THE APPLICATION OF DUAL CODING THEORY IN MULTI-REPRESENTATIONAL VIRTUAL MATHEMATICS ENVIRONMENTS

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This mixed method study compared mathematics achievement in two third-grade classrooms using two different representations: virtual manipulatives which has dual codes of visual and symbolic representations, and physical manipulatives for instruction in fraction addition and balancing algebraic equations. The research employed a within-subjects crossover repeated measures design, administered pre and post tests and gathered examples of students' work. An Analysis of Variance showed statistically significant differences in achievement in favor of the virtual manipulative fraction treatment. An analysis of students' written work showed that the virtual environment supported students' learning of the algorithmic process by providing captured procedures displaying pictorial and numeric representations.

The use of and the ability to translate among multiple representational systems has been shown to influence students' abilities to model and understand mathematical constructs (Cifarelli, 1998; Fennell & Rowan, 2001; Goldin & Shteingold, 2001;; Lamon, 2001; Perry & Atkins, 2002). This ability requires the learner to use various cognitive structures for processing a variety of inputs during the learning process. The purpose of this paper is to examine the application of Dual Coding Theory (Clark & Paivio, 1991) in multi-representational virtual mathematics environments. In particular, the present study investigates the nature of learners' algorithmic thinking processes as they explore mathematical tasks with dynamic electronic objects, or *virtual manipulatives* (Moyer, Spikell & Bolyard, 2002).

THEORETICAL FRAMEWORK

Cognitive science has influenced educational researchers with theoretical models which explain encoding of information through different modes of representations. Dual Coding Theory (DCT), proposed by researchers in the field of educational psychology and based on Cognitive Information Processing Theory, is the assumption that information for memory is processed and stored by two interconnected systems and sets of codes (Clark & Paivio, 1991). These sets of codes include visual codes and verbal codes, sometimes referred to as symbolic codes, which can represent something arbitrarily, such as letter, numbers and words. According to the Dual Coding Theory, being presented with both nonverbal and verbal codes, which are functionally independent, can have additive effects on recall. Rieber (1994) reports that it is easier to recall information from visual processing codes than verbal codes because visual information is accessed using synchronous processing, rather than sequential processing. Many research are found applying Dual Coding Theory to literacy and multimedia. Rieber notes, “adding pictures (external or internal) to prose learning facilitates learning, assuming that the pictures are congruent to the learning task,” and, “children do not automatically or spontaneously form mental images when reading” (1994, p.141). Based on the DCT, Mayer (1992) discusses an instructional design principle that is called the contiguity principle, which states that the effectiveness of multimedia instruction increases when words and pictures are presented together. In mathematics, dual coding theory has been long in use for textbooks and other instructional mediums use visual representations along with symbolic representations. Clark and Campbell (1991) used dual coding theory to develop a general theory of number processing. The theory emphasizes the concrete basis of number concepts and the roles of associative imagery in performing numerical operations. The most basic application of dual coding processes is used to teach children the names of numerals and then their meanings by associating them with groups of objects or their pictures. Pyke (2003) used the dual coding theory to study the effects of symbols, words and diagram on eighth grade students engaged in a problem solving task. Result showed that students' use of symbols, words and diagram contributed to the different strategies used to solve the task and revealed different kinds of cognitive processes.

Ball (1992) expressed the caution that students do not automatically make the connection between their actions with the physical manipulatives and their actions with symbols. Kaput's (1989) explanation for this disconnect was that the cognitive load imposed during the activities with physical manipulatives was too great for students. He stated that the problem with physical manipulatives is that people cannot keep record of everything. In essence, students are unable to track all of their actions with the manipulatives and fail to see the connection between these actions and the manipulation of symbols. Applying dual coding theory to this study allows researchers to examine whether representational connection between numeric and visual (pictures) forms has an additive effect on understanding mathematical concepts, in particular the algorithmic process, since two mental representations are available contiguously.

