

## CONDENSED MATTER PHYSICS

## Giant coercivity and enhanced intrinsic anomalous Hall effect at vanishing magnetization in a compensated kagome ferrimagnet

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Ferrimagnets that can be driven to magnetic compensation show promise for use in spintronics as they exhibit a finite anomalous Hall effect at zero magnetic field without having a substantial magnetic moment. Compensated ferrimagnet spintronic devices with both a large anomalous Hall effect and a high coercivity would be simultaneously easy to read and difficult to erase. The kagome ferrimagnet  $\text{TbMn}_6\text{Sn}_6$  has been reported to host a large intrinsic anomalous Hall effect. Here, we demonstrate that doping the Mn sites with Cr drives the system toward magnetic compensation. For nearly compensated compositions at low temperatures, giant coercive fields exceeding 14 T are observed. Additionally, Cr doping markedly enhances the intrinsic anomalous Hall effect, which can be attributed to a shift in the Fermi level. Our results extend the range of unique magnetic states observed in kagome materials, demonstrating that chemical doping is an effective strategy to tune and realize these states.

## INTRODUCTION

Compensated ferrimagnets have recently emerged as a promising material platform for spintronics applications, combining advantageous properties of both ferromagnets and antiferromagnets (1–3). In antiferromagnets, the net moment is zero by symmetry. This feature leads not only to faster switching times, reduced stray fields, and resistance against external fields but also to difficulty to read and write magnetic domains. A related class of Luttinger-compensated ferrimagnets (4) also features exact compensation, albeit not by symmetry but by virtue of Luttinger theorem.

These can be insulators, or half-metals (metallic in one spin channel). A more promising possibility is chemically compensated ferrimagnets, which are metallic in both spin channels. There, the net magnetization can be tuned to zero by controlling the chemical composition of their inequivalent spin sublattices. A great advantage here is that the net magnetic moment can be tunable by chemical composition or temperature so that it is small, but not zero, thus allowing domain control by external fields. Another important advantage is that despite near-zero magnetization, compensated ferrimagnets allow for large (on the order of hundreds meV) exchange splitting. Consequently, they can exhibit finite anomalous Hall effect (AHE), providing a relatively straightforward reading mechanism.

A well-known class of compensated ferrimagnets is the rare-earth transition-metal alloys (5). In these materials, the magnetic moments of rare-earth and transition-metal ions are antiferromagnetically coupled,

and the net magnetic moment can be controlled by tuning their relative concentrations. The magnetization of the rare-earth and transition-metal moments has different temperature dependencies. As a result, zero net magnetization occurs at a compensation temperature, denoted as  $T^*$ , which can be smoothly modulated by chemical composition. While amorphous materials have dominated the study of rare-earth transition-metal compensated ferrimagnets, crystalline compounds have received comparatively less attention. Unlike amorphous materials, crystalline materials host well-defined Bloch energy bands, characterized by Berry curvatures capable of generating large intrinsic AHEs. Consequently, the search and design of materials with large Berry curvature has also emerged as a key focal point in spintronics research (6).

Here, we report the realization of compensated ferrimagnetism in a rare-earth transition-metal intermetallic compound,  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$ .  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  belongs to the  $RT_6X_6$  ( $R$  = rare earth,  $T$  = transition metal,  $X$  = Si, Ge, Sn) family, known for its kagome lattice structure and intriguing band structure features such as flat bands, van Hove singularities, and Dirac points (7–9). Notably, the ferrimagnet  $\text{TbMn}_6\text{Sn}_6$  exhibits a large intrinsic AHE, which was initially attributed to Berry curvature arising from a gapped two-dimensional (2D) Dirac point above but near the Fermi level (10). This picture, if correct, would preclude Cr doping (hole doping) as a knob to tune the net magnetization without decreasing the intrinsic AHE, since that would shift the Fermi level away from the Dirac point. Furthermore, the 2D nature of the Dirac point would limit the large AHE to one particular geometry. Fortunately, it was later shown (11–13) that the AHE is accumulated over the entire Brillouin zone (BZ) and does not depend dramatically on the Fermi level; it was also shown that the AHE in similar compounds is 3D (14), which is also favorable for applications.

Here, we show that the substitution of Cr for Mn reduces the magnetic moment of the transition-metal site, which eventually turns  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  into a compensated ferrimagnet. Near compensation, we observed a divergence of the coercive field at low temperatures. Furthermore, contrary to the initially advocated 2D Dirac point scenario, Cr substitution substantially enhances intrinsic AHE, likely due to the tuning of the Fermi level to various Berry

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curvature hotspots (13). Our findings pave the way for the rational design of a compensated ferrimagnet with a large intrinsic AHE.

