What Do We Know about the Teaching and Learning of Algebra in the Elementary Grades?

This research brief brings together the research on experiences students should have with generalization in the elementary grades. We address the question: Which algebraic concepts might be stressed in elementary school to lay a foundation for later success in algebra?

The algebra-related research conducted with elementary school students has included the following areas: the properties of number operations, numeric equalities, change and patterns, and relationships between quantities. The majority of this work does not introduce young students to conventional algebraic notation; rather, it relies on the use of language and other representations to express algebraic ideas.

The research dealing with properties is based on the hypothesis that *if students understand their arithmetic such that they are able to explain and justify the properties they are using as they carry out calculations, they will have learned some fundamental foundations of algebra.* For example, third- and fifth-grade students have been found to develop and justify generalizations such as: "When you add zero to a number you get the number you started with; when you subtract a number from itself, you get zero; when multiplying two numbers, you can change the order of the numbers" (Carpenter, Franke, and Levi 2003). This finding indicates that though students do not use algebraic notation in their answers, they are still able to express general, algebraic properties about the number system.

It has also been indicated that elementary school students can be introduced to algebraic reasoning through numerical expressions, using numbers as quasi-variables—for example, number sentences such as 87 – 39 + 39 = 87, which are true whatever number is taken away and then added back. As with the work above involving mathematical properties, the *activity with quasi-variables allows teachers to help students build bridges from existing arithmetic knowledge to algebraic thinking without having to rely on knowledge of algebraic symbols* (Fujii 2003).

The ever-increasing body of research involving change and patterns reflects the fact that pattern finding in single-variable situations is becoming more common in the elementary curricula. Much of this research, which has been carried out with students from about the second grade onward, deals with geometric and numeric pattern building and often tabular and informal graphical representations. On geometric patterns, research has suggested mixed results (Carraher and Schliemann 2007). Although going back and forth between patterns with geometric shapes and their numerical representation in tables can lead to general insights, *too early a focus on the numeric values in tables can inhibit the richness of the process of generalizing from the geometric data* (Moss et al. 2005). Similarly, rushing students to represent patterns with letter symbols can be counterproductive. The research on patterns suggests that *it is generally more profitable for young students to remain for long periods of time in exploring aspects of the generality in their patterns than to be exposed too quickly to the symbolic representation of this generality* (Radford 2006)—for, in actual fact, these symbolic representations do not get used until much later when students begin work in symbol manipulation.

Up to this point, all the research described in this brief has focused on students' learning at the presymbolic level; however, not all such research does. The Russian-based approach developed by Davydov and his colleagues (Davydov et al. 1999) emphasizes the teaching of algebra with symbols right from the first grade. The Russian curriculum does not use experience with number as the basis for developing algebra but instead uses relationships between quantities as the foundation (Schmittau and Morris 2004). In an American adaptation of the Davydov approach, students as young as third graders engage in searching out relationships among quantities across contextualized situations and in "solving" related equations using literal symbols (Dougherty 2003).

In summary, the current body of research related to the development of algebraic reasoning at the elementary school level emphasizes that arithmetic can be conceptualized in algebraic ways and that building an understanding of algebra begins within the learning of arithmetic. The research also describes ways in which this emphasis can be capitalized on to *encourage young students to make algebraic generalizations without necessarily using algebraic notation.* These studies point to promising avenues for developing the conceptual underpinnings of students' later activities in algebra.

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