

INDEX

- Absorbing barriers, 103
- Ample service, *see* Service, ample
- Analyticity, of generating functions, 100, 127
- Anderson-Darling (AD) test, 411
- Aperiodic state, 37
- Applications, 2, 3
 - aircraft, 3
 - airline call center, 110
 - airline ticketing, 174
 - assembly line, 271
 - automobile inspection, 80
 - bank drive-in, 135
 - car polishing, 92
 - car wash, 109, 125, 174
 - carry-out restaurant, 216
 - communications, 3, 107, 110, 273
 - computers, 3, 108
 - correspondence courses, 111
 - hair salon, 63, 109, 150, 288
 - heating oil distribution, 136
 - hospitals, 3, 72
 - inventory, 111, 174
 - machine repair, 85, 112, 197, 228
 - maintenance, 105, 108, 112, 215, 217
 - manufacturing, 87, 121, 136, 173, 400
 - production, *see* Applications, manufacturing
 - quality control, 121, 136
 - rate switching, 91
 - reliability, *see* Applications, maintenance; Applications, machine repair
 - safety, 107
 - scheduling, 3
 - short-order counters, 104, 105
 - spares provisioning, 311
 - supermarket, 184
 - tanker docking, 109
 - television viewing, 84
 - tool crib service counter, 328
- Approximations, 293, 343
 - diffusion, 352
 - heavy traffic, 347
 - network, 356
 - process, 346
 - saturated systems, 351
 - system, 345
 - using bounds, 343
- Arbitrary arrivals and/or service, *see* General models
- Arrivals
 - arrival pattern of customers, 3

Fundamentals of Queuing Theory, Fourth Edition.

By D. Gross, J. F. Shortle, J. M. Thompson, and C. M. Harris
Copyright © 2008 John Wiley & Sons, Inc.

- arrival-point probabilities, 64, 79, 90, 259, 304
- balking, 3, 95
- batches, 3, 45, 117, 244
- bulk, *see* Arrivals, batches
- constant, 13, 129
- deterministic, 13, 129
- discrete, 263, 293, 337
- distribution, 3
- empirical, 337
- Erlang, *see* Erlang arrivals
- exponential, *see* Exponential arrivals
- general, 259
- hyperexponential, 273
- impatience, 4
- interarrival times, 3
- jockeying, 4
- nonstationary, 4
- phases, 137
- rate, 3
- reneging, 3, 96
- state-dependent, 85, 95
- stationary, 4
- Averages, *see* Expected value
- Backward equations, 28, 117
- Balance equations
 - detailed, 53, 304
 - flow, 50, 54
 - global, 52, 304
 - local, *see* Balance equations, detailed
 - stochastic, 188
- Balking, 3, 95, 243, 254
 - distribution, 95
 - function, 95
- Batches
 - $G/M^{[Y]}/1$, 269
 - $M/M^{[Y]}/1$, 123
 - $M^{[X]}/G/1$, 244
 - $M^{[X]}/M/1$, 117
 - arrivals, 3, 45, 117, 244
 - constant distribution, 120
 - distribution, 118
 - geometric distribution, 120
 - input, 117
 - service, 4, 123, 269
- Bayes' theorem, 79, 90
- Bessel functions, 100, 102
- Beta distribution, 407
- Bilevel hysteretic control, 315
- Birth-death process, 29
 - multidimensional, 134, 146
 - quasi, 283
 - steady-state, 49
- Birth-death queueing models, 49–115
- Blocking, 185
- Bookkeeping, 12, *see also* Simulation, book-keeping
- Bootstrapping, 412
- Bounds, 330
 - multiserver queues ($G/G/c$), 341
 - single-server queues ($G/G/1$), 332
- Bromwich inversion integral, 387
- Brownian motion, 349
- Bulk, *see* Batches
- Busy cycle, 102, 104
- Busy period, 10, 102
 - $G/G/1$, 104
 - $G/M/1$, 269
 - $M/G/1$, 239
 - $M/M/1$, 102
 - $M/M/c$, 102
- Busy probability, 12
- Buzen algorithm, 198
- Capacity, 3, 5, *see also* Truncated queues
- Cauchy criterion, 377
- Central limit theorem, 349
- Channels, 5, *see also* Service
- Chapman-Kolmogorov (CK) equations, 26
- Chapman-Kolmogorov (CK) equation, 244
- Chapman-Kolmogorov (CK) equations, 51, 117
- Characteristic equation, *see* Operator equation
- Characteristic function, 447
- Characteristics of queueing process, 3
- Chebyshev's inequality, 352
- Chi-square distribution, 447
- Chi-square goodness-of-fit test, 408
- Classification of queues, 3
- Closed networks, 195
- Coefficient of variation (CV), 46, 344
- Communication of states, 37
- Completely random processes, 19
- Compound Poisson, 22, 118
- Computers, 3, 108
- Confidence intervals (CIs), *see* Confidence statements
- Confidence statements
 - ρ for $M/M/1$, 321
 - simulation output, 416
- Congestion, measures of, 9
- Conservation, 143, 304
- Constant arrival rate, *see* Arrivals, constant
- Constant failure rate (CFR), 406
- Constant service rate, *see* Service, constant
- Continuity of queues, 339
- Continuous parameter Markov chain, 118