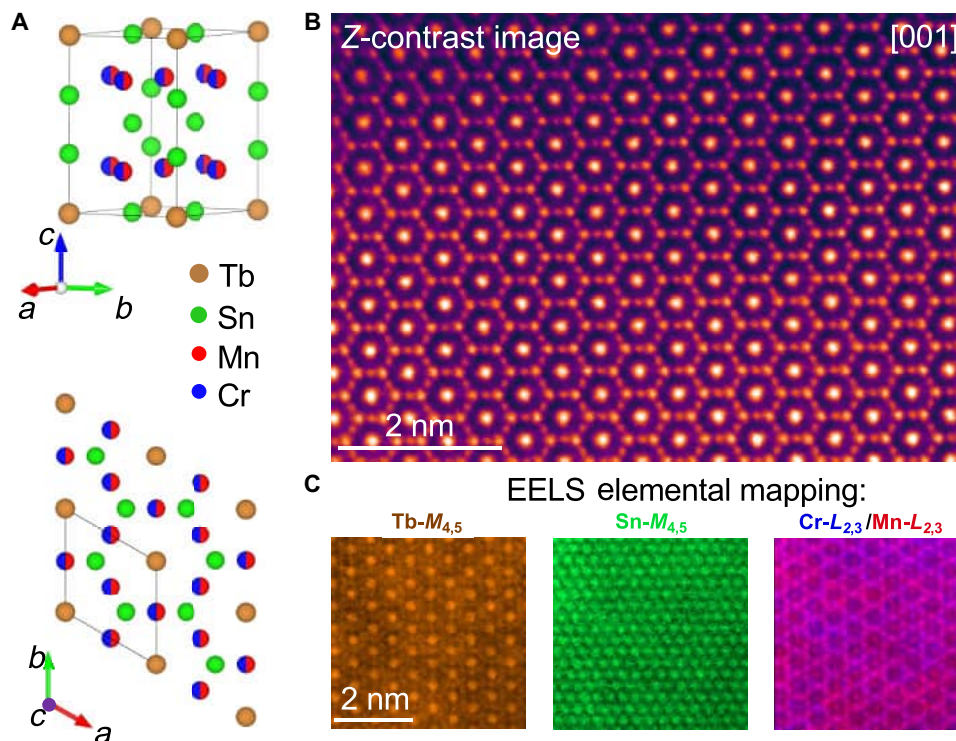
## RESULTS

### Basic characterization

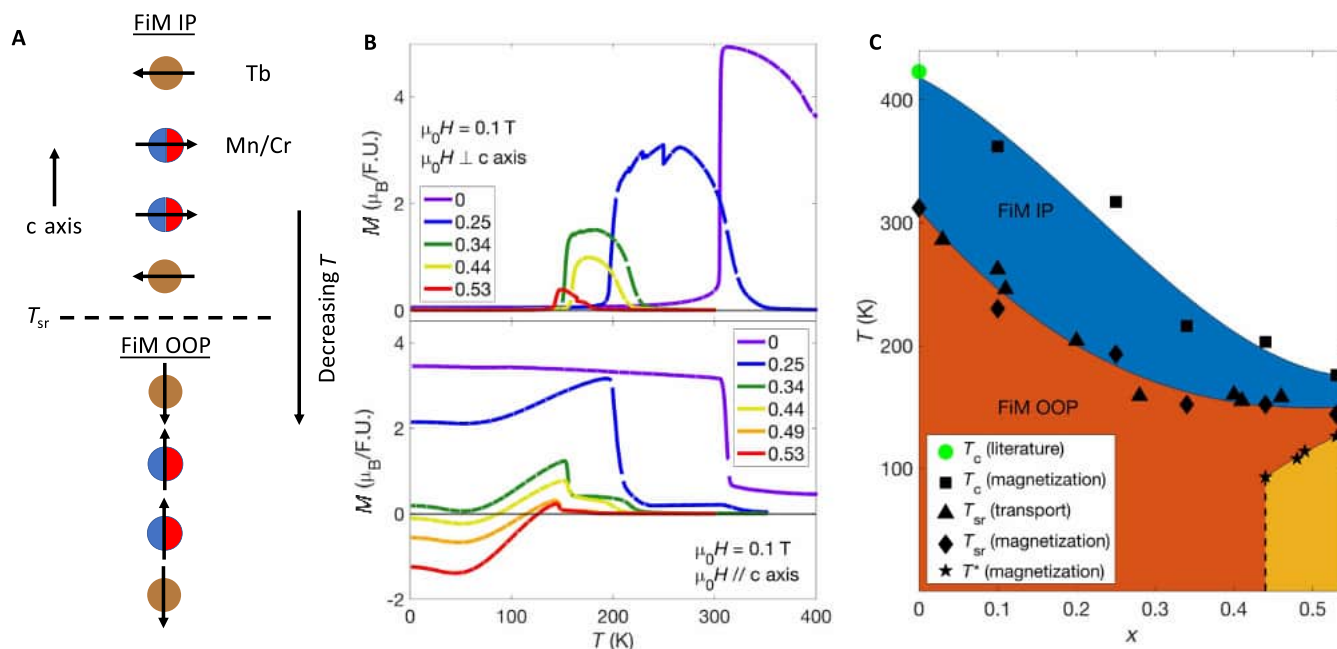
High-quality single crystals of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  with  $x$  up to 0.55 were grown using a Sn flux method. The magnetization and magneto-transport properties of these crystals were measured by the vibration sample magnetometry (VSM) option of a Quantum Design Physical Property Measurement System (PPMS) Dynacool and standard AC electrical transport techniques, respectively. Further details can be found in Materials and Methods. Figure 1A shows the crystal structure of  $\text{TbMn}_6\text{Sn}_6$ , which consists of two layers of kagome lattices formed by Mn atoms and one layer of triangular lattice composed of Tb atoms, viewed along the  $ab$  plane (top) and  $c$  axis (bottom). This crystal structure is preserved with Cr doping, as exemplified in the Z-contrast atomic-resolution scanning transmission electron microscope (STEM) image shown in Fig. 1B where the  $ab$ -plane kagome motif can be observed in an  $x = 0.55$  sample. Additionally, atomic-resolution electron energy-loss spectroscopy (EELS) was used to map out the elemental distribution in this sample, which is presented in Fig. 1C. Notably, the Cr substitution for Mn is relatively homogeneous even at the nanometer scale.

$\text{TbMn}_6\text{Sn}_6$  orders ferrimagnetically at 423 K ( $T_C$ ) (15). Within the layers, the Mn moments and the Tb moments order ferromagnetically, but the Tb and Mn layers couple antiferromagnetically, resulting in a ferrimagnetic structure where Tb and Mn moments are antiparallel

[schematically shown in the top portion of Fig. 2A (FiM IP)]. Neutron scattering studies on polycrystalline samples have revealed that at base temperature the size of Mn moment is 2.4 bohr magnetons ( $\mu_B$ ) and the size of Tb moment is 8.6  $\mu_B$ , leading to a net 5.8  $\mu_B$  per formula unit along the direction Mn moments (15). It should be noted that the saturated magnetization of single crystals of  $\text{TbMn}_6\text{Sn}_6$  from bulk magnetization measurements (10, 13, 16) is  $\approx 4 \mu_B$  per formula unit (F.U.), which is consistent with our results but less than the value extracted from neutron scattering; this discrepancy has previously been noticed (16) and may be due to inherent differences between polycrystalline and single-crystal samples. Figure 2B shows the temperature dependence of the magnetization of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  with a small in-plane (top) and out-of-plane (bottom) magnetic field of 0.1 T. The ferrimagnetic ordering temperature of the  $x = 0$  sample exceeds the instrument limit (400 K); thus, it is not visible within this dataset. Nevertheless, a sudden decrease in in-plane magnetization and an increase in out-of-plane magnetization are observed at 309 K ( $T_{sp}$ ). The abrupt change in magnetization is associated with a spin reorientation transition (10), where the Tb and Mn moments switch from a high-temperature in-plane orientation to a low-temperature out-of-plane orientation (FiM OOP), as shown in the bottom portion of Fig. 2A. This transition can be attributed to the competition between the  $c$ -axis uniaxial anisotropy favored by the Tb moments and the easy-plane anisotropy preferred by the Mn moments. At high temperatures, the Tb moments fluctuate more due to a weaker exchange interaction, allowing the easy-plane anisotropy of the Mn moments to dominate. At low temperatures, the scenario is reversed, and the  $c$ -axis uniaxial anisotropy of the Tb moments prevails (13, 17).



