little trouble graphing due to decimals and so downloads were .50 for the last plan instead of .40.

Based on some of these misconceptions and novel ideas that came up during the lesson, teachers generated a list of modifications to enhance the lesson. One was to promote math talk in the classroom, teachers created discussion cards that could help guide small group discussion (i.e. You have $50 which plan? You are downloading x amount of songs, which plan would you choose? At which amount do all three plans meet? (system of equations). In addition, teachers thought paired work would also promote talking. They revisited the original task and discussed how E-musical plan which was $4/month with $.40 per download might be changed to a more compatible "mental math" friendly numbers to $4/month with $.50 per download. To promote algebraic connections throughout the year, teachers would revisit this lesson when using graphing calculators and simultaneous equations.

FINAL THOUGHTS

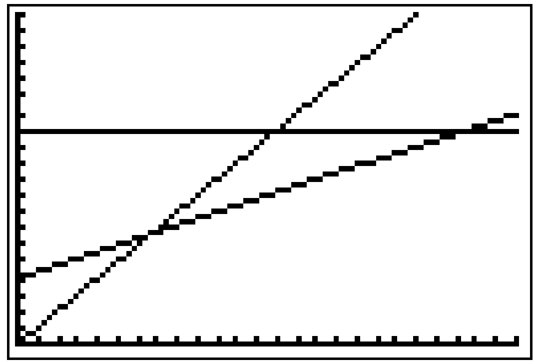
For the Math 8 students, completion of the chart (cost per plan based on the number of downloads) proved most helpful in their analysis of which plan was best under various circumstance. Determination of the linear functions ($c = d$, $c = .4d + 4$, and $c = 13$) and sketching their corresponding graphs was most effective for the algebra students in their analysis. Students were able to see that for a given amount of downloads each plan varied in whether it was the most cost effective. They could also see the 'break even' point and realized that at certain points two plans were both equally cost effective. The graph of the plans really helped them tie all of this information together and made it visual for them so they could better understand the material. There was a question asking the students which representation helped them decide which plan is best - the verbal description, table, or graph. Several students from the Math 8 class said the table, but some said the graph. It was noticeable that the algebra students were using algebraic thinking when they created the formula or lines from the description of each plan. The math 8 students were using algebraic thinking as they used the patterns to make up their tables. Both sets of students used algebraic thinking when using the tools they had developed to answer the questions. The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes that representations serve as tool for communicating, justifying, sense making and connecting ideas by stating, "Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others. They allow students to recognize connections among related concepts and apply mathematics to realistic problems" (p. 67). In this Lesson Study, teachers learned the importance of representational fluency on building rules to represent linear functions as they collaborated on the lesson planning, teaching and reflecting process.

Multiple Representations

Verbal expressions were created together as a class.

Table

Number of Downloads	i-sound	Rhaps	e-musical
0	0	13	4
1	1	13	4.40
2	2	13	4.80
3	3	13	5.20
4	4	13	5.60
5	5	13	6
6	6	13	6.40
7	7	13	6.80
8	8	13	7.20
9	9	13	7.60
10	10	13	8
11	11	13	8.40
12	12	13	8.80
13	13	13	9.20
14	14	13	9.60
15	15	13	10
16	16	13	10.40
17	17	13	10.80
18	18	13	11.20
19	19	13	11.60
20	20	13	12
21	21	13	12.40
22	22	13	12.80
23	23	13	13.20
24	24	13	13.60
25	25	13	14



- **Algebraic Formulas**

$y = x$ (i-Sound)

$y = 13$ (Rhaps)

$y = 4 + .4x$ (e-musical)

Figure 1. Planning for different representations

Name: ______________________
Date: ____________ **Period:** _______

MP3 Plan Memberships
You have decided to buy a plan for your mp3 purchases and have three choices of plans. The three plans are outlined below.

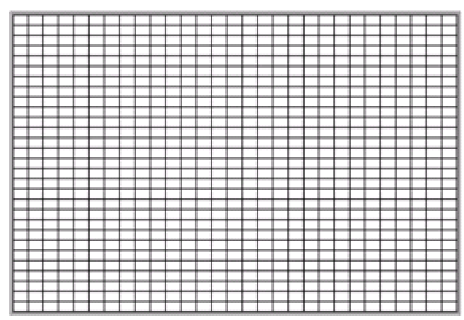
i-Sound	**Rhaps**	**e-musical**
$1 for each download	$13 a month unlimited downloads	$4 monthly membership fee and $0.40 per download

1. Which plan would you buy? Why?

2. Write an equation to represent the cost, *c*, for each plan based on the number of downloads, *d*, you purchase.
   a. i – Sound: ______________________________
   b. Rhaps: ______________________________
   c. e – musical: ______________________________

Complete the table and graph the cost of each plan.

Number of Downloads	i-sound	Rhaps	e-musical
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			



Part D. **<u>Cost effectiveness and adequacy of resources</u>**

- Provide detailed budget narrative explaining the expenditure of funds to the program's objectives.

  Part D: Cost Effectiveness and Adequacy of Resources

The following chart lists the budget codes, amounts awarded, expenditures to date, and balances remaining. A narrative follows the chart.

Budget Code	Amount Awarded	Expenditures to Date	Balance Remaining
Personal Service and Employee Benefits (1100)	$24,090.00	$23,994.20	$95.80
Contractual Services: Consultants and Travel Employee Benefits (1200)	$13,850.00	$13,850.00	$571.22
Supplies and Materials (1300)	$9,182.00	$8,372.18	$809.82
Transfer Payments for Participants or Substitutes (1400)	$22,108.00	$21,096.00	$1,012.00
Continuous Services (1500)	$0.00		
Indirect Cost Recovery	$3,770.00	$3,635.60	$134.40
TOTAL	$73,000.00	$70,376.76	$2,623.24

Personal Service and Employee Benefits (1100):
The principal investigator for the grant, Dr. Jennifer Suh, as well as one CO-PI, Dr. Seshaiyer, were paid for their work during the Summer Institute and the follow-up lesson study cycle. The graduate research assistant was paid for his work on behalf of the Summer Institute. The other CO-PI, Dr. Hjalmarson, was paid for working on the assessment, Teacher Content Knowledge of Algebra Assessment.

Contractual Services: Consultants and Travel Employee Benefits (1200):
Two consultants for the Summer Institute, Spencer Jamieson and Patti Freeman, have been paid for their work. The evaluator, Dr. Bolyard, completed her evaluation and was paid.
We used $2,000 of this budget (from Subsistence/Per Diem for project directors) line item to be used for Conference Fees instead, because the NCTM Registration fees were higher than originally anticipated. We anticipate using only $1,000 of the $3,000 allotted for Subsistence/per Diem for project directors. Thus, the other $2,000 stayed in the same Budget Code, just used for a different purpose which was already part of the project. That increases the amount available for Conference Fees to $8,250 (which includes $6,250 already approved.) Conference registration reimbursements were paid.

Supplies and Materials (1300):

$9,000 was budgeted and $8,372.18 was spent on supplies and materials. The Principal Investigator and Co-PI's made purchases for the participants for materials for the professional development.

Transfer Payments for Participants or Substitutes (1400):
All transfer payment for participants and substitutes were paid in the amount of $21,096.00 from the original requested amount of $22,108.00 with a remaining balance of $1012.

Stipends were paid. We formally requested that the remaining $1,000 in unused stipends be moved to Budget Code 1200 to increase the amount available for Conference Fees. (See previous explanation under that section.) $12,108 was approved for tuition for the participants in this project. Due to cost increases at George Mason University, the actual amount was $12,296.00. Thus, we exceeded the budgeted amount by $188.00 but was able to cover the difference in the same budget category.

Indirect Cost Recovery:
Indirect Cost Recovery totaled $$3,635.60 from the original $3770.00 with a remaining balance of $134.40.

In sum, the project ACT NOW was able to provide a successful professional development opportunity due to SCHEV's support. We are grateful for your support.

Appendices

APPENDIX A

ACT NOW ROSTER

Last Name	First Name	School	Grade
Webb	Andrew	Belvedere	5
Rosenthal	Joanne	Blackwell	7
Bell	Melissa	Bren Mar Park	5
Orantes	Cristina	Brookfield	SUM
Valonis	Julia	Bull Run	6
Lopez	Ana	Burke Center (SPEC)	7
Miles	Joshua	Cameron	5
Gadley	Jamie	Eagle View	3
McAfee	Pauline	Forest Edge(SPEC)	5
Markov	Sarah	Fort Belvoir	3
Miller	Karen	Fort Belvoir	5
			5
Klarevas	Steven	Franklin	8
Johnson	Andrea	Frost	7
Ruel	Gerard	Groveton	5
ClarkAshton	Susie	Gunston	4
Pruitt	Cornelia	Hayfield	7
Wilson	Lora	Hayfield	7
Robinson	Zari	Herndon	5
Lowe	Georgianne	Herndon	6
Sweetser	Lindsay	Hollin Meadows(math resource)	4
Corpus	Phillip	Holmes	7
Florio	Angela	Irving	8
Cardon	Aimee	Key	8
Turel	Diane	Lake Braddock	7
Hornfeck	Robert	Lanier	7
Postlethwait	Heather	Longfellow	7
McGuinness	Denise	Lorton Station	3
Walker	Cynthia	Mt. Vernon Woods	3
Harris	Gregory	Mt. Vernon Woods	6
Sampson	Elizabeth	Mt. Vernon Woods	SUM
Goodheart	Pat	Oak View	4
Baldwin	Beth	Poplar Tree	5
Ziegler	Crista	Poplar Tree	6
Hickman	Patricia	Robinson (SPEC)	7
Monroy	Andrea	Rose Hill	6
Reinecker	Donald	Sleepy Hollow	SUM
Stevens	Angela	Timber Lane	SUM
Wieser	Ellen	Westlawn	5
Rutecki	Lucy	Woodlawn	6
Hawkins	Tia	Woodlawn	SUM

State Council of Higher Education for Virginia

No Child Left Behind (NCLB) Act
Improving Teacher Quality State Grants
Title II, Part A, Subpart 3
2008-2009 Project Director Summary of Participants

1. Professional level upon entrance into program:
   a. Teacher, pre-service ______
   b. Teacher, in-service 37
   c. Administrator (all categories) ______
   d. Paraprofessional ______
   e. Other (Explain) ______
   TOTAL: 37

2. Highest degree earned:
   a. Baccalaureate 18
   b. Masters 19
   c. Doctorate ______
   d. Other ______
   TOTAL: 37

3. Licensure status:
   a. Certified 34
   b. Not Certified 2
   c. Provisional 1
   d. Emergency ______
   TOTAL: 37

4. Years of experience: TOTAL: 440 AVG: 12 years

5. Number of hours beyond Baccalaureate degree: TOTAL: 1157 AVG: 33 hrs

6. Where do you teach?
   a. Public school division 37
   *(**List ALL division codes)* 029
   b. Private school (Specify name) ______
   c. Not currently teaching ______
   d. Preparing to teach
   TOTAL: 37