METHODOLOGY

Procedures

The present study employed a within-subjects crossover repeated measures design to examine the research questions (Campbell & Stanley, 1963). All subjects participated in both treatments using virtual and physical manipulatives, which allowed each student to serve as his or her own comparison during the analysis. To avoid any residual effects, researchers introduced two different mathematics units, fractions and algebra, as the topics of study. The researchers chose concepts that traditionally are taught using an algorithm like adding fractions with unlike denominators and balancing equations to examine ways manipulative representations can serve as conceptual supports in helping student understand how and why those procedures work.

In the first phase of the study, Group One participated in fraction lessons using the physical manipulatives while Group Two participated in fraction lesson using the virtual manipulatives. In the second phase, each group received the opposite condition. That is, Group One received algebra instruction using virtual manipulatives and Group Two received algebra instruction using physical manipulatives (see Figure 1). A pretest on fraction and algebra concepts was administered at the beginning of the study. Students learned fraction content using virtual or physical manipulatives during the first unit. During the second unit on algebra, students switched treatment conditions and learned algebra content. Fractions and algebra content tests were administered at the end of each unit.

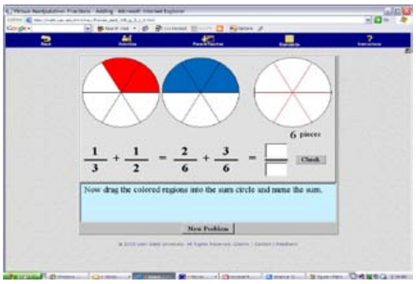
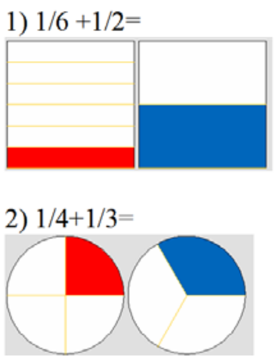
 Virtual Manipulative Fraction applet	 Deluxe Fraction Circles
 Virtual Manipulative Algebra Balance	 Hands-On Equations®

Figure 1: Instructional materials for fraction and algebra unit

Participants and Data Sources

The participants in this study were 36 third grade students in two classes at the same elementary school. The student demographics included 83% White, 11% Asian, 3% African American, and 3% Hispanic. There were 22 boys and 14. Students at this school were placed in mathematics achievement groups through standardized testing methods. The students selected for this study were in the middle achievement group working on a third grade level in mathematics. Intact classes were randomly assigned to treatment groups.

The data sources used for this study were both qualitative and quantitative. The quantitative data included pretest and posttest scores of students' mathematics content knowledge. The research-designed tests had three sections with a total of 20 items. The first section had items with dual modes where students were presented with pictorial and numerical representations, the second section had only numerical representations and the third section had two word problems which asked students to draw a picture, represent the problem with a number sentence and to explain their solutions strategies(See figure 2).



2/4+3/8=

2/5+3/10=

Figure 2. Examples of dual coded (pictorial and numeric) and single coded (numeric only) test items.

The qualitative data included field notes, students' written work, student interviews and classroom videotapes. Students' written work contained drawings, solution procedures, and numeric notations. These qualitative data were examined and categorized along dimensions of students' solution strategies. Student interviews, field notes, and classroom videotapes were used to examine the representations that students used to solve problems in both treatment environments. The qualitative results allowed researchers to further examine and interpret the results of the quantitative findings.

RESULTS

The results of all tests were entered into SPSS and descriptive statistics for each treatment group are presented in Table 1.