**Fig. 1. Atomic-scale visualization of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  kagome lattice.** (A) Atomic models of the  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  lattice viewed along the  $ab$  plane in the top panel and along the  $c$  axis in the bottom panel. The kagome layers have a mixed occupancy of Cr dopants and Mn (depicted in blue and red, respectively). (B) Z-contrast atomic-resolution STEM image of an  $x = 0.55$  crystal viewed along the  $c$  axis. (C) Color maps of elemental distribution by atomic-resolution EELS. Left, Tb ( $M_{4,5}$  edge); middle, Sn ( $M_{4,5}$  edge); right, mixed distribution of Cr and Mn ( $L_{2,3}$  edges).



**Fig. 2. Spin reorientation transition and the phase diagram of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$ .** (A) Orientation of magnetic moments of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  in the high-temperature ferrimagnetic in-plane (FiM IP) (top) and low-temperature ferrimagnetic out-of-plane (FiM OOP) (bottom) phases. (B) Low-field magnetization as a function of temperature of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  for a variety of  $x$  with an in-plane (top) or out-of-plane (bottom) magnetic field of 0.1 T applied. The sudden decrease/increase of in-plane/out-of-plane magnetization indicates the spin reorientation transition at  $T_{\text{sr}}$ . F.U., formula unit. (C)  $x$ - $T$  phase diagram of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  as determined from magnetization and transport measurements.

As shown in Fig. 2B, the temperature dependence of the in-plane magnetization for Cr-doped samples closely resemble that of the parent compound except that  $T_{\text{c}}$ ,  $T_{\text{sr}}$ , and the magnitude of the magnetization steadily decrease with increasing  $x$ . This is consistent with a previous study on polycrystalline samples, where Cr doping reduced the average moment size of the transition metal, thereby suppressing the overall ferrimagnetic ordering temperature (18). However, the out-of-plane magnetization exhibits a more intriguing behavior. While the features at  $T_{\text{c}}$  and  $T_{\text{sr}}$  are consistent with the in-plane magnetization data, the out-of-plane magnetization demonstrates a strong temperature dependence below  $T_{\text{sr}}$ , contrasting with the nearly constant magnetization observed in the parent  $\text{TbMn}_6\text{Sn}_6$  over this temperature range. For the Cr-doped samples, the out-of-plane magnetization decreases with temperature, reaching a local minimum near 60 K. For  $x$  greater than 0.4, the magnetization continues to decrease even after reaching zero at  $T^*$ , resulting in negative value for temperatures below  $T^*$ . This anti-alignment of the moment with the external magnetic field has been observed in other ferrimagnets driven to magnetic compensation by changing temperature (19).

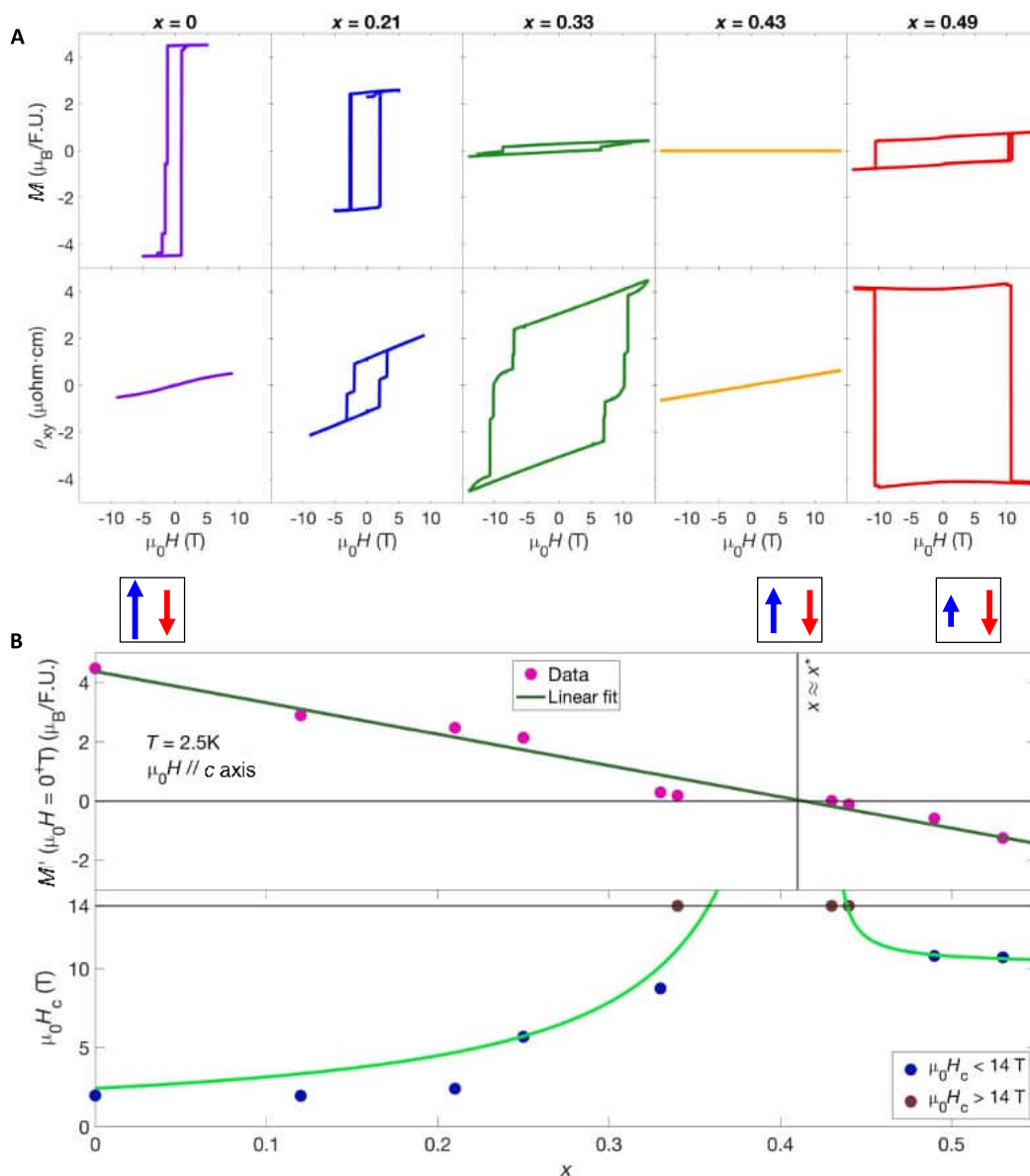
Using the magnetization data presented here and the resistivity data presented in note S1, along with the literature value of  $T_{\text{c}}$  for  $x = 0$  (15, 16), an  $x$ - $T$  phase diagram was constructed and is shown in Fig. 2C. It can be seen that both  $T_{\text{c}}$  and  $T_{\text{sr}}$  decrease rapidly with increasing  $x$  in the low  $x$  regime, but become relatively constant with further doping above approximately  $x = 0.3$ . For higher Cr concentration,  $T^*$  emerges, delineating the boundary of a region where the magnetization is anti-aligned with the external field during the cool-down process. In the following sections, we will examine the hysteresis loop of both the magnetization and the anomalous Hall resistivity as a function of doping and temperature. Our results reveal that this