7. Total number of pre-service teachers served? 0
   By school level:
   Elementary _____
   Middle School _____
   High School _____

8. Total number of paraprofessionals served? 0
   By school level:

Elementary _____
Middle School _____
High School _____

9. Total number of in-service teachers served? 37
By school level:
Elementary 23
Middle School 12
High School 0
Not specified 2

10. Total number of administrators served? 0
By school level:
Elementary _____
Middle School _____
High School _____

11. Endorsement area:

a.	English	5
b.	Mathematics	15
c.	Reading or Language Arts	3
d.	Science	3
e.	Foreign Languages	0
f.	Civics and Government	1
g.	Economics	1
h.	Arts	1
i.	History	6
j.	Geography	3
k.	Other	25

12. This ITQ project core academic area (s) is/are:

a.	English	
(b.)	Mathematics	37
c.	Reading or Language Arts	
d.	Science	
e.	Foreign Languages	
f.	Civics and Government	
g.	Economics	
h.	Arts	
i.	History	
j.	Geography	
k.	Other	

13. The percentage of increase in core content areas:

a.	English	
b.	Mathematics	
c.	Reading or Language Arts	
d.	Science	
e.	Foreign Languages	
f.	Civics and Government	
g.	Economics	
h.	Arts	
i.	History	
j.	Geography	
k.	Other	

14. Please list all the subjects taught: Language Arts, Math, Science, Social Studies, Health, Algebra 1, Geometry

15. Number of school districts served? 1

16. Number of high-need K-12 school districts served? 1

17. Number of schools served? 34

18. Number of K-12 students impacted? app. 4,100

19. Number of higher education faculty working with a grant? 3

20. Number of pre-service (2 years of classroom experience) teachers impacted? 0

21. Indicate the purpose for participating in this ITQ project activity: (Total each response)
   a. Professional development 36
   b. Retraining for a teacher not currently endorsed in this subject. If retraining, did this activity allow you to complete necessary certification?
      Yes ____ No 1
   c. Other (Explain) ____
      TOTAL: 37

22. Indicate race/ethnicity:

a. Black, non-Hispanic	6	
b. White, non-Hispanic	25	
d. Hispanic	4	
e. Asian/Pacific Islander	1	
f. American Indian/Alaskan Native	0	
g. Other (indicate)	1	White and Asian/Pacific Islander
TOTAL:	37	

23. Indicate gender:

a. Male	7
b. Female	30
TOTAL:	37

24. Did this activity take into account the needs of historically underrepresented and underserved students/groups?

c. Yes	28
d. No	1
e. Not Sure	8
TOTAL:	37

Table 1. Fairfax County Public School
NEEDS ASSESSMENT DATA

Grade 3 Mathematics SOL:	% of Students Passing or Failing					
	2004 2005		2005 2006		2006 2007	
	Passed	Failed	Passed	Fail	Passed	Fail
All Students	89	11	92	8	90	10
Black	75	25	89	11	78	22
Hispanic	78	22	83	17	80	20
Students with Disabilities	75	25	79	21	75	25
Limited English Proficient	82	18	85	15	83	17

Grade 4 Mathematics SOL:	% of Students Passing or Failing					
	2004 2005		2005 2006		2006 2007	
	Passed	Failed	Passed	Fail	Passed	Fail
All Students			79	21	83	17
Black			57	43	66	34
Hispanic			58	42	66	34
Students with Disabilities			56	44	62	38
Limited English Proficient			62	38	69	31

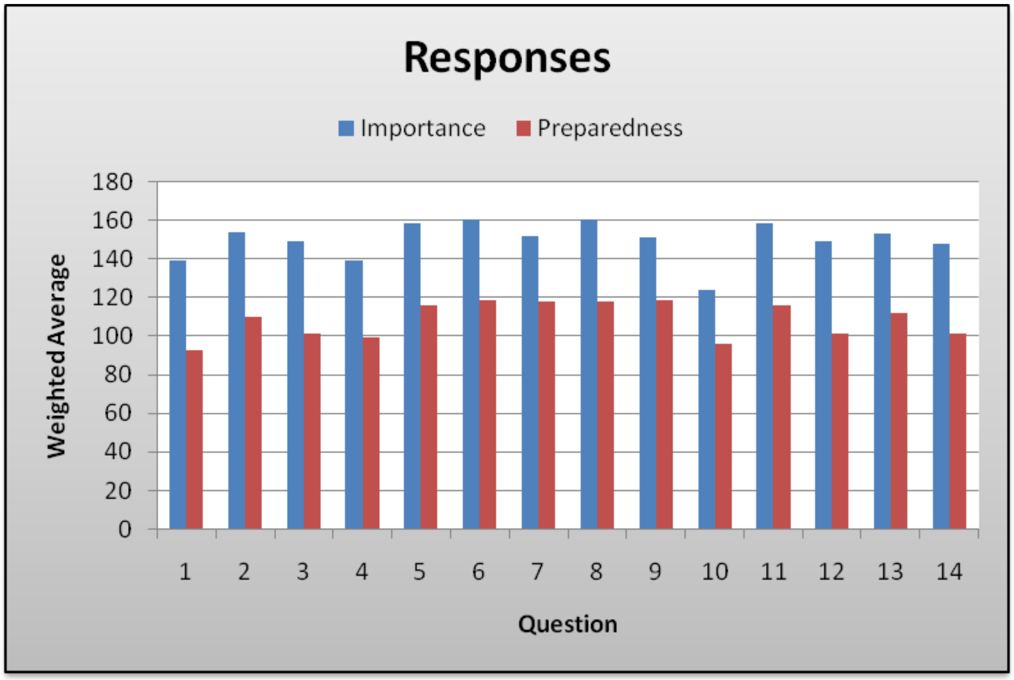
Grade 5 Mathematics SOL:	% of Students Passing or Failing					
	2004 2005		2005 2006		2006 2007	
	Passed	Failed	Passed	Fail	Passed	Fail
All Students	84	16	83	17	85	15
Black	65	35	67	33	73	27
Hispanic	69	31	68	32	72	28
Students with Disabilities	58	42	62	38	67	33
Limited English Proficient	73	27	72	28	76	24

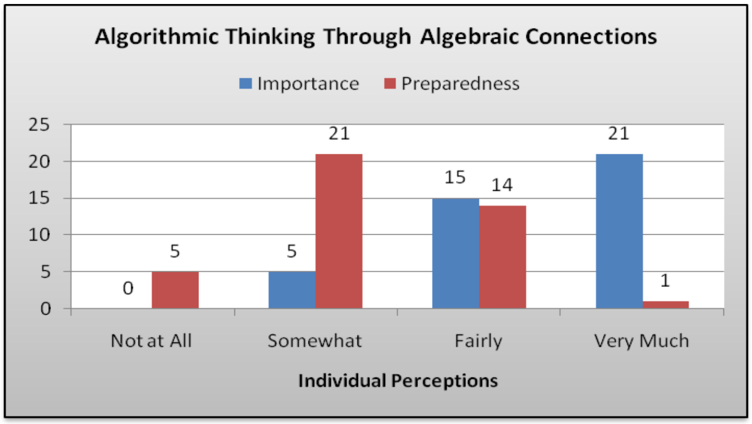
Grade 6 Mathematics SOL:	% of Students Passing or Failing					
	2004 2005		2005 2006		2006 2007	
	Passed	Failed	Passed	Fail	Passed	Fail
All Students			69	31	71	29
Black			43	57	48	52
Hispanic			46	54	49	51
Students with Disabilities			39	61	45	55
Limited English Proficient			49	51	53	47

Grade 7 Mathematics SOL:	**% of Students Passing or Failing**					
	2004 2005		**2005 2006**		**2006 2007**	
	Passed	**Failed**	**Passed**	**Fail**	**Passed**	**Fail**
All Students			60	40	66	34
Black			35	65	39	61
Hispanic			33	67	40	60
Students with Disabilities			28	72	34	66
Limited English Proficient			34	66	44	56

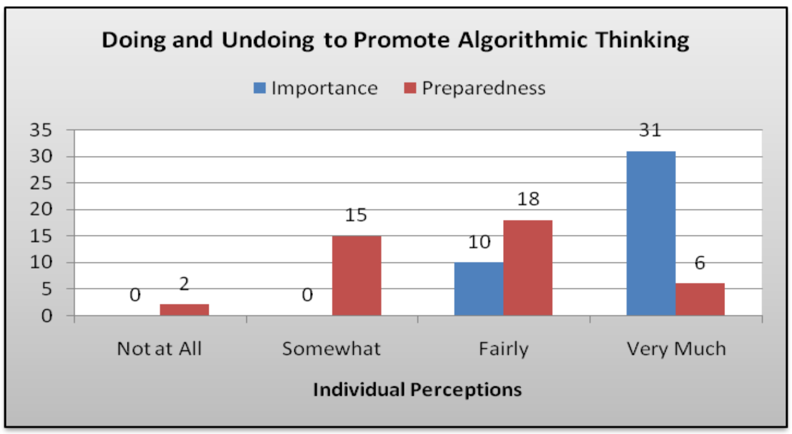
APPENDIX B**. Survey to Pre and post assessment**
Preparedness for Math Instructional Practices
(Administered pre survey August 3, 2008 post survey Dec 3, 2008)

ACT NOW DATA

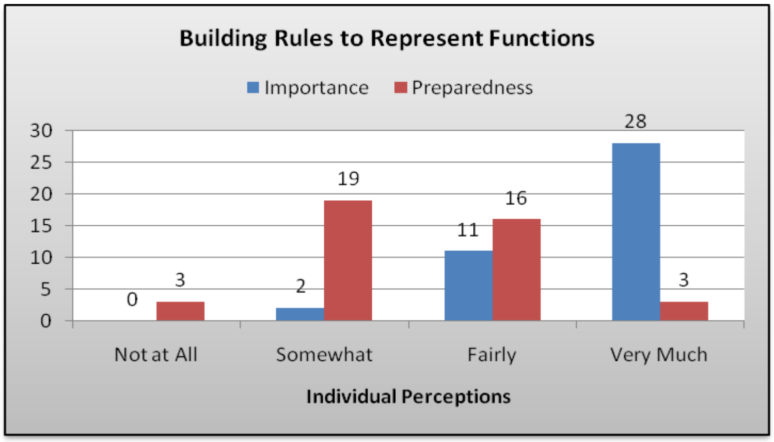




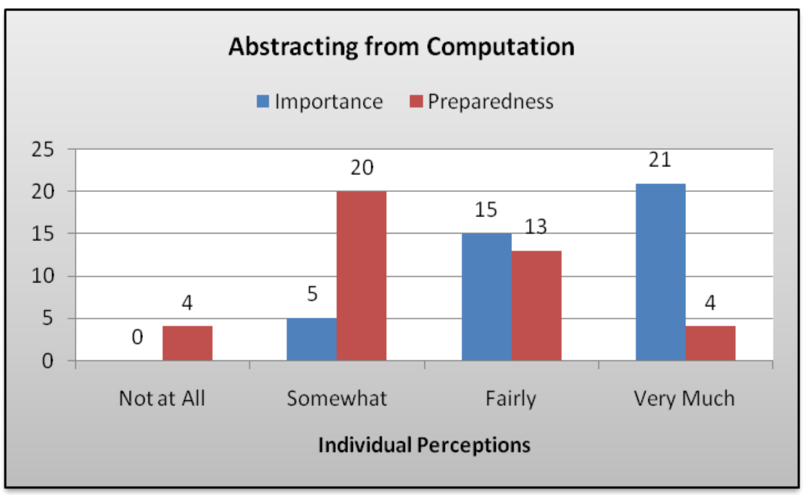
1. Developing students' algorithmic thinking through algebraic connections			15. Preparedness	
Not Important	0	0%	5	12%
Somewhat Important	5	12%	21	51%
Fairly Important	15	37%	14	34%
Very Important	21	51%	1	2%
Total	41	100%	41	100%
Weighted Average	139		93	