- Continuous parameter Markov process, 24
- Continuous parameter stochastic process, 24
- Control of queues, 306, 313
- Cost models, 105, 121, 238, 308–313
- Coxian distribution, 345
- Customer, 2
- Customer impatience, *see* Impatience
- Cyclic queues, 209

- Data, 317, 399, 405, 407, 408, 416
- Decomposition, 188, 249
- Decreasing failure rate (DFR), 406
- Delay, *see* Waiting times
- Departures, 13, 14, 181, 192, 223, 243, 246, 257
 - departure-point probabilities, 225, 232, 287, 304, 370
 - departure-point state dependence, 246
 - relation to arrival-point probabilities, 304
 - relation to general-time probabilities, 232, 304
- Description of queueing processes, 2
- Descriptive models, 306
- Design and control of queues, *see* Design of queues or Control of queues
- Design of queues, 306
 - cost models, *see* Cost models
 - economic models, *see* Cost models
- Detailed balance, 53, 304
- Deterministic arrivals, *see* Arrivals, deterministic
- Deterministic service, *see* Service, deterministic
- Difference equations, 17, 28, 50, 54, 467, 483
- Differential equations, 17, 28, 97, 467
 - partial, 28, 99
- Differential-difference equations, 17, 97, 99
- Diffusion approximation, 352
- Discipline, 4, 64
 - FCFS, 4
 - general (GD), 306
 - LCFS, 5, 301
 - priority, 5, 141
 - RSS, 5, 301
- Discouragement, 95
- Discrete-event simulation, *see* Simulation
- Discrete-parameter Markov process, 24
- Discrete-parameter stochastic process, 24
- Distribution
 - balking, 95
 - Bernoulli, 447
 - beta, 407, 447
 - binomial, 447
 - busy period, *see* Busy period
 - chi-square, *see* Chi-square
 - composite, 64
 - compound Poisson, 22, 118
 - Coxian, 345
 - deterministic, 129
 - discrete, 117, 228, 263, 293, 400
 - empirical, 228, 337, 399, 410, 412
 - Engset, 273
 - Erlang, *see* Erlang
 - exponential, *see* Exponential distribution
 - F, 321, 411
 - gamma, 128, 447
 - Gaussian, *see* Normal distribution
 - generalized Erlang, 290, 345
 - generalized hyperexponential, 346
 - geometric, 55, 120, 189, 447
 - hyperexponential, 47, 131, 273, 405, 447
 - mixed-exponential, 47, 289, 414
 - multiple Poisson, 22, 118
 - negative binomial, 114, 447
 - normal, *see* Normal distribution
 - phase type, *see* Phase-type distribution
 - Poisson, *see* Poisson distribution
 - rectangular, 412
 - selection, 399, 404
 - table of, 447
 - uniform, 20, 400, 412, 447
 - waiting times, *see* Waiting times
 - Weibull, 406
- Dynamic programming, 308, 314

- Economic models, 105, *see* Cost models
- Empirical distributions, *see* Discrete distributions
- Engset formula, 273
- Ensemble average, 35, 40
- Entropy, 24
- Equilibrium, *see* Steady-state
- Ergodicity, 34, 233
- Erlang, 2, 447
 - $E_j/E_k/1$, 140
 - $E_k/M/1$, 137
 - $M/E_k/1$, 133
 - arrivals, 137, 140
 - B formula, 22, 81, 169, 257
 - C formula, 70, 345
 - distribution, 19, 47
 - first formula, 81
 - generalized, 290, 345
 - generation of random variates, 414
 - loss formula, 22, 81, 257
 - parameter estimation, 403