unusual behavior arises from magnetic compensation driven by Cr doping and thermal fluctuations of Tb moments.

### Chemical doping–induced magnetic compensation and giant coercivity

We begin by examining the doping dependence at the base temperature. Figure 3A shows the magnetization (top row) and  $\rho_{xy}$  (bottom row) as functions of the magnetic field parallel to the  $c$  axis at 2.5 K for five representative Cr concentrations. The magnetization of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  shows a clear hysteresis loop characteristic of a hard ferrimagnet with uniaxial anisotropy. As the Cr concentration increases, the saturated magnetization decreases, reaching a value near zero at a composition  $x^* \approx 0.43$ , before increasing again for  $x > x^*$ . The reduction in saturated magnetization is accompanied by a considerable increase in the coercive field. A previous neutron scattering study on low- $x$   $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  revealed that Cr doping at the Mn site reduces the moment on that site without substantially changing the moment on the Tb site (18). Extending this observation to higher dopings suggests that at  $x^*$  the total magnetization from the Mn/Cr and Tb moments becomes equal, resulting in a net magnetization of zero. As  $x$  increases beyond  $x^*$ , the total magnetization of the Tb moments surpasses that of the Mn/Cr moments, causing the net magnetization to rise again, aligning with the Tb moment.

To confirm this hypothesis, we plot  $M'$  in the top panel of Fig. 3B, where  $M' = M$  for  $x < x^*$  and  $M' = -M$  for  $x > x^*$ . Here,  $M$  represents the experimentally measured zero field magnetization after the application of a large positive field to polarize the moments. The assumption is that  $M'$  corresponds to the net magnetization of the Mn/Cr moments minus the net magnetization of the Tb moments.



**Fig. 3. Magnetic compensation and giant coercivity through chemical doping.** (A) Magnetization (top row) and  $\rho_{xy}$  (bottom row) as a function of magnetic field for a variety of  $x$  at 2.5 K. Plots in the same column are from the same  $x$ , which is indicated by the text on top of the column. (B) Top:  $M'$  (described in the main text) as a function of  $x$  at 2.5 K with decreasing magnetic field. Bottom:  $\mu_0 H_c$  as a function of  $x$  from magnetization measurements.

$M'$  decreases roughly linearly as a function of  $x$  while (by construction) crossing zero at  $x^*$ . A linear fit of these data indicates that the average moment decreases by roughly  $1.8 \mu_B$  per Cr substitution, in good agreement with the previous neutron scattering study (18). Since each Mn ion has a moment of  $2.4 \mu_B$  in  $\text{TbMn}_6\text{Sn}_6$ , this implies that each Cr ion in  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  carries a moment of  $0.6 \mu_B$  if the Mn moments remain the same regardless of doping. This is quite similar to the value of  $0.48 \mu_B$ , which was found to be the moment per Cr ion in  $\text{TbCr}_6\text{Ge}_6$  via neutron scattering measurements (20). Thus, Cr is likely in a low-spin state in these materials.

To gain further insight, we performed density functional theory (DFT) calculations to investigate the magnetic state of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$ . We found that for  $\text{TbCr}_2\text{Mn}_4\text{Sn}_6$  (with both DFT + U frozen core

$\text{Tb}^{3+}$ ), the lowest energy states correspond to Cr forming 1D in-plane antiferromagnetic chains. Thus, a potential major contribution to the magnetic moment reduction with Cr doping is due neither to dilution of the Mn sublattice by nonmagnetic ions nor to suppression of Mn moments, but by the tendency of Cr appearing next to each other in the plane to form antiferromagnetic clusters. The results of these calculations are shown in note S2.

The bottom panel of Fig. 3B shows the coercive field,  $\mu_0 H_c$ , extracted from the magnetization measurement. The coercive field approximately follows the inverse of the zero-field magnetization, consistent with the flipping of the magnetization when the Zeeman energy  $\mu_0 H_c M$  overcomes the domain depinning energy (21, 22).