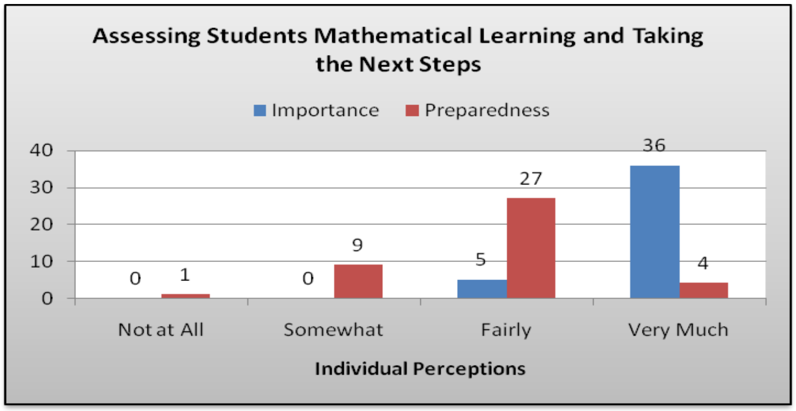
2. Using doing and undoing to promote algebraic thinking			16. Preparedness	
Not Important	0	0%	2	5%
Somewhat Important	0	0%	15	37%
Fairly Important	10	24%	18	44%
Very Important	31	76%	6	15%
Total	41	100%	41	100%
Weighted Average	154		110	



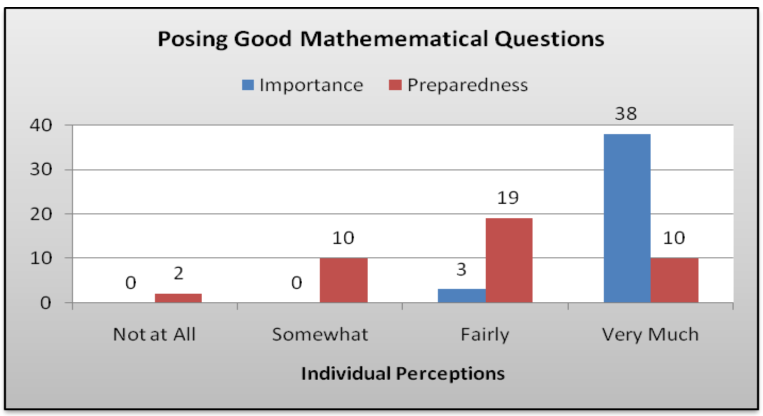
3. Building rules to represent functions			17. Preparedness	
Not Important	0	0%	3	7%
Somewhat Important	2	5%	19	46%
Fairly Important	11	27%	16	39%
Very Important	28	68%	3	7%
Total	41	100%	41	100%
Weighted Average	149		101	



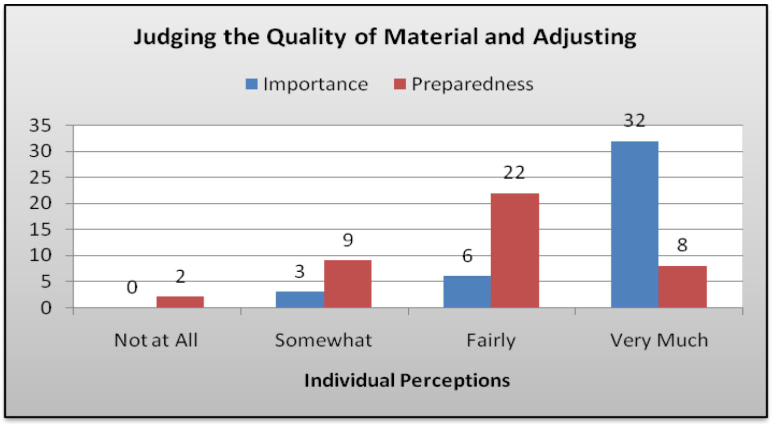
4. Abstracting from computation			18. Preparedness	
Not Important	0	0%	4	10%
Somewhat Important	5	12%	20	49%
Fairly Important	15	37%	13	32%
Very Important	21	51%	4	10%
Total	41	100%	41	100%
Weighted Average	139		99	



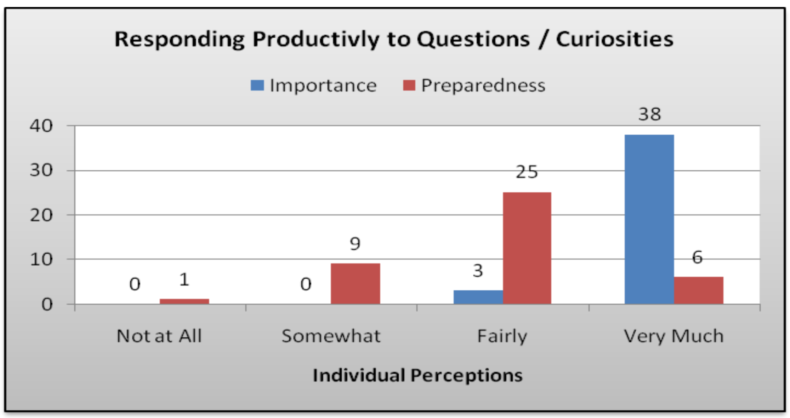
5. Assessing students' mathematical learning and taking the next steps.			19. Preparedness	
Not Important	0	0%	1	2%
Somewhat Important	0	0%	9	22%
Fairly Important	5	12%	27	66%
Very Important	36	88%	4	10%
Total	41	100%	41	100%
Weighted Average	159		116	



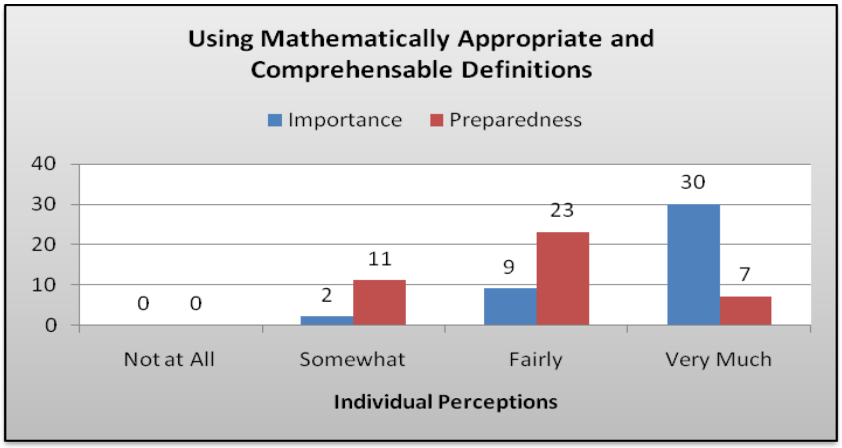
6. Posing good mathematical questions and problems that are productive for students' learning.			20. Preparedness	
Not Important	0	0%	2	5%
Somewhat Important	0	0%	10	24%
Fairly Important	3	7%	19	46%
Very Important	38	93%	10	24%
Total	41	100%	41	100%
Weighted Average	161		119	



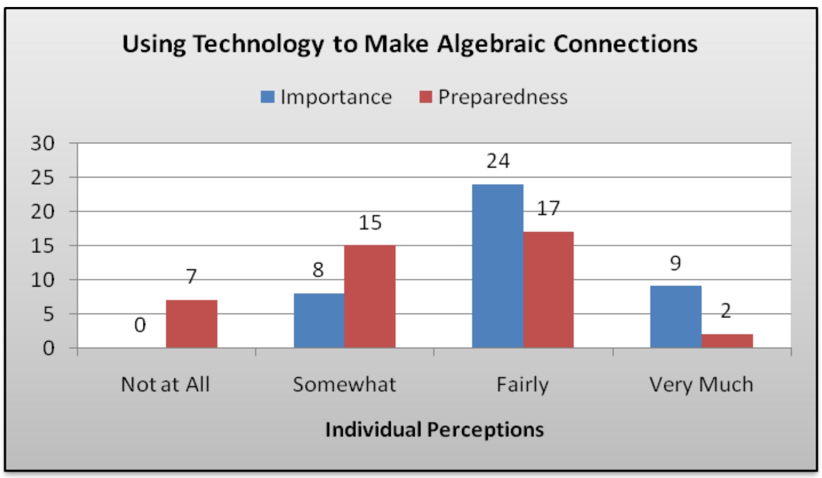
7. Making judgments about the mathematical quality of instructional materials and modify as necessary.			21. Preparedness	
Not Important	0	0%	2	5%
Somewhat Important	3	7%	9	22%
Fairly Important	6	15%	22	54%
Very Important	32	78%	8	20%
Total	41	100%	41	100%
Weighted Average	152		118	



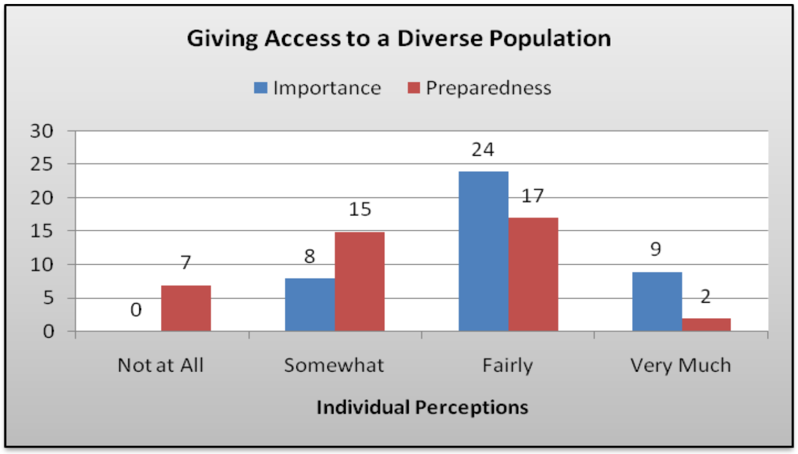
8. Responding productively to students' mathematical questions and curiosities.			22. Preparedness	
Not Important	0	0%	1	2%
Somewhat Important	0	0%	9	22%
Fairly Important	3	7%	25	61%
Very Important	38	93%	6	15%
Total	41	100%	41	100%
Weighted Average	161		118	