- relation to the exponential, 129
 - service, 133, 140, 278
- Estimation, 317, 402
 - distribution selection, 404
 - maximum likelihood, 318, 402
 - method of moments, 402
 - parameters, 318, 402
- Euler summation, 390
- Euler's method, 378
- Excess of renewal process, 221
- Expected value
 - busy cycle, 103
 - busy period, 103, 240, 269
 - Little's formula, *see* Little's formula
 - queue size, 10
 - queue size of nonempty queues, 61
 - system size, 10
 - table of, 447
 - waiting times, 10
- Exponential arrivals, 16, 53–103, 117, 123, 133, 143, 150, 156, 181, 297, 318
- Exponential distribution, 16, 20, 447
 - Markovian property, 20
 - memorylessness property, 20
 - parameter estimation, 402
 - random-variate generation, 414
 - relation to the Erlang, 129
 - relation to the Poisson, 16
- Exponential service, 16, 53–103, 117, 123, 137, 143, 150, 156, 180, 259, 277, 318
- F distribution
 - confidence intervals, 321
 - test for exponentiality, 411
- F-test, 411
- Failure rate, 405
 - constant, 406
 - decreasing, 406
 - increasing, 406
- Feedback, 6, 189
- Finite dimensional distribution, 33
- Finite queues
 - capacity limits, 5, 76, 81, 241
 - source limits, 85, 244
- First come, first served (FCFS), 4, 7
- First passage time, 38
- Flow balance, 54, 304
- Fokker–Plank equation, 354
- Forward equations, 28, 117
- Foster's method, 234
- Fourier-series method, 387
- Gamma distribution, 128, 447
- Gamma function, 19
- Gauss–Seidel technique, 373
- Gaussian distribution, *see* Normal distribution
- General arrivals, 259–270, 277–294, 304
- General arrivals and service, 284
- General queue discipline, 153
- General service, 154, 219–258, 284–294
- Generalized Erlang, 290, 345
- Generalized hyperexponential, 346
- Generating functions, 56, 455
 - Moment, 460
 - moment, 44, 447
 - Probability, 464
 - probability, 447
- Geometric distribution, 55, 120, 189, 447
- Geometric series, 55
- Global balance, 52, 304
- Goodness-of-fit tests
 - Anderson–Darling (AD), 411
 - Chi-square, 408
 - F, 411
 - Kolmogorov–Smirnov (KS), 410
- Hazard rate, *see* Failure rate
- History of queueing theory, 2
- Hyperexponential distribution, 47, 131, 273, 405, 447
- Hysteretic control, 315
- Idle period, 102
- Idle time, 8, 12
- Imbedded Markov chains, 30, 219, 259, 297
- Imbedded SMP, 298
- Impatience, 3, 95, 243
- Increasing failure rate (IFR), 406
- Induction, 52
- Inequalities, *see* Bounds
- Infinite divisibility, 279
- Infinite number of servers, 84, 255
- Infinitesimal generator, 29
- Initial state, 31
- Input, *see* Arrivals
- Insensitivity, *see* Invariance
- Inspection paradox, 221
- Intensity matrix, 29
- Interarrival times, *see* Arrivals
- Interdeparture process, 181, 243, 256
- Invariance, 87, 257, 258
- Inventory control, 111, 174
- Irreducibility, 37
- Irreducible chain, 37
- Iteration