	Group 1: Pretest	Group 1: Posttest	Group 2: Pretest	Group 2: Posttest
	Physical Manipulatives		Virtual Manipulatives	
Fraction	12.50 (SD=15.00)	45.55 (SD=17.05)	13.00 (SD=14.50)	75.55 (*SD* = 19.91)
	Virtual Manipulatives		Physical Manipulatives	
Algebra	30.00 (SD=12.00)	83.33 (SD = 14.34)	22.00 (SD=14.00)	80.00 (SD = 20.16)

Table 1. Mean for the Pretest and Posttest by Treatment Type and Mathematics Content (N=36)

The results showed that students from both conditions had very little prior knowledge on either topic, fractions or algebra. There were no significant differences in the two student groups in terms of achievement at the beginning of the study. Posttest scores indicated differences among the groups and an ANOVA was performed for further analysis. Results from the ANOVA produced a significant main effect for manipulative types, $F(3,68) = 15.03$, $p < .001$ which indicated that students' scores depended on the manipulative type they used. Results from the ANOVA also produced a significant main effect for mathematics concept, $F(3,68) = 24.11$, $p < .001$, which indicated that students performed significantly better on the algebra posttests than the fraction posttests. There was a significant interaction effect, which indicated that the effect of the manipulative treatment on the dependent variable was different depending on the mathematics concepts, $F(3,68) = 9.62$, $p < .01$. The Bonferroni multiple comparison indicated that significant results existed between the physical fraction circle group compared to the other three treatment groups.

To further understand the physical fraction circle group results, researchers analysed learners' performance on the test items. For this investigation, we applied the framework of Dual Coding Theory to examine the single and dual representational test items. Results of this analysis are presented in Table 2.

Performance on the Representational test items (I)	Performance on the Representational test items (J)	Mean Difference (I-J)	*p*
Physical Manipulative- Dual coded (visual and numeric)	Physical Manipulative – Single coded	36.11	.001 ***
	Virtual Manipulative – Dual coded	-27.77	.033 *
	Virtual Manipulative – Single coded	-12.50	1.00
Physical Manipulative –Single coded (numeric only)	Physical Manipulative – Dual coded	-36.11	.001 ***
	Virtual Manipulative – Single coded	-63.88	.000 ***
	Virtual Manipulative – Single coded	-48.61	.001 ***

* p <.05. **p<.01. p<.001***

Table 2: Bonferroni Multiple Comparisons on Fraction Posttest by Representational Test Items

Results showed several significant differences among the dual and single coded test items in the two treatment environments. Participants in the physical manipulatives treatment group scored higher on the dual coded test items (which included both visual and numeric information) than the single coded test items (which included only numeric items). The second row shows that the physical manipulative group performed significantly lower on the single coded numeric items compared to all other fraction test items in both groups. Even though, the physical treatment group performed better overall on the dual coded items than single coded numeric test items, the virtual manipulatives treatment group performed significantly better in the dual coded test items than the physical treatment group. This may be attributed to the feature in the virtual fraction applet environment that provided dual coding of pictures and numeric representations contiguously on the screen during class practice that was not present in the physical fraction environment. The dual coded nature of the virtual environment provided learners' opportunity to interact with the dynamic visual representations and input numeric representations that corresponded to the algorithmic process of renaming and adding fraction with unlike denominators.

Based on these statistical results, we further examined the qualitative data to determine the sources of these differences. On the fraction posttest, Group One who worked with the physical manipulatives relied more on pictures to solve the single coded problem but found it limiting when they encountered fractions that were difficult to illustrate For example for problems like ¼ + 1/5, where both fractions needed to be renamed before being added, drawing these two fractions into common fractions was not intuitive. These test items became complex for students because it was harder for them to illustrate their answers since they had to divide the fraction pieces equally into 20 fractional pieces. This suggests that although, physical models and visual can be helpful when initially learning fraction concepts and visualizing fractions, over-reliance on pictures may be limiting when students need to solve more complex fraction problems.