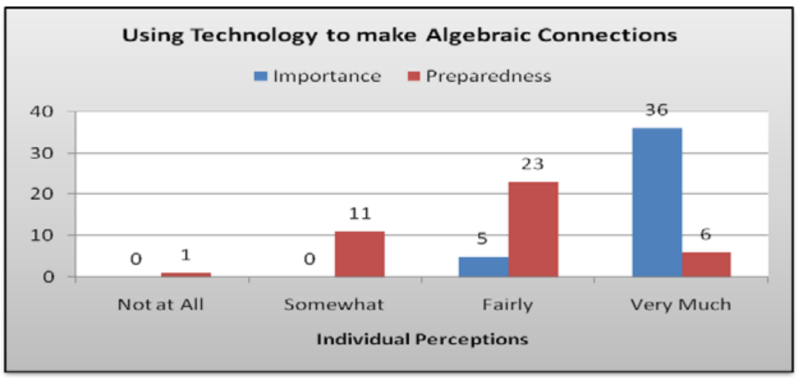
9. Using mathematically appropriate and comprehensible definitions with students.			23. Preparedness	
Not Important	0	0%	0	0%
Somewhat Important	2	5%	11	27%
Fairly Important	9	22%	23	56%
Very Important	30	73%	7	17%
Total	41	100%	41	100%
Weighted Average	151		119	



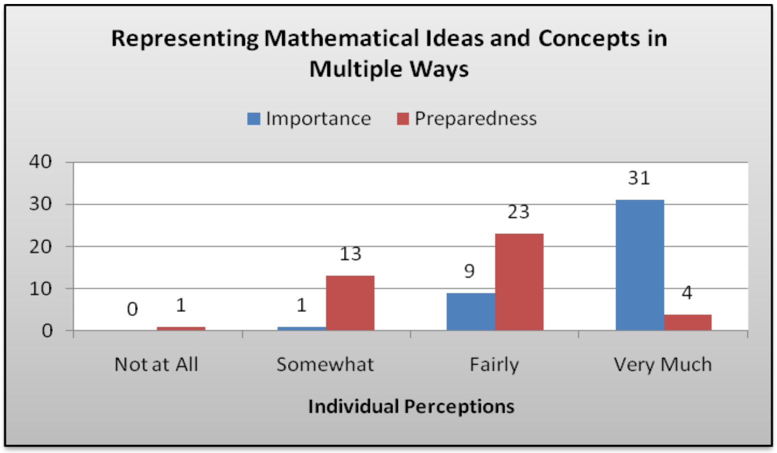
10. Using technology with students to make algebraic connections			24. Preparedness	
Not Important	0	0%	7	17%
Somewhat Important	8	20%	15	37%
Fairly Important	24	59%	17	41%
Very Important	9	22%	2	5%
Total	41	100%	41	100%
Weighted Average	124		96	



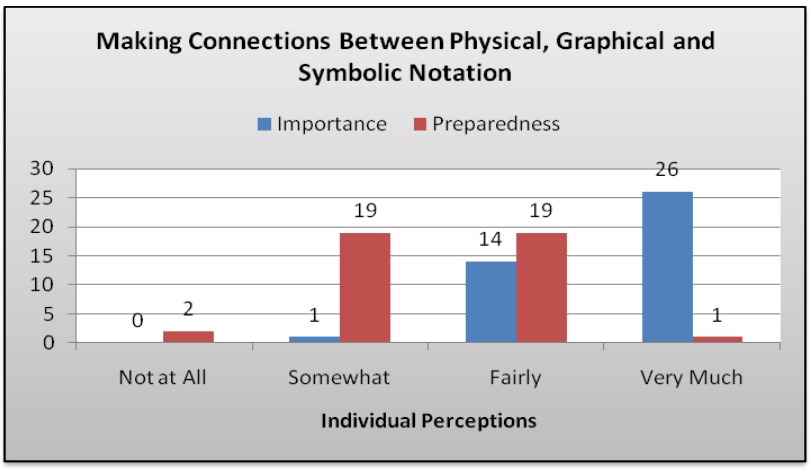
11. Giving access for mathematical learning to all members of a diverse population.			25. Preparedness	
Not Important	0	0%	1	2%
Somewhat Important	0	0%	11	27%
Fairly Important	5	12%	23	56%
Very Important	36	88%	6	15%
Total	41	100%	41	100%
Weighted Average	159		116	



12. Identifying and making algebraic connections among various mathematical topics.			26. Preparedness	
Not Important	0	0%	3	8%
Somewhat Important	1	2%	14	35%
Fairly Important	13	32%	22	55%
Very Important	27	66%	1	2%
Total	41	100%	40	100%
Weighted Average	149		101	

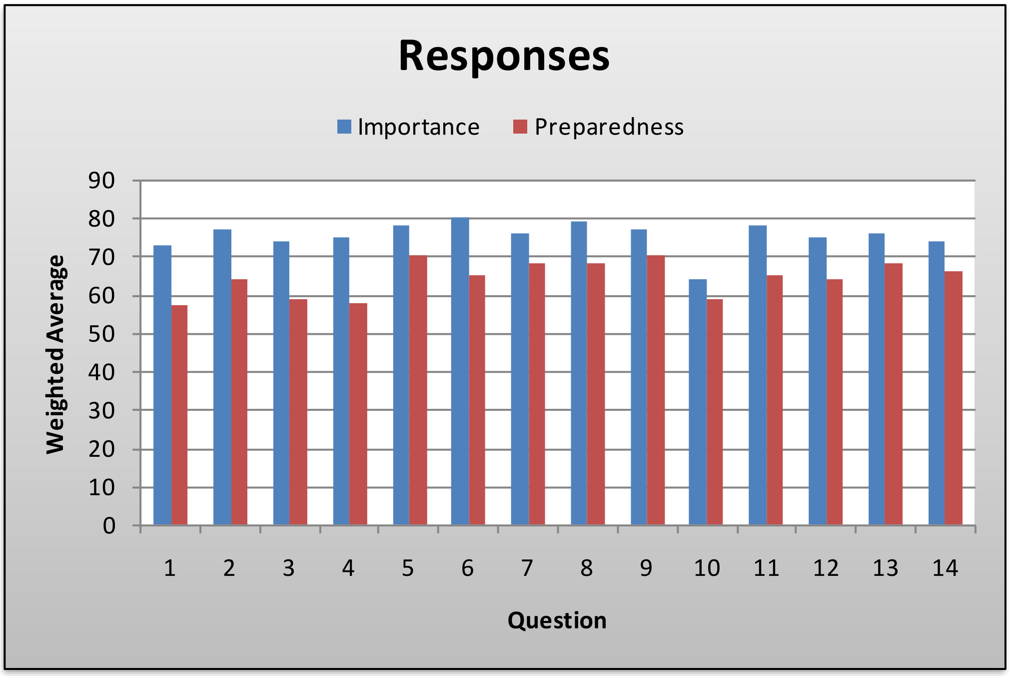


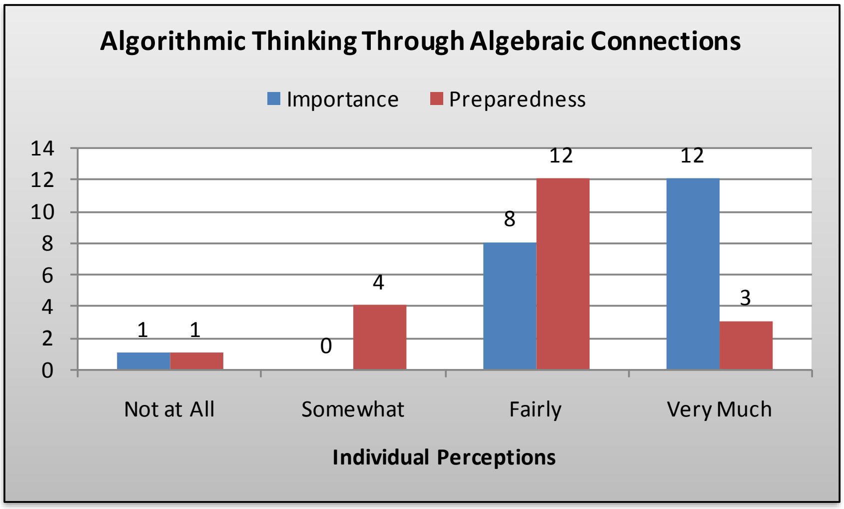
13. Representing mathematical ideas and concepts carefully in multiple ways.			27. Preparedness	
Not Important	0	0%	1	2%
Somewhat Important	1	2%	13	32%
Fairly Important	9	22%	23	56%
Very Important	31	76%	4	10%
Total	41	100%	41	100%
Weighted Average	153		112	



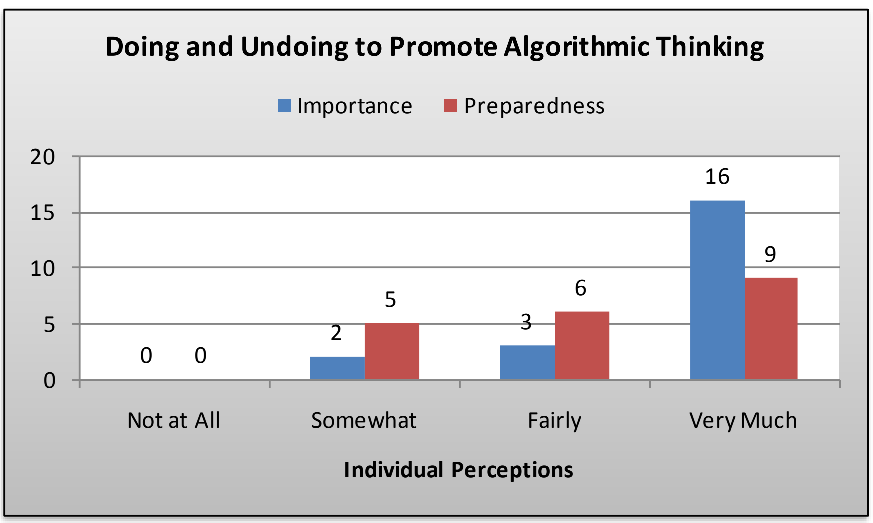
14. Making connections between physical, graphical models and symbolic notation.			28. Preparedness	
Not Important	0	0%	2	5%
Somewhat Important	1	2%	19	46%
Fairly Important	14	34%	19	46%
Very Important	26	63%	1	2%
Total	41	100%	41	100%
Weighted Average	148		101	