- for solving steady-state difference equations, 54
 - solution techniques, *see* Numerical methods
- Jackson networks
 - closed, 195
 - open, 187
 - properties, 180
- Jacobi technique, 373
- Jockeying, 4, 7, 96
- Khintchine, *see* Pollaczek-Khintchine formulas
- Kolmogorov equations, 28
- Kolmogorov-Smirnov (KS) test, 410
- Laplace transforms, 99, 455
 - 2-sided, 285
 - table, 456
- Laplace-Stieltjes transforms (LST), 456
- Last come, first served (LCFS), 5, 7, 301
- Last in, first out (LIFO), *see* Last come, first served (LCFS)
- Level crossing, 250, 305
- Limit results, 347
- Limiting behavior, 31, 39
- Limiting distribution, 31, 39
- Lindley's equation, 14, 285
- Line
 - delay, *see* Waiting times
 - size, *see* Queues
- Little's formulas, 10, 12, 304
 - applications of, *see* specific models
 - for higher moments, 236, 255
- Local (detailed) balance, 53, 304
- Long-run behavior, 31
- Loss systems, 5, 76, 81, 241, 255
- Machine repair models, 85
- Markov chain, 24, 25
 - continuous parameter, 27
 - decision process, 308
 - discrete-parameter, 26
 - ergodic theory, 34, 233
 - imbedded, *see* Imbedded Markov chains
 - uniformized, 385
- Markov decision problems, 308
- Markov process, 24
 - long-run behavior, 31
- Markov renewal process, 25, 296
- Markovian property, 20
- Matrix geometric, 281
- Maximum-likelihood estimation (MLE), 318, 402
- Mean, *see* Expected value
- Mean value analysis, 200, 201
- Measures of effectiveness, 8, 59
- Memorylessness, 20
- Method of moments (MOM), 402
- Mixed-exponential distribution, 7, 47, 289, 346, 414
- Models
 - selection of, 6
 - summary table of models treated and type of results, 449
- Moment generating functions, 44, 447, 460
- Multiple channels, 5
- Multiple customer classes, 143, 150, 156, 194
- Multiple Poisson, 22, 118
- Multiple queues, 6
- Multistage queueing system, 6, 181
- Negative binomial distribution, 114
- Network of queues, 179-213
 - approximations, 356
 - closed Jackson networks, 195
 - cyclic queues, 209
 - extensions of Jackson networks, 210
 - feedback, 189
 - mean-value analysis, 201
 - multiple customer classes, 194
 - non-Jackson networks, 212
 - open Jackson networks, 187
 - queue output, 181
 - routing probabilities, 180
 - series, 181
 - series with blocking, 185
 - traffic equations, 188
- Nonhomogeneous Poisson process, 22, 256
- Nonstationary queues, 4, 347
- Normal distribution, 42, 354, 416, 447
 - generation of variates, 414
 - test for, 410
- Notation, 7
- Number
 - in queue, *see* Queues
 - in system, *see* System
- Numerical integration, 378, 389
 - Euler, 378
 - predictor-corrector, 379
 - Runge-Kutta, 378
 - Taylor, 378
 - trapezoidal rule, 389
- Numerical methods, *see also* Simulation, 369
 - steady-state solutions, 370
 - successive substitution, 262, 279, 282
 - transform inversion, 385
 - transient solutions, 378

- Offered load, 12
- Open networks, 187
- Operator equation, 57, 260, 277, 339, 477
- Optimization of queues, 306, 399, 420
- Order statistics, 23, 46
- Output, *see* Departures

- Parallel channels, *see* Multiple channels
- Parameter estimation, *see* Estimation, parameters
- PASTA, 20, 220, 233, 263
- Performance, 8, 59
- Phase-type distributions, 130, 277, 281, 346
- Phases
 - of arrivals, 137
 - of service, 129, 133
- PK formula, *see* Pollaczek-Khintchine formula
- Poisson distribution, 447
- Poisson process, 16
- Poisson relation to the exponential distribution, 16
- Policy iteration, 308
- Pollaczek-Khintchine formula, 220, 344
- Predictor-corrector methods, 379
- Preemption, 5, 142, 156
- Prescriptive models, 306
- Priorities, 5, 141
- Probability distributions, *see* Distribution
- Probability generating functions, *see* Generating functions, 464
- Process approximations, 346
- Product form solution, 188
- Pseudorandom numbers, 412
- Pure birth process, 30

- QtsPlus software, 40
- Quasi birth-death process, 283
- Queues
 - advanced Markovian, 117–171
 - batch, *see* Batches
 - bulk, *see* Batches
 - characteristics of, 3
 - cyclic, 209
 - deterministic (constant rates), 13
 - discipline, *see* Discipline
 - Erlangian, *see* Erlang
 - expected (average) number in, 10
 - feedback, 6, 189
 - finite, *see* Finite queues
 - history, 2
 - imbedded Markov models, 219–270
 - length, 9
 - Markovian, 49–115
 - networks, *see* Network of queues
 - non-Markovian, 277
 - notation, 7
 - optimization, *see* Optimization of queues
 - output, *see* Departures
 - parameter estimation, *see* Estimation, parameters
 - priority, 141
 - properties, 2, 3
 - retrial, *see* Retrial queues
 - self-service, 84, 184
 - series, 181
 - size, 5, 9
 - statistical analysis, 317, 416
 - tandem, 181
 - waiting time, 10

- Random numbers, 412
 - common, 419
 - generation of, 412
 - pseudo, 412
 - uniform (0-1), 412
- Random selection for service, 5, 7, 301
- Random variates, 412
- Random walks, 347, 349, 353
- Randomization technique, 381
- Rate control and switching, 91
- Rate matrix, 29
- Rectangular distribution, 412
- Recurrence time, 38
- Recurrent state, 38
- Recursive computation, 89
- Recycling, 6
- Regeneration points, 245
- Remaining service time, 152, 220, 303, 342
- Remaining work, 306, 342
- Reneging, 3, 96
- Renewal process, 23, 296
- Renewal theory, 23, 221, 237
- Repair, *see* Machine repair
- Residual service time, 154, 220, 237, 270, 303
- Residual time of renewal process, 154, 221
- Retrial queues, 157–171
 - multiserver, 168
 - with impatience, 162
- Reversibility, 53, 182
- reversibility, 257
- Root finding, 140, 141, 262, 278, 279, 293, *see also* Numerical methods, 339
- Rouché's theorem, 100, 262, 278, 279, 291, 295
- Routing probabilities, 180
- Runge-Kutta methods, 378