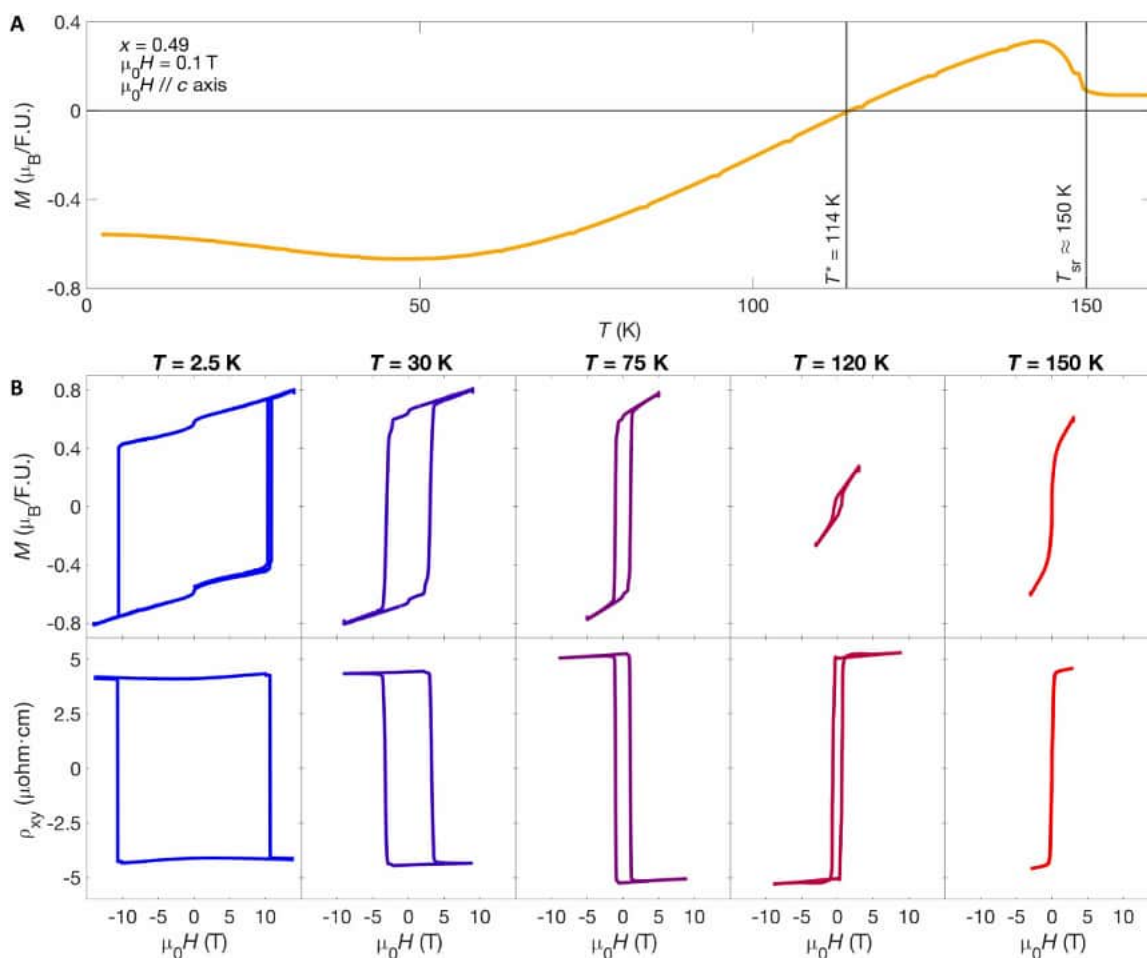
Near  $x^*$ , the coercive field exceeds 14 T, the maximum field achievable by the PPMS used. To our knowledge, this is the largest coercive field ever observed in a crystalline material (23). This massive coercivity is a result of the small magnetic moment observed combined with the large magnetocrystalline anisotropy of the Tb ions at low temperatures (13, 24). These observations are consistent with Tb(Mn<sub>1-x</sub>Cr<sub>x</sub>)<sub>6</sub>Sn<sub>6</sub> behaving as a nearly compensated ferrimagnet, exhibiting a giant coercive field at low temperatures between  $0.30 \lesssim x \lesssim 0.45$ , where the net Tb moments and the net Mn/Cr moments nearly cancel each other.

In contrast to magnetization,  $\rho_{\text{AH}}$  increases with Cr concentration. For samples very close to  $x^*$ , the hysteresis in  $\rho_{\text{AH}}$  cannot be detected by magnetic field sweep at base temperature simply because the coercive field exceeds 14 T. However, when the sample is field-cooled from high temperature, a large  $\rho_{\text{AH}}$  can still be observed even as  $M$  approaches zero (see fig. S12E in note S6). For  $x > x^*$ , the AHE switches sign, i.e.,  $\rho_{\text{AH}}$  switches from positive to negative when the magnetization is positive. Given that the energy bands near the Fermi level are primarily composed of transition-metal d-orbitals, the sign change of the AHE is consistent with the AHE being driven by the transition-metal sites. In-depth analysis of the AHE will be presented in the “Enhancement of AHE with doping” section.

### Temperature-induced magnetic compensation

In addition to chemical substitution, temperature is another effective parameter for controlling magnetic compensation. Figure 4A displays the previously shown out-of-plane magnetization as a function of temperature for a sample with  $x = 0.49$ . These data were collected as the sample is cooled under a 0.1-T  $c$ -axis magnetic field. At the spin reorientation transition temperature  $T_{\text{sr}} = 150$  K, the magnetization abruptly increases as all moments reorient from in-plane to out-of-plane. Below  $T_{\text{sr}}$ , the magnetization reaches a maximum and then decreases with decreasing temperature. It crosses zero at  $T^* = 114$  K and remains negative for temperatures below  $T^*$ .

To gain a deeper understanding of this behavior, we measured the field dependence of magnetization and Hall resistivity  $\rho_{xy}$  across a range of temperatures, as shown in the top and bottom rows of Fig. 4B, respectively. Each column represents data taken at the same temperature, which is noted above the column. In the 2.5 K data, in addition to the large hysteresis loop, a small additional paramagnetic contribution can be observed near  $\mu_0 H = 0$  T in the magnetization versus field data. This contribution is highly sample dependent and only noticeable in samples with  $x$  greater than 0.4, suggesting that it may result from either disordered spins or a minor impurity phase. As the temperature increases,  $\mu_0 H_c$  decreases substantially (the temperature



**Fig. 4. Temperature dependence of magnetization and sign reversal of  $\rho_{\text{AH}}$ .** (A) Low-field magnetization with an out-of-plane magnetic field for  $x = 0.49$  as a function of temperature. (B) Magnetization (top row) and  $\rho_{xy}$  (bottom row) as a function of magnetic field at a variety of temperatures for the same sample. Plots in the same column are taken at the same temperature, which is indicated by the text on top of the column.

dependence of  $\mu_0 H_c$  for multiple samples is detailed in note S3). Near  $T^*$  ( $\approx 120$  K), the hysteresis loop in  $\rho_{xy}$  reverses direction, whereas the magnetization vanishes. As temperature increases above  $T^*$ , the magnetization increases again. The temperature dependence of the magnetization and the AHE mirrors the doping dependence at base temperature, indicating that the magnetic compensation can also be induced by temperature sweep. Instead of a reduction in average Mn/Cr moments due to Cr substitution, the magnetic compensation at  $T^*$  is driven by the reduction of the average Tb moments at elevated temperatures due to thermal fluctuations, which is the same mechanism that drives the spin reorientation transition (13). Such temperature-driven compensation is commonly seen in other rare-earth transition-metal alloy system, where sign switching of  $\rho_{AH}$  has been observed (25).