ACT NOW – POST SURVEY 2

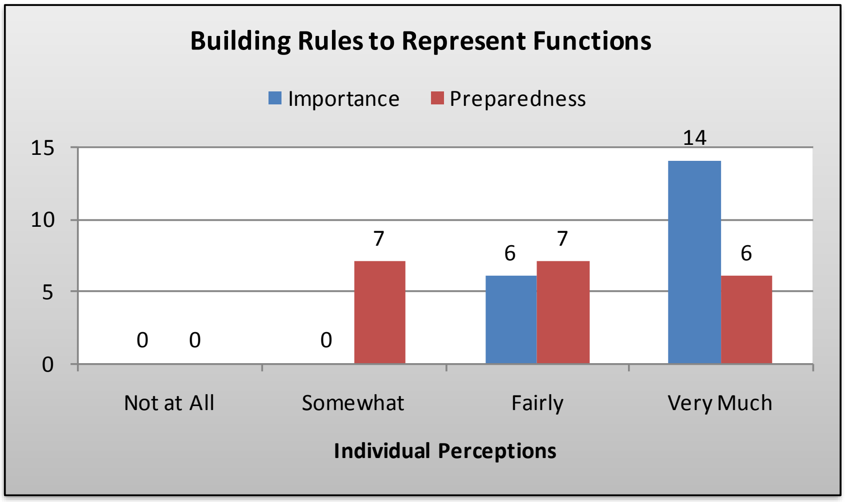




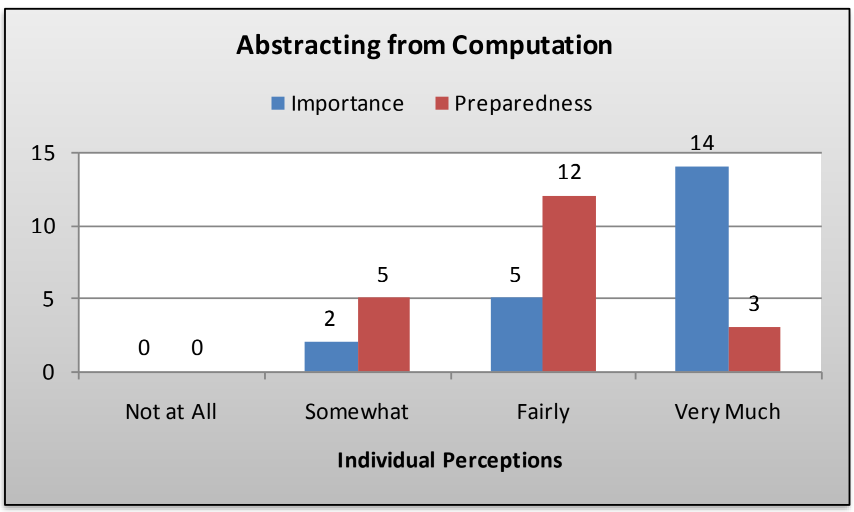
1. Developing students' algorithmic thinking through algebraic connections			15. Preparedness	
Not Important	1	0%	1	12%
Somewhat Important	0	12%	4	51%
Fairly Important	8	37%	12	34%
Very Important	12	51%	3	2%
Total	21	100%	20	100%
Weighted Average	73		57	



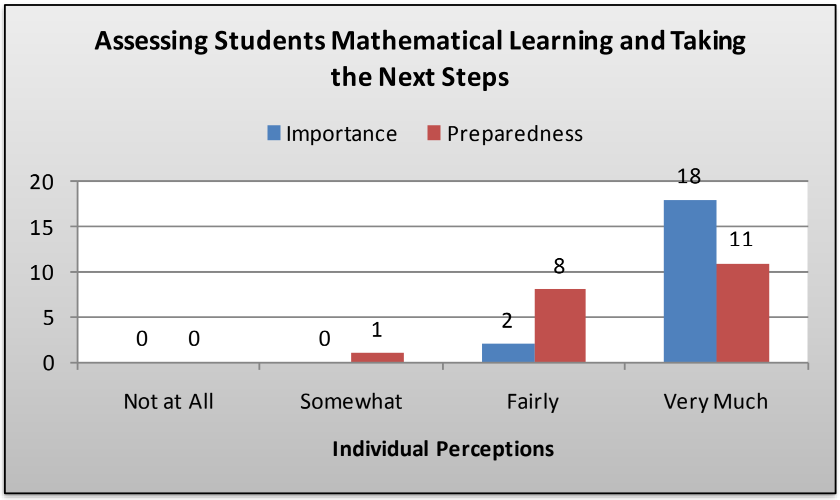
2. Using doing and undoing to promote algebraic thinking			16. Preparedness	
Not Important	0	0%	0	5%
Somewhat Important	2	0%	5	37%
Fairly Important	3	24%	6	44%
Very Important	16	76%	9	15%
Total	21	100%	20	100%
Weighted Average	77		64	



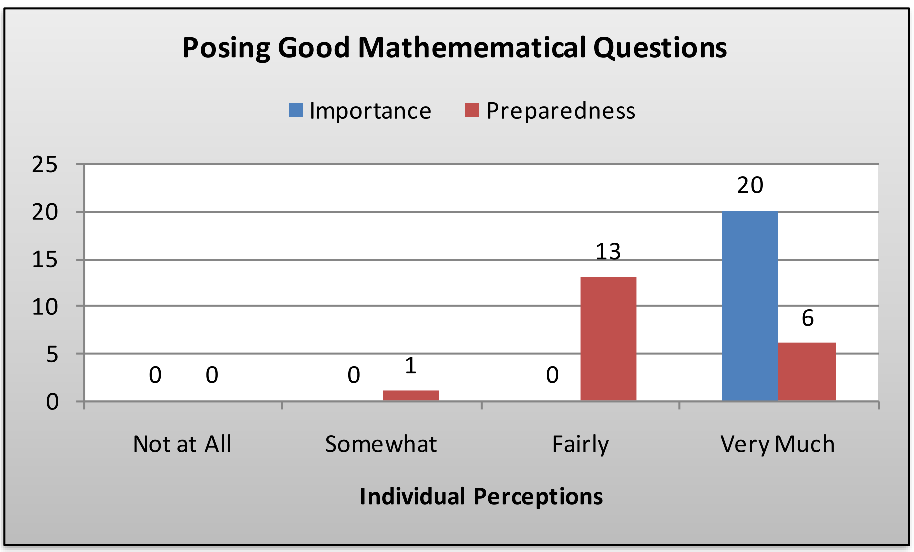
3. Building rules to represent functions			17. Preparedness	
Not Important	0	0%	0	7%
Somewhat Important	0	5%	7	46%
Fairly Important	6	27%	7	39%
Very Important	14	68%	6	7%
Total	20	100%	20	100%
Weighted Average	74		59	



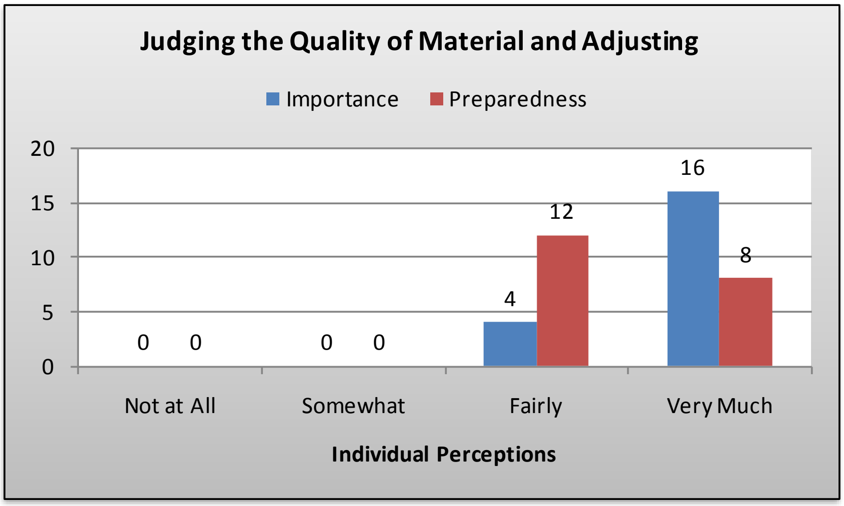
4. Abstracting from computation			18. Preparedness	
Not Important	0	0%	0	10%
Somewhat Important	2	12%	5	49%
Fairly Important	5	37%	12	32%
Very Important	14	51%	3	10%
Total	21	100%	20	100%
Weighted Average	75		58	



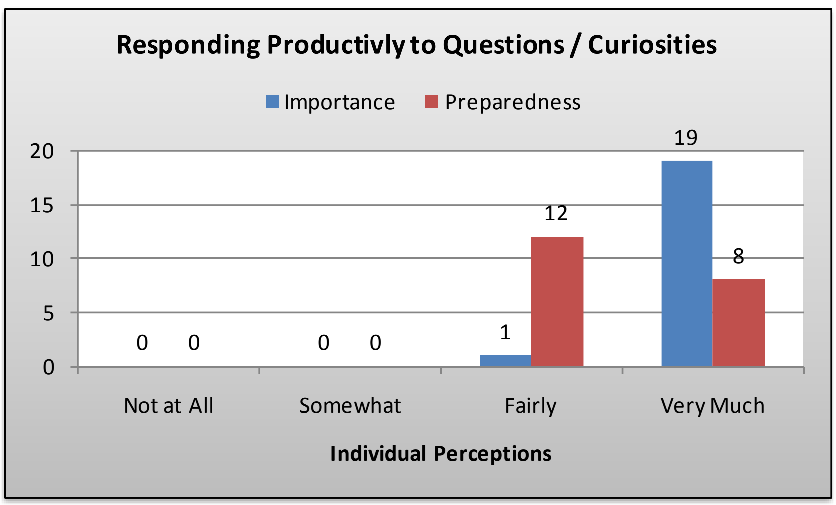
5. Assessing students' mathematical learning and taking the next steps.			19. Preparedness	
Not Important	0	0%	0	2%
Somewhat Important	0	0%	1	22%
Fairly Important	2	12%	8	66%
Very Important	18	88%	11	10%
Total	20	100%	20	100%
Weighted Average	78		70	



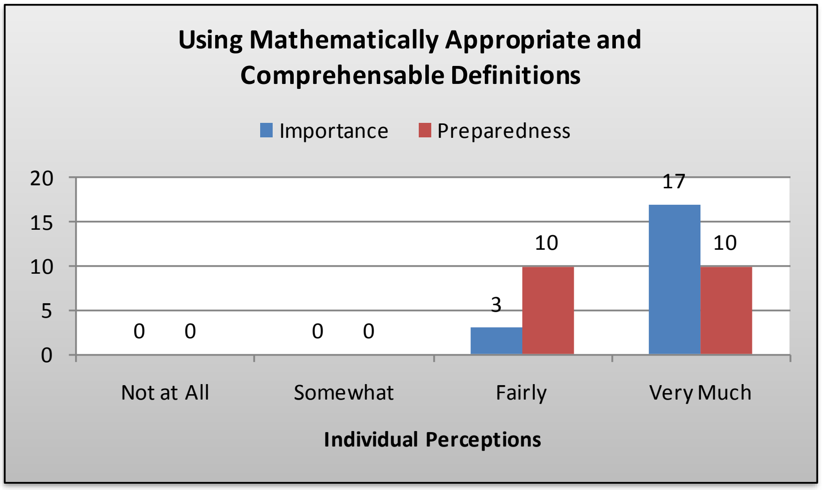
6. Posing good mathematical questions and problems that are productive for students' learning.			20. Preparedness	
Not Important	0	0%	0	5%
Somewhat Important	0	0%	1	24%
Fairly Important	0	7%	13	46%
Very Important	20	93%	6	24%
Total	20	100%	20	100%
Weighted Average	80		65	