In the virtual fraction group, more students used an algorithm that showed an understanding of the algorithmic process of renaming then combining fractions modelled on the applet by the linked representation feature indicating better transfer of learning. Most students who successfully answered the numeric items changed the unlike fractions into fractions with common denominators, as was modelled by the virtual fraction applet(e.g. 3/4+ 1/8= 6/8 + 1/8= 7/8). In addition, there was a marked difference in students' explanation of their solutions on the word problems. Most students in Group One, who used the fraction circles, explained their process using the picture that they drew to illustrate the problem. One student explained, "I drew a picture and took the half and I put it in the third." Although, the student had the correct answer, there was no evidence of the renaming process. However, most students in Group Two drew pictures, wrote the correct number sentence and used the formal algorithmic approach to solve the problem by renaming each fraction to have common denominators. Some examples of their explanations are shown in figure 8.

- "I said to myself 2, 4, 6 and 3, 6, 9 and got my common denominator."
- "I found a multiple of 2 and 3."
- "I multiplied the [number of divided parts] by 2 for 1/3 which equals 2/6 and I divided 6 in half which is 3/6 and then I added 2/6 and 3/6 which equals 5/6."

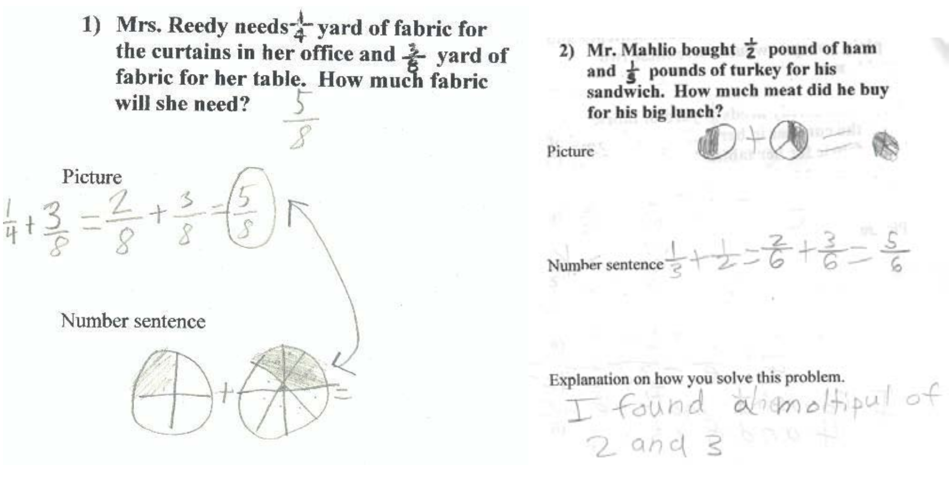


Figure 3. Examples of student solutions on a fraction word problem with pictorial and numeric representations.

The proper use of the virtual fraction applet provided students with the conceptual knowledge and the procedural knowledge of adding fractions with unlike denominators. It promoted algorithmic thinking because students learned the procedure while building a conceptual foundation for fraction addition with unlike denominators using the interactive dynamic visual and numeric representations. Kaput(1992) stated that constraint-support structures built in to computer based learning environments “frees the student to focus on the connections between the actions on the two systems[notation and visuals], actions which otherwise have a tendency to consume all of the students cognitive resources even before translation can be carried out” (p.529). The dual coded virtual fraction environment offered many meta-cognitive opportunities such as keeping record of users’ actions and of the transformation of numeric notation which allowed for learners to use their cognitive capacity to observe and reflect on the connection and the relationship among the representations, thereby promoting algorithmic thinking. The physical fraction environment proved to be ineffective because the learners cognitive resources were expended on keeping track of fraction pieces, finding equivalent fraction using an equivalence mat, and recording it on paper. The task became a cognitive overload and the demands on the learners did not leave any cognitive resources to observe relationships between actions on the physical manipulatives to the symbolic manipulation. Results from the test items also suggest that students do much better on tests when given dual codes of visual and symbolic representations. Building mental images for symbolic and numeric representations is an important skill for students to improve in their mathematical understanding. This study suggests that dual coded representations in virtual manipulatives environments and models may be more effective in teaching different cognitive processes, especially concepts where stored and captured procedures can develop algorithmic thinking.

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