- Sample path, 10, 13, 143, 232, 305
- Scheduling rules, 149, *see also* SPT rules
- Self-service queues, 84, 184
- Semi-Markov process, 25, 296
- Series queues, 181
- Served in random order (SIRO=RSS), 5, 7, 301
- Service
 - ample, 84, 255
 - batches, 123
 - bulk, 123
 - channels, 5
 - constant, 13, 129, 231, 294
 - deterministic, 13, 129, 222, 231, 271, 294, 395
 - discrete, 228, 293, 337
 - distribution, 4
 - empirical, 228, 337
 - Erlang, *see* Erlang
 - exponential, *see* Exponential service
 - patterns, 4
 - phase type, 277, 281
 - phases, 129, 133
 - rate, 4
 - remaining, 220
 - self-service, 84
 - shortest processing time rule, *see* SPT rule
 - stages, 6
 - state-dependent, 4, 91, 246, 252
 - time, 4
 - unequal rates, 147
 - vacation, 246
- Simulation, 398–421
 - bookkeeping, 401, 415
 - confidence statements, 416
 - credibility, 421
 - data generation, 402, 412
 - elements of a simulation model, 399
 - input modeling, 402
 - languages and packages, 415
 - nonterminating, 416
 - optimization, 399, 420
 - output analysis, 416
 - random number generation, 412
 - random variate generation, 402, 412
 - steady-state, 417
 - terminating, 416
 - transient effects, 417
 - validation, 420
 - variance reduction, 419
 - verification, 421
 - warmup period, 418
- Single channels, 5
- Social optimization, 311
- Software
 - QtsPlus, 40
- Sojourn time, 191
- Spares, 88
- SPT rule, 149, 150, 153, 154
- Squared coefficient of variation (SCV), 221, 358
- Stages, *see* Phase
- Starting state, *see* Initial state
- State dependence
 - arrivals, 85, 95
 - service, 4, 91, 246, 252
- Stationarity, 4
- Stationary distribution, 31
- Stationary equations, 32
- Statistical equilibrium, 39
- Statistical inference, 317
- Steady state, 31
 - arrival-point probabilities, 64, 259
 - birth–death process, 49
 - departure-point probabilities, 225
- Stieltjes integral, 226
- Stieltjes transforms, 235, 456
- Stochastic balance, 304
- Stochastic matrix, 27
- Stochastic orderings, 333
- Stochastic process, 24
 - birth–death, 29, 49
 - Markov chain, 25
 - Markov process, 24
 - Poisson, 16
- Successive substitution, *see* Numerical methods
- Symbols, 7
- System
 - approximations, 345
 - capacity, 5
 - number in, 10
 - size, 5
- t distribution, 416
 - confidence intervals, 416
 - paired-*t* test, 419
- Tandem queues, 181
- Taylor series method, 378
- Telephony, 2
- Time averages, 20, 34
- Traffic equations, 188
- Traffic intensity, 9
 - estimation, 321
- Transforms, 455
 - Laplace, 99, 455
 - numerical inversion, 385

- Stieltjes (Laplace–Stieltjes), 235, 456
- Transient analysis, 97
 - $G/M/1$, 270
 - $M/G/1$, 244
 - $M/G/\infty$, 255
 - $M/M/\infty$, 101
 - $M/M/1$, 98
 - $M/M/1/1$, 97
 - simulation, 416
- Transient behavior, *see* Transient analysis
- Transient state, 38
- Transition probabilities, 26, 226, 260
- Truncated queues, 5, 76, 81, 241, 257, 269

- Uniform distribution, 20, 400, 412, 447
- Uniformized imbedded Markov chain, 385
- Unlimited service, *see* Service, ample
- Utilization rate (ρ), 9

- Vacation in service, 246
- Value iteration, 308
- Variance
 - table of, 447
- Variance reduction techniques, 416, 419
- Virtual idle time, 274
- Virtual waiting time, 191, 250, 263, 305, 347

- Waiting times, 8
 - distributions, 64
 - expected value, 10
 - higher moments, 235
 - recursion, 14, 284, 330
 - virtual, 191, 250, 263, 305, 347
- Weibull distribution, 406
- Wiener process, 354, 355
- Wiener–Hopf integral equation, 285
- Work backlog, 191, 305
- Work conserving, 305