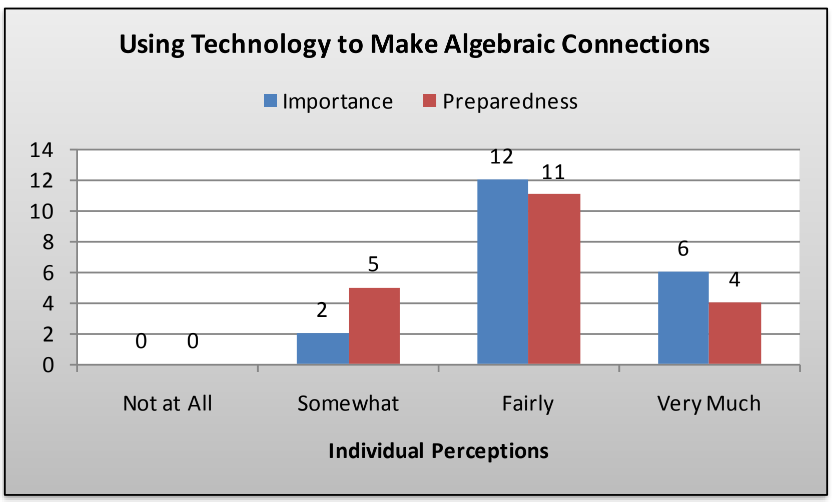
7. Making judgments about the mathematical quality of instructional materials and modify as necessary.			21. Preparedness	
Not Important	0	0%	0	5%
Somewhat Important	0	7%	0	22%
Fairly Important	4	15%	12	54%
Very Important	16	78%	8	20%
Total	20	100%	20	100%
Weighted Average	76		68	



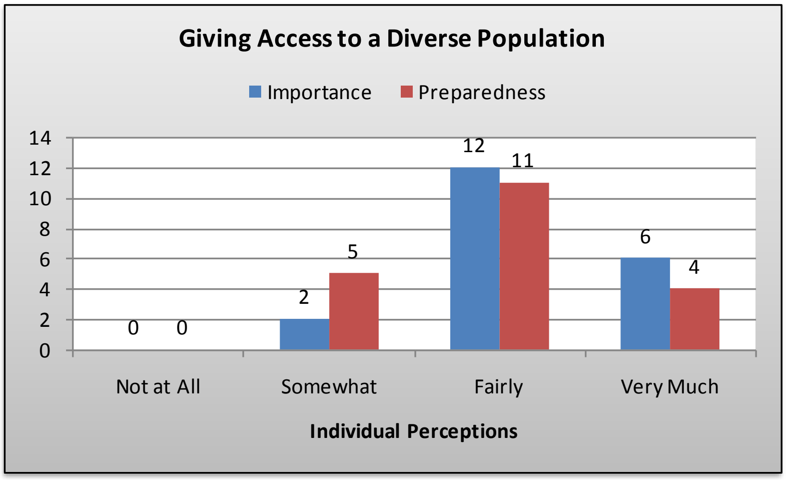
8. Responding productively to students' mathematical questions and curiosities.			22. Preparedness	
Not Important	0	0%	0	2%
Somewhat Important	0	0%	0	22%
Fairly Important	1	7%	12	61%
Very Important	19	93%	8	15%
Total	20	100%	20	100%
Weighted Average	79		68	



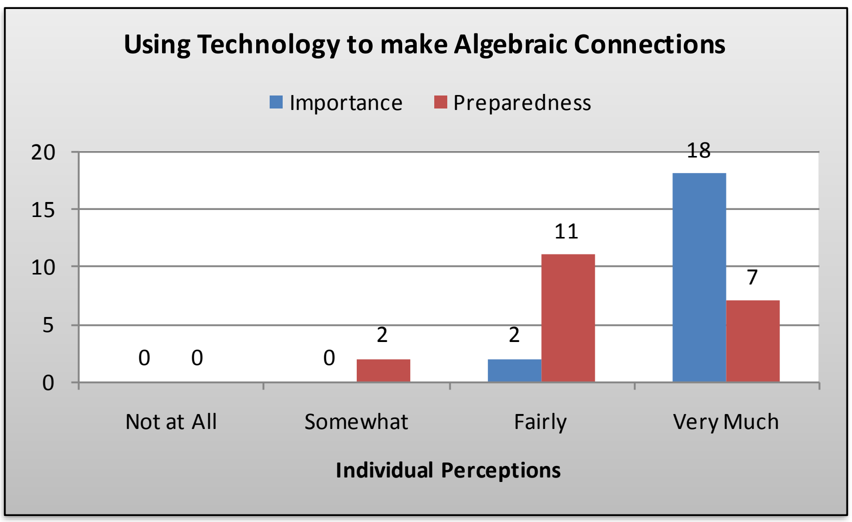
9. Using mathematically appropriate and comprehensible definitions with students.			23. Preparedness	
Not Important	0	0%	0	0%
Somewhat Important	0	5%	0	27%
Fairly Important	3	22%	10	56%
Very Important	17	73%	10	17%
Total	20	100%	20	100%
Weighted Average	77		70	



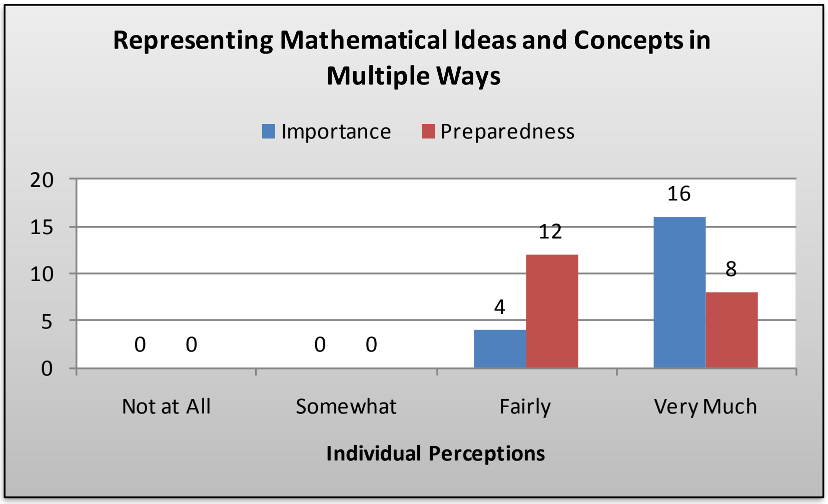
10. Using technology with students to make algebraic connections			24. Preparedness	
Not Important	0	0%	0	17%
Somewhat Important	2	20%	5	37%
Fairly Important	12	59%	11	41%
Very Important	6	22%	4	5%
Total	20	100%	20	100%
Weighted Average	64		59	



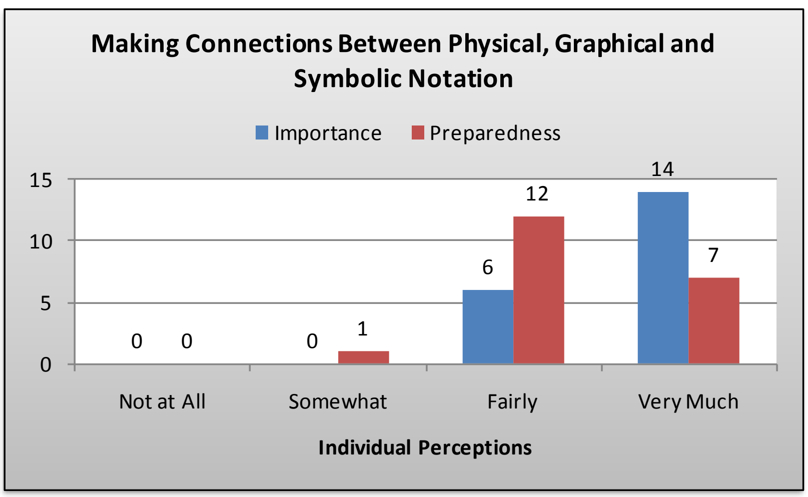
11. Giving access for mathematical learning to all members of a diverse population.			25. Preparedness	
Not Important	0	0%	0	2%
Somewhat Important	0	0%	2	27%
Fairly Important	2	12%	11	56%
Very Important	18	88%	7	15%
Total	20	100%	20	100%
Weighted Average	78		65	



12. Identifying and making algebraic connections among various mathematical topics.			26. Preparedness	
Not Important	0	0%	0	8%
Somewhat Important	0	2%	1	35%
Fairly Important	5	32%	14	55%
Very Important	15	66%	5	2%
Total	20	100%	20	100%
Weighted Average	75		64	



13. Representing mathematical ideas and concepts carefully in multiple ways.			27. Preparedness	
Not Important	0	0%	0	2%
Somewhat Important	0	2%	0	32%
Fairly Important	4	22%	12	56%
Very Important	16	76%	8	10%
Total	20	100%	20	100%
Weighted Average	76		68	



14. Making connections between physical, graphical models and symbolic notation.			28. Preparedness	
Not Important	0	0%	0	5%
Somewhat Important	0	2%	1	46%
Fairly Important	6	34%	12	46%
Very Important	14	63%	7	2%
Total	20	100%	20	100%
Weighted Average	74		66	

Table 3. Evidence that project fostered high quality professional development

EVALUATION OF THE SUMMER INSTITUTE:

2008 GMU-FCPS Act Now Program (Sponsored by SCHEV)
Final-Program Survey Results

Note: 39 teachers participated in the Final Program Survey.
Numerical Rating:
- **5** Strongly Agree
- **4** Agree
- **3** Neutral
- **2** Disagree
- **1** Strongly Disagree

Final- Program Survey	mean	St.dev
1. My overall perspective about algebraic thinking has been positively impacted by this program.	**4.74**	**.5**
2. The program has provided me with an intellectually rewarding experience.	**4.74**	**.5**
3. The program has greatly enhanced my interest in mathematical and algebraic thinking.	**4.62**	**.67**
4. The program has helped me to understand the algebraic habits of mind that one must develop to solve real-world problems.	**4.62**	**.59**
5. The group interaction with my lesson study group has been helpful.	**4.69**	**.47**
6. The technology and software tools presented have been helpful.	**4.54**	**.6**
7. The guest seminars and presentations helped me to learn about the broader impact of algebraic thinking.	**4.58**	**.6**
8. The assignments and reflection papers helped me to keep a check on my progress in the program.	**4.55**	**.65**
9. The program has improved my capability to think and learn independently.	**4.46**	**.72**
10. The instructors in the program were friendly, accessible and helpful.	**4.97**	**.16**
11. The program was well coordinated and the daily activities were well structured.	**4.41**	**.72**
12. The program has motivated me to see the vertical connections in algebraic thinking between K3-5 to K6-8.	**4.54**	**.72**
13. At the end of eight days, I feel I have gained an appreciation of the importance of algebraic thinking and the implications to middle grades students.	**4.67**	**.7**
14. Overall the program has been a successful and enjoyable learning experience for me.	**4.82**	**.45**

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Comments/feedback that you would like to share about the class below.
[Participant names, if recorded, are indicated in brackets.]
23 teachers offered comments.