With these insights, we can now explain the magnetization reversal observed in the field-cooled measurements. In samples with  $x > \approx 0.4$ , the Mn/Cr moments have decreased sufficiently that the Tb moments can match them in magnitude at  $T^*$ . As the temperature continues to decrease in these samples, the net Tb moment becomes larger in magnitude than the Mn/Cr moment, causing the net magnetization to become anti-aligned with the external field. Additionally, the coercive field rapidly increases with decreasing temperature and exceeds the external field below  $T^*$ . Consequently, the magnetization becomes trapped in the reverse configuration, leading to an overall negative value of magnetization.

### Enhancement of AHE with doping

We now turn to a more quantitative analysis of the AHE. Figure 5A and B shows the field dependence of Hall resistivity  $\rho_{xy}$  of samples with  $x = 0.28$  and  $0.43$  measured at various temperatures. Similar plots of samples with other Cr concentrations can be found in note S4. All samples exhibit pronounced hysteresis and a well-defined AHE at zero field. For higher Cr concentrations, the AHE changes

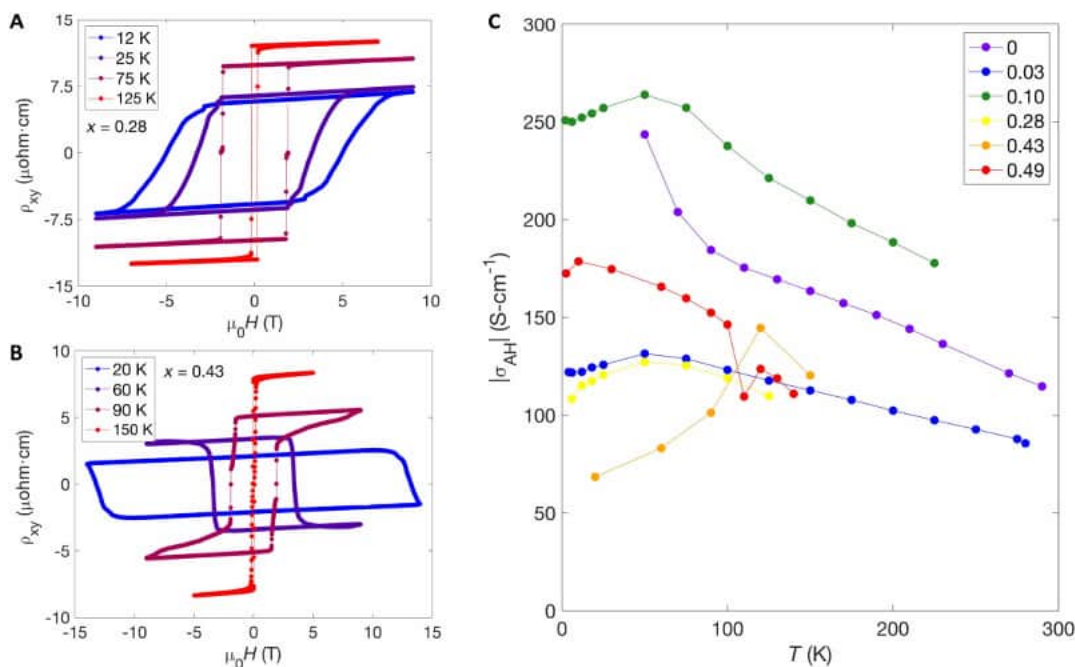
sign as a function of temperature as a result of magnetic compensation, as discussed in the previous section.

We calculated the anomalous Hall conductivity (AHC),  $\sigma_{AH}$ , using the relation  $\sigma_{AH} = -\rho_{AH} / (\rho_{xx,0}^2 + \rho_{AH}^2)$ , where  $\rho_{xx,0}$  is the zero-field longitudinal resistivity. Figure 5C presents the AHC as a function of temperature for various Cr concentrations. To facilitate comparison across different samples without the influence of sign switching, we plot only the absolute value  $|\sigma_{AH}|$ . The results for  $x = 0$  are consistent with previous reports (13). Cr doping does not substantially alter  $|\sigma_{AH}|$ , which changes by less than an order of magnitude across all measured Cr concentrations. Additionally, this quantity is not strongly dependent on temperature.

There are two contributions to the AHE: the intrinsic and extrinsic contributions. The former is originated from the Berry curvature of the band structure, while the latter is due to the skew scattering and side-jump mechanisms. Separating the intrinsic and extrinsic contributions of the AHE is a challenging task. Traditionally, this is done by fitting the AHC versus longitudinal conductivity to the function  $\sigma_{AH} = a\sigma_{xx}^2 + c$ , where  $c = \sigma_{AH}^{int}$  is the intrinsic contribution, and the  $a\sigma_{xx}^2$  term corresponds to extrinsic skew scattering and side-jump contributions (26). However, it has been shown that this function does not adequately fit the AHC in  $\text{TbMn}_6\text{Sn}_6$ . An additional nonstandard term, tentatively associated with spin fluctuations as discussed in (13), can be included to achieve a better fit, leading to an equation of the following form

$$\sigma_{AH} = a\sigma_{xx}^2 + c + d\sigma_{xx}^{-1} \quad (1)$$

We fitted the  $\sigma_{AH}$  versus  $\sigma_{xx}$  data for each sample, as shown in Fig. 6A and B for low and high doping levels, respectively, to extract  $\sigma_{AH}^{int}$  and  $\sigma_{AH}^{ext}$ . The dashed lines represent fits to the standard anomalous Hall equation, while the solid lines include an additional term proportional



**Fig. 5. AHE of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$ .** (A and B)  $\rho_{xy}$  as a function of  $\mu_0 H$  at several temperatures for  $x = 0.28$  and  $0.43$ , respectively. (C) AHC  $|\sigma_{AH}|$  as a function of temperature for a variety of dopings of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$ .

to  $\frac{1}{\sigma_{xx}}$  (Eq. 1). This additional term decisively improves the fit, especially for higher dopings.

The doping dependence of the fit parameters is presented in Table 1 and plotted in Fig. 7. It can be seen that the  $a$  coefficient associated with the skew scattering changes sign and increases dramatically as Cr concentration increases. This can be explained by the increase of disorder scattering due to Cr doping, and it is consistent with the substantial drop of residual resistance ratio (RRR), as shown in Fig. 7A. The extracted nonstandard parameter ( $d$ ) increases roughly an order of magnitude over the measured dopings as shown in Fig. 7B, which is consistent with the enhanced Tb fluctuations corresponding to reduction of  $T_{sr}$  with increasing  $x$ . Also shown in this panel is a substantial enhancement of the extracted parameter  $c$  ( $= \sigma_{AH}^{int}$ ) in orange. To further validate these fits, analyses using the resistivity form of the AHE equations are detailed in note S4 and yield similar results.