- I really believe in the power of this type of course! The presenters/instructors along with classroom discourse was amazing! The time was short and sweet!! A key for teacher summertime learning! Thank you all again for your effort, time, care and love of mathematics learning and teaching. [Tia Hawkins]

- Thank you all for the valuable experiences. You are all master teachers who I will strive to emulate. I hope there will be a "2nd ACT" class! I'll be the 1st to sign up! (I'd even be willing to pay for it!) [Beth Baldwin]

- I need some time to process all the information, because the class has moved a little too fast for me, but it has been very helpful. [Ana Lopez]

- Teach and hold this course again and encourage ALL teachers to participate regardless of subject taught. [Phillip Corpuz]

- Thank you for this class. It has given me useful experiences that I can use in my classroom.

- This was a wonderful growth for me – I'm encouraged to take more of these grad classes – which I've been afraid of up till now.

- The instructors were all great and did their best to help all students. [Andrea Johnson]

- I enjoyed the many different perspectives from the teachers. I am very excited to begin the year with the ideas presented in this class! [Andrea Monroy]

- Overall, this was a great experience. It really helped me to get excited about math again. [Andrew Webb]

- I was hoping to see more technology that we could incorporate in our classrooms.

- THANK YOU for enjoyable learning experience. [Pat]

- 1) Work consecutively through problems – start with third grade problems and move up to 8th grade. 2) When presenting new technologies spend less time on them during class – I had already used many of them so I didn't need the time. 3) The pace of the class was very slow, spend less work time on each problem.

- Expected more 'tech stuff' and work on websites, but much preferred the paper and pencil and discussion of problems. Suggest: For each classwide problem, find an online manipulative or website that illustrates or helps you solve the problem. [Crista Ziegler]
  When this class started I thought "NO WAY!" Kudos you found the way! [Zarle]

- I would have liked more connections with technology and how to use it in class as described in the course description. Also, more feedback on my grade would have been nice. Overall, though, it was a very powerful experience. (Note, this person, next to item 8 on the survey, wrote: more feedback, please.)

- I really enjoyed the diversity of the background of the instructors – everyone had something to bring to the class. I hope there will be m ore courses like this one offered in the future. Thank you for putting so much time and energy into the course. [Lucy Rutecki]

- Continue to offer this course to other teachers. [Donald Reinecker]

- Great course! I'm excited about taking this model back to my third grade team and instructional coach. [Denise McGuinness]

- Loved this class/would like a Part II next summer. [Georgie Lowe]

- Thank you for helping us find our “Algebra Eyes.”
- What a dynamic team of instructors! They complemented each other so well. I would recommend this class to all.
- I really enjoyed having the opportunity to see how different people approach the same problems. It helped me see how my own students would work on a problem, in a way that would vary from the “right way”. [Angela Florio]

Table 4: Impact on Teachers' Instructional Practices and Students Learning

ALGEBRAIC CONNECTIONS AND TECHNOLOGY NOW 12/3/08

With your new "Algebra eyes and ears", how has it impacted your classroom teaching practices? Share some examples/strategies of algebraic connections and using technology in your classroom. Have you noticed any changes in YOUR STUDENTS' LEARNING?

	IMPACT ON TEACHERS' INSTRUCTIONAL PRACTICES	IMPACT ON STUDENT LEARNING
Lakeyta Smith	I approach my lessons and teaching with the emphasis on problem solving using the strategies to solve problems without giving the students the information. I feel that they own their information and their knowledge. I also have to give myself time to allow students to think for themselves without rushing.	I've noticed students want to do the work on their own and they do not want my help. Students feel the need to share their ways of solving the problems. They are excited when they see others solved it the same way or if they were the only one to solve it a certain way.
Angela Stevens	I began to look at problems differently. In my position I was able to influence teachers by reviewing problems and strategies that they could present to their students. More thought and preparation has improved teaching the lessons.	Students are looking for patterns. The use of vocabulary is improving. Students are beginning to make connections.
Susie Clark Ashton	Identifying more patterns Connecting number patterns, letters, and numbers	Students more engaged Students beginning to see more patterns
Andrew Webb	Began to implement more problem-based lessons to allow for all types of algebraic thinking. Started to teach basics of math to help relate to the structure of underlying ideas.	Less hesitant to fail and more willing to take risk. Students understand why they need to know the basics to have fun with the problem-based lesson.
Lora Wilson	Group warm ups with word problems. Discussion of vocabulary before we start the unit.	More willing to try the problems in different ways and discuss in groups. Students have a chance to pull up previously learned material.
Connie Pruitt	I do not give notes at beginning any longer. We discover solving solutions together by pulling on prior knowledge with doing and undoing. Then after lesson we summarize with notes for all to refer to. More "problem solving" oriented to help students think…think…think!	Whole class and small group participation is increased and more ownership is increased and more ownership in the meaning of the information – more connections are being made. By not limiting them to using a particular strategy, they are starting to risk being wrong…knowing they'll have opportunity to reflect and rework. Less problems yet more math is being completed.
Rob Hornfeck	Rather than presenting info for the students to memorize, I try to present a pattern leading up to the concept.	This year, students seem to be asking more specific questions, rather than saying "I don't get it." They're identifying specific items on where they're having trouble. Also, kids seem to be better at completing my thoughts; they're anticipating what will come up next.
Diane Turel	The biggest transformation for me was giving my students more freedom to solve problems in a manner that is most comfortable to them. That the methodology is not as important as the discovery process and learning from each other. For example, in my ARI classes we have application problems that are designed to focus on particular strategies of problem solving. I have been using these problems and allowing them the choice of how they solve the problems and during discussion of their answers, that they placed on the board, I highlighted the particular strategy that the problem was originally designed to represent.	I have seen an increase of confidence in my 8th grade students. They couldn't believe that they have the skills to not only solve the problems, but also be able to describe their process.
Aimee Cardon	Student-centered classroom more often. Give students more opportunities to reflect.	Responsibility for learning increases. Allows them to share different ideas with each other so they can see the way others are thinking.
Phillip Corpuz	Introduced the concept of variables earlier than the pacing guide. Provided scaffolding techniques to allow students to think about using and making tables. Introduced methods on how to recognize patterns by trial and error.	This concept was not difficult for my students and allowed them to build a rule to represent a function. Learning has improved as reflected in their ability to make and use tables. My students are no longer afraid to try something different.

Ana Lopez	My teaching practices have been transformed because I am more aware of the problems I present to the students. The intent is more profound and more meaningful. I am actively picking problems or rewriting them to make the students be interested. The collaborative planning and the enhancement of the lesson once we taught it the first time, makes us be more efficient, be prepared for reluctant learners.	Students are more engaged when the problems are meaningful. When students see the patterns or connections, they are more inclined to keep trying. Even if they do not get the right answer, the process is meaningful and fun. Students get immediate feedback and recognition when they are struggling.
Andrea Monroy	The most important change in my teaching is using a "problem" as my main focus and letting it lead to the multiple representations of solving it. I have found it fascinating to see how not all kids were able to come up with a final answers, but when you pulled all of their work collectively, you can see the progress and understanding(s) that they are making.	The students have really impressed me with their cooperative learning and with their willingness and determination to solve their problems.
Lindsay Sweetser	Abstracting from computations	Look for connections between problems Make generalizations about computation – try to derive rule or procedure for problem types
Lucy Rutecki	Ask questions that promote looking for patterns and trends Focus on presenting real life tasks for my students Have implemented algebra sections of calendar math so students see patterns consistently and on a daily basis - Daily variable - Daily patterns Promote connections among concepts and problems	Enjoy sharing and presenting solution strategies More focused on noticing patterns More focused on making generalizations Engaged and on task Enjoy problems that involve them as characters Notice connections and similarities among problems
Crista Ziegler	Posting the star on a poster reminds me to ask students for alternative solutions I try to model alternative strategies for problems. Increased the difficulty level/complexity of the problems I present for class/homework…even though it may only be 1 problem.	Students know it's okay to try different solution strategies, they don't all have to do the same thing.
Georgie Lowe	QUESTION as a means of helping students move forward. WAIT for students to work (instead of jumping in with my help) Share the teaching! Built my confidence	Like to try new things Learning how to explain their own solutions LISTEN to each other Effectively work in groups – draw on each student's strengths "collective knowledge" Kids realize that learning is life-long (since I'm in a class)
P. McAfee	I have gotten away from a lot of drill & practice. There is a more conscientious effort to do problem solving. The students are just as or even more enthusiastic as they experience this. What my students are missing is the experience of being in a large group setting with students expressing multiple ways of solving the problem. This will encourage me to provide opportunities for them to be "included" in a larger group.	I hope this is making an impact on their learning. As I continue to provide them with more experiences I'm sure I will see evidence.
[Not identified]	Doing and undoing – this is now a strategy I use with my students. Model thinking, let the students try different strategies and let them cross out thinking if it's not working. I have taught my students not to erase so I can see their thinking.	They are used to solving problems now by doing and undoing. Taking risks and abandon different strategies when they aren't working for them.
Ellen Richard	Extending "Problem Solver" problems – batting cages every 3 days and every 5 days Doing and undoing – Asking kids for flexibility in their thinking – think about solutions, but also different ways to get solutions Using Illuminations & NLVM websites more – make "0" website Multiple representations	Got kids to expand thinking and move away from concrete numbers toward algebraic "n" days/times Getting kids to not only compose, but decompose numbers (What multiplication sentence gets 48 as a product, instead of 6 x 8 = ?) Forces kids to think 2, 3, 4 steps ahead because to make 0, all pieces have to fit together Kids recognize that there are multiple solutions to make 0
Julia Valonis	Have used more problem solving problems with similar connections Organizers to show different ways to represent A lot more of building rule to represent function	My students are demonstrating more of an abstract way of thinking math. They are purposefully looking for connections between the problems such as how is this like the problem Mrs. Pike showed us last week.

	Using NCTM activities with the applets to illustrate the math Connecting a graphing component with equations	They are also looking more at sequence of numbers and analyzing for rules.
Jerry Ruel	Each Friday I try to assign one of the problems we did over the summer or that Jenn has sent to us. The students work with partners and we focus on multiple representations. We look for patterns and then a rule. If possible, we try to graph the solution.	My students have difficulty solving word problems and understanding what variables are. These activities seem to help with the process as the student talks about their approaches.
Karen Miller	Using my new found knowledge has enabled me to give my students a lot more flexibility when solving problems. I give the students more logic problems and more complex word problems to foster the thinking not just solving for the answers.	My students have much more open minds to math than before. They used to see it as just drill and practice, now they are much more excited with the practice.
Andrea Johnson	Looking at connecting students interests to algebraic concepts Not using fractions and decimals when introducing a new concept Using manipulatives more often so students can discover	Student were much more interested, they asked questions and were INVOLVED Students understood concepts easier than when they were buried in calculations. Students are making connections to algebra while "playing."
Steve Klarevas	I have integrated more technology into my classroom, both software and hardware, including Geometer's Sketchpad and the Airliner and tablet PC. I have begun giving a Problem of the Week in my Geometry Honors class; it is a weekly deep thought problem. I have utilized more group work.	The students have taken to the technology as young people tend to do! The Sketchpad software's ability to animate figures – to make lines, figures, etc. dynamic – has certainly helped the students to understand and see the material. The students also enjoy using the Airliner because it is fun and pulls them into the lesson. The Problem of the Week has provoked some VERY SERIOUS thought. ☺ The students have taken more ownership of their math education and have collaborated in teaching each other. And as the thought goes, if someone can teach it, then they must truly know it.
Donald Reinecker	One of the benefits of algebra eyes and ears has been the fun of looking for patterns in everyday math. I have been able to always look for these patterns in problem solving. It has been quite interesting because now the teachers I coach also look for algebraic patterns in their problem solving.	Students find these "algebraic eyes and ears" interesting because the bug is catching. I have had students ask me where is the algebraic pattern after we complete problem solving. It's refreshing to find students who question.
Heather Postlethwait	The course that I am teaching this year is very different than the courses I have taught before. The course has more algebra in the content so I don't know if what I brought from this course is impacting my classroom or is it the course content. I am hoping that the combination of this course and the content that I am teaching is making the difference.	Students have tried the practice problems that we did in class. I gave the handshake, the mango and several other problems to my students as extra credit. They attacked the problem with such verver (?) and intensity that I decided to keep doing the problems each week. My students would like to rename my class from Math 7 Honors to Almost Algebra.
Zarita Robinson	My math teaching has been changing during my career. The way I learned math was very different than the way I have evolved in my teaching. My teaching is changing from teacher/algorithm directed to student/problem solving driven. When I plan I always try to incorporate algebra concepts which is a big change from previous years. Now that I have the awareness I think I will be able to create those habits of mind.	My students are changing as well. They now look for "patterns" before they "freak" when they see a problem. They are now aware of connections and try different ways or approaches to problem solving. That is a BIG improvement from "I can't do this" or "I don't get it."
Angela Florio	I have been trying to connect more concepts in Math8 to a larger overarching Algebra focus. I have been trying to incorporate more Algebra applications so that my students will have a chance to see the Algebra around them. I have been trying to help my students to see patterns visually and algebraically so that they can build rules to extend the patterns. I did this as part of my functions unit.	By helping my students see the overarching Algebra in other concepts, I think they are beginning to form their own connections. I think that the use of more applications in Algebra helps my kids take the abstract and move it into the concrete and practical uses for what we are learning. I think that by introducing patterns and rule making, my students were able to see Algebra visually which they may not have before. It also helped them learn how to experiment with numbers.

Pat Goodheart	I introduced having “Algebra eyes and ears” in our math classroom. I even purchased big “Algebra” sunglasses to motivate the students. When they meet in small groups. When a student talks with the small group, he/she wears the glasses and tries to use math language/strategies to problem solve. I also give the small group the problem solving strategies on cards on a ring. As a student think outloud about the strategy they are using to problem solve, the card helps to focus the group and talk about the process. Having this type of activity at least once a week is an alternative to drill and practice. They are applying the skills or concepts. They are practicing to problem solving and applications. (?) Use our Smartboard for more interactive learning/students are using NLVM activities.	Students are becoming more comfortable with communicating their thinking. Students are EXPECTED to talk about their math thinking. Looking for more patterns of thinking because I am telling them that math is beautiful with many patterns and “habit-forming.” It is coming very slowly.

FINAL FEEDBACK "How has the course impacted your teaching"

Phillip R. Corpuz
HOLMES MIDDLE SCHOOL
(1) This course has helped me identify and make algebraic connections among different math topics for students.
(2) It has helped me understand the intrinsic value in developing algorithmic thinking early in students to help them make algebraic connections in problem-solving as they develop and move up in their math skills and concepts.
3)This course emphasized the importance of teaching children how to build rules to represent functions through careful analysis of "doing" and "undoing".

Crista Ziegler
1. Helping students to think about math problems in multiple ways - this course reinforced the idea that a picture or table solution is valid & appropriate and that we needn't always be striving for the algebraic forumula.
2. Using technology - any opportunity to see new websites in action or to get teachers together with the express purpose of sharing their technology tips and applications is going to improve classroom instruction.
3. Preparing lessons more creatively - the Lesson Study, while time consuming, influences how I plan and prepare my lessons now. It's forced me to use fresh ideas and incorporate more varied approaches in my teaching methods.
4. It inspired me to tell my husband to buy a Wii. My kids think I'm a much cooler mom now b/c I play it a lot!

Steve Klarevas
1. Posing good mathematical questions and problems that are productive for students' learning. WHY? Because the many problems posed to us have turned into wonderful classroom resources for me. And also because the course has motivated me to go out and find more such questions to provoke serious student thinking.
2. Representing mathematical ideas and concepts carefully in multiple ways. WHY? When I first started teaching I gave my students one way to do each type of problem, because that's where my comfort zone was. But as the years have gone on I have realized the value of multiple techniques to solve any problem; so that if one way doesn't work, you're not stuck. ACT NOW simply reinforced that notion to me.
3. Making connections between physical, graphical models and symbolic notation. WHY? I think this ties into my #2. Also, ACT NOW reminded me that we don't all tackle a problem by jumping right into algebra-honors-level symbolic notion. It's good to know and be appreciative of where other people are coming from.

Ana Lopez
I feel this class has better prepared me to ask better questions to foster algebraic thinking. I also fell better prepared to be judgemental about the resources available and really pick the ones that are best suited for my students. And the use of technology across all differences and diversities.

Cynthia Walker
Numbers 10,11,and 13 were areas that I was weak in and through this class I am more confident in these areas.

Connie Pruitt

#1~I feel that I can make better algabraic connections with all types of problems. By using all types of problems I was able to identify ways to pull out and highlight the underlying algebra.
#2~Even though I do not feel experienced in making the technological connections I have been able to increase my knowledge on how to combine techology and algebra.
#3~I can't pull just one more out. I have seen so many ways of applying algebra thinking for my students that I feel better prepared in numerous ways. With each lesson I prepare, I think more across the board. I try as much as I can to enhance the information; whether it be with graphs, or formulas or even just combining the verbal with some pictures to enhance/define the problem. Thank you so much for all of this.

Representing mathematical concepts and ideas in multiple ways. There are numerous ways to get to the same answer.
Posing good math questions and problems that are productive to student learning, I have good ?s to encourage quality thinking for reading now I have good ?s to encourage algebraic thinking as well as problem solving.

Giving mathematical access to diverse learners..given our student population in FCPS, we all are faced with this challenge daily..we all learn in different ways.

Andrea Monroy
The ACT NOW course has shown me how easy it is to make algebraic connections in the lower grades. I also am more knowledgable with the multiple representations (symbols, table, graph, concrete, and verbal) that are used in algebraic learning. The lesson study was the best part! It was fascinating to see how a lesson that we worked on together in the summer worked out exactly as we had planned, months later. I can't wait to do my next lesson study and share this experience with others!

Georgie Lowe
I am now better able to make connections across various methods of solving problems because we did that in our class this

summer.
I feel more equipped to ask the right questions since the teachers this summer asked us the right questions when we were working in our groups, and carrying out a lesson study showed the importance of questionings to lead students.

Ellen (Wieser) Richard
1. Doing and Undoing - I think the ACT course has helped me to really see the importance of working through a problem forwards and backwards, and, more importantly, helping my students have the flexibility in thinking
2. Using technology - After having been exposed to the many wonderful online resources (NLVM, Illuminations, etc), I feel more confident incorporating these types of technology into my class, as yet another way to stretch their thinking.
3. Representing Problems/Solutions in Multiple Ways - After taking the ACT class, I see more the benefit of have graphical, pictoral, verbal, and written representations. This levels the playing field for the different types of learners and also allows the students to perhaps see problems in a different light.
Other notes: I think the class was fascinating and I loved having time to really dissect the different types of problem. The lesson study was fantastic and helped me learn so much about my students, as well as my own teaching. Thank you!

Beth Baldwin
I feel most prepared to teach my students to use multiple representations, to notice patterns, and build rules. I think it was most helpful to solve problems collaboratively in the summer class and to share our solutions. It was interesting to see the various ways my classmates solved the problems.
I thoroughly enjoyed the 8 day class, although it has been a challenge to do the follow up classes and homework while I am teaching full time. I understand the rationale for having multiple grade levels together for the class, but I think I would have gotten more out of it if it had been geared to 3-6th grade. The homework and test practice were extremely challenging for me. The lesson study was valuable and enlightening. I wish teachers could collaborate on more lessons in that way, but time is always the read block. My last comment is that it is very difficult to work with teachers from multiple schools for the lesson study. There was not enough time given to do it in class (and my group members were off task) so it was necessary for me to do most of it at home. I did not get any feedback from my team when I sent the lesson to them, or any support when I was unable to teach the lesson. I think my emotional investment in the lesson declined after that and I was not really interested in being involved with the final presentaton. I guess that's the way it is in any group. Perhaps if more time was given in class, we could have finished it together and been more cohesive. I hate to end with the negative, so I want to say thank you to Dr. Suh and the other professors who opened my eyes to algebraic habits of mind! I particulary appreciate Dr. Suh's continued communication, compassion, and resources!

Susie Clark Ashton
I can't see the items to remember exactly, but... I think it's important to give students various experiences problem solving, talking about the problem solving, and then repeated instances of a similar problem to practice the skills learned.

Every time I've worked on this course I've felt energized to continue working with my students to help them understand math in ways I didn't when I was their age.

The lesson study experience is powerful. Brought the idea to a teammate, we worked together to plan, teach and revise the Piggy bank lesson. As a result of her experience, she contacted her professor from her pre-service learning and the professor wants to know more about lesson study! We are both excited about working more together to plan more lessons and I hope to enlist a few more teachers to our endeavor.

Melissa Bell I feel that using technology it's an important component in bridging the gap. It's a great way to get students hooked into an idea that typically they would have no interest in. I also think modeling various strategies is vital in any classroom. It helps students realize that there is more than one way to solve a problem. I also think it's important to use the math vocabulary with students even if you have to put in other terms for them to understand. At least they're being exposed to the vocabulary and will hopefully make a connection later on in their math career.