This extracted  $\sigma_{AH}^{int}$  with increasing  $x$  is plotted again in orange in Fig. 7C. Also shown is a simultaneous development of a large negative  $\sigma_{AH}^{ext}$  with increasing  $x$  (results shown for data taken at 50 K). Because of the opposite signs of the intrinsic and extrinsic contributions, the overall  $|\sigma_{AH}^{total}|$  does not vary substantially with doping, and thus can be completely missed barring a proper scaling analysis.

## DISCUSSION

TbMn<sub>6</sub>Sn<sub>6</sub> has attracted appreciable attention due to reports of a large intrinsic AHE, initially attributed to quasi-2D gapped Dirac points, a characteristic feature of the kagome lattice band structure (10). However, this interpretation has been challenged by recent calculations, which suggest that the relevant Dirac points are far from the Fermi

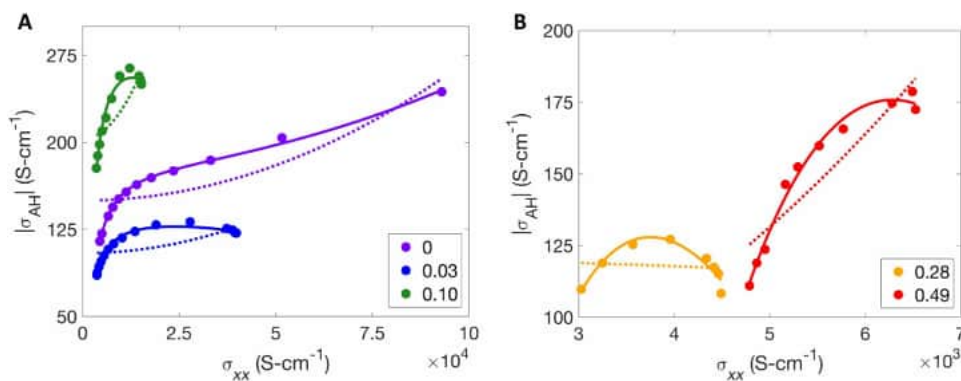
level and that the intrinsic AHE primarily arises from the anti-crossing of energy bands at other locations in momentum space (12, 13, 21). These studies further indicate that intrinsic AHC could be substantially enhanced if the material is hole-doped, a condition that can be achieved through Cr substitution.

The pronounced doping dependence of  $\sigma_{AH}^{int}$  observed in this work sheds more light on this debate. According to the gapped Dirac point model proposed in (10),  $\sigma_{AH}^{int}$  should decrease with Cr doping, as Cr doping lowers the Fermi level away from the gapped Dirac points, where the Berry curvature is concentrated. In contrast, the observed increase in  $\sigma_{AH}^{int}$  with Cr doping (i.e., hole doping) aligns better with scenarios proposed in (13, 12, 21). To verify this, we calculate  $\sigma_{AH}^{int}$  as a function of hole doping by rigid band shift of the Fermi level of TbMn<sub>6</sub>Sn<sub>6</sub>, as shown Fig. 7D. The good agreement between the experiment (orange curve in Fig. 7C) and the calculation (Fig. 7D) unambiguously confirms that multiple anti-crossing features in the band structure contribute to the intrinsic AHE.

In summary, we successfully grew Cr-doped single crystals of TbMn<sub>6</sub>Sn<sub>6</sub> and studied their magnetic and magnetotransport properties. Cr doping tunes the ferrimagnetic state toward magnetic compensation with a giant coercive field. Additionally, with increased Cr doping,  $\sigma_{AH}^{int}$  becomes substantially larger, in strong agreement with first-principles calculations. This work offers a pathway for identifying and synthesizing compensated ferrimagnets with large intrinsic AHCs.

## MATERIALS AND METHODS

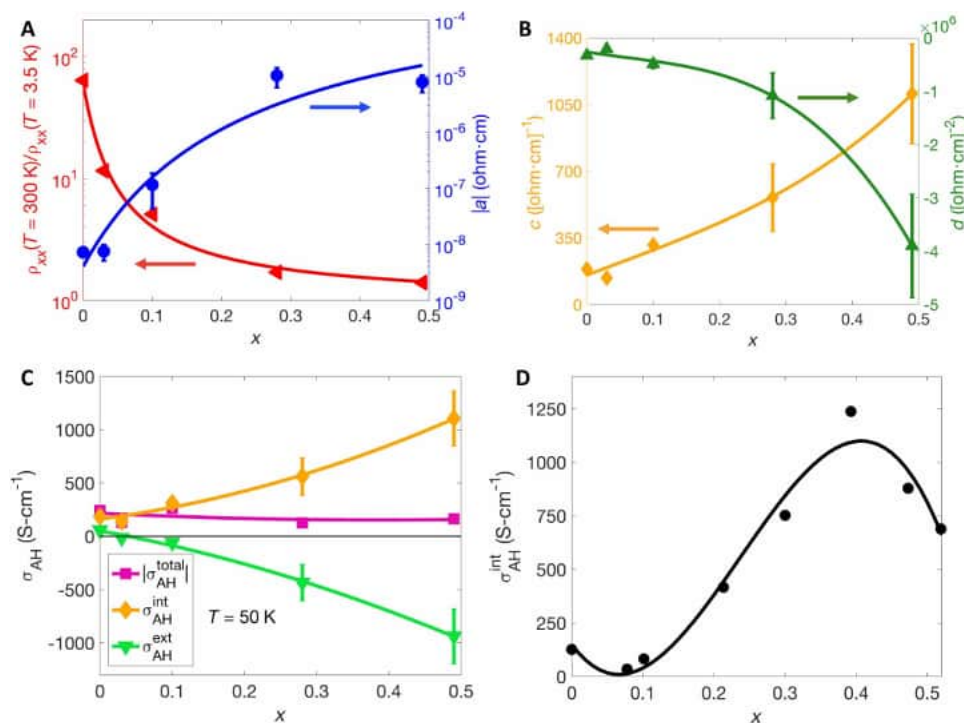
Single crystals of Tb(Mn<sub>1-x</sub>Cr<sub>x</sub>)<sub>6</sub>Sn<sub>6</sub> were synthesized utilizing a flux method similar to those previously reported for TbMn<sub>6</sub>Sn<sub>6</sub>



**Fig. 6. Fitting of AHC.** (A and B)  $|\sigma_{AH}|$  as a function of  $\sigma_{xx}$  for low (A) and high (B)  $x$ . Dashed lines correspond to fits utilizing the standard anomalous Hall fitting function, and solid lines show fits that include the extra term described in the main text.

**Table 1. Extracted parameters from conductivity fitting of AHE.**

$x$	$a$ (ohm-cm)	$c$ [(ohm-cm) <sup>-1</sup> ]	$d$ [(ohm-cm) <sup>-2</sup> ]
0	$(7.21 \pm 0.64) \times 10^{-9}$	$186 \pm 2.8$	$(-3.26 \pm 0.22) \times 10^5$
0.03	$(-7.48 \pm 2.45) \times 10^{-9}$	$140 \pm 3.5$	$(-2.04 \pm 0.18) \times 10^5$
0.10	$(-1.16 \pm 0.73) \times 10^{-7}$	$313 \pm 20.4$	$(-4.87 \pm 0.88) \times 10^5$
0.28	$(-1.03 \pm 0.41) \times 10^{-5}$	$562 \pm 177$	$(-1.08 \pm 0.43) \times 10^6$
0.49	$(-7.86 \pm 2.72) \times 10^{-6}$	$1107 \pm 262$	$(-3.90 \pm 0.97) \times 10^6$



**Fig. 7. Doping dependence of intrinsic and extrinsic AHE parameters.** (A) Residual resistivity ratio and fit parameter  $a$  (extrinsic contribution) as a function of  $x$ . (B) Fit parameters  $c$  (intrinsic contribution) and  $d$  (spin fluctuation contribution) as a function of  $x$ . (C)  $|\sigma_{\text{AH}}|$  and extracted  $\sigma_{\text{AH}}^{\text{int}}$  and  $\sigma_{\text{AH}}^{\text{ext}}$  at 50 K as a function of  $x$ . (D) Calculated  $\sigma_{\text{AH}}^{\text{int}}$  as a function of  $x$  using a rigid band model.

(10, 13). Mixtures of Tb pieces (99.999%), Cr powder (99.99%), Mn powder (99.95%), and Sn shot (99.999%) were loaded into Canfield crucible sets (27) with atomic ratios Tb:(Cr, Mn):Sn 2.5:15:97.5 and then vacuum-sealed in quartz tubes. These were heated up to 1150°C in 12 hours, held at this temperature for 12 hours, and then cooled to 600°C in 150 hours. Then, the growths were decanted in a centrifuge to separate the excess flux from the crystals. The doping concentration  $x$  of each crystal of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  used for measurements was determined using energy-dispersive x-ray spectroscopy (EDX) with a Sirion XL30 scanning electron microscope. Each crystal was polished before EDX measurements to remove residual flux on the surface of the crystal, and at least eight points on each crystal were measured. The measured spread in  $x_{\text{EDX}}$  for each crystal used in this study [and most of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  in general] was limited to a few hundredths, indicating that the doping is reasonably homogeneous throughout the crystal. A plot of  $x_{\text{EDX}}$  as a function of  $x_{\text{nom}}$  (the nominal  $\frac{\text{Cr}}{\text{Cr}+\text{Mn}}$  included in a growth) for a number of growths is presented in note S5 along with other EDX information. Throughout this paper, the measured  $x_{\text{EDX}}$  is referred to as  $x$  for simplicity. Using the growth technique outlined here, crystals of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  with  $0 < x < 0.55$  were able to be grown. Some crystals with  $x \gtrsim 0.3$  had Sn inclusions that could be detected via transport measurements (shorting of the resistivity below Sn's superconducting transition) and/or seen under a microscope. Measurements performed on these samples are not reported. It should be noted that for relatively high  $x_{\text{nom}}$  ( $\geq 0.5$ ), secondary phases of Cr (cubic crystals) and  $\text{Tb}_3\text{Sn}_7$  (plate-like crystals) were also found in the growths, and growths with  $x_{\text{nom}} > 0.6$  did not provide any crystals.

Transport measurements were performed on samples that were polished and cut by a wire saw to be bars with dimensions roughly 1 mm by 0.4 mm by 0.05 mm. Silver paste and gold wires were used to make five point (Hall pattern) contacts. These measurements were performed in a Quantum Design Dynacool PPMS with standard lock-in techniques in temperatures ranging from 1.7 to 400 K and in magnetic fields up to 14 T. To eliminate any contributions from contact misalignment, the in-line and Hall resistivities were symmetrized and anti-symmetrized with magnetic field, respectively.

Magnetization measurements were performed with the vibrating sample magnetometer option of the PPMS. Samples of the same size as the transport samples were attached to a quartz paddle with GE varnish such that the magnetic field was either out-of-plane (i.e., parallel to the  $c$  axis) or in-plane.

Electron microscopy imaging and spectroscopy were carried out by an aberration-corrected STEM in a Nion UltraSTEM 100 (28). The electron microscope was operated with an acceleration voltage of 100 kV, using a semi-convergence angle of 32 mrad. High-angle annular dark field (HAADF) images, also known as  $Z$ -contrast images due to the intensity being proportional to atomic number, were acquired with an inner (outer) semi-angle of 80 (200) mrad. EELS data were acquired with a collection semi-angle of 48 mrad.

The DFT calculations were performed using a full-potential linear augmented plane wave (FP-LAPW) method, as implemented in wien2k (29, 30). Spin-orbit coupling (SOC) was included using a second variational method. The generalized gradient approximation by Perdew, Burke, and Ernzerhof (PBE) (31) was used for the exchange-correlation potentials. To generate the self-consistent potential and charge, we used  $R_{\text{MT}} \cdot K_{\text{max}} = 8.0$  with Muffin-tin (MT) radii  $R_{\text{MT}} =$



2.8, 2.4, 2.4, and 2.5 atomic units (a.u.) for Tb, Mn, Cr, and Sn, respectively. The calculations are performed with 4800  $k$ -points in the full Brillouin zone (FBZ) and iterated until the charge differences between consecutive iterations are smaller than  $10^{-4}e$  and the total energy differences are lower than  $10^{-2}$  mRy/cell. The strongly correlated Tb-4f electrons are treated using the DFT +  $U$  ( $U = 10$  eV) method with the fully localized limit (FLL) double-counting scheme (32). TbMn<sub>6</sub>Sn<sub>6</sub> consists of six equivalent Mn atoms, which are replaced by one, two, or three Cr atoms depending on the doping concentration. We constructed one, three, and three configurations by replacing on one, two, and three Mn atoms to Cr atoms, respectively, and compared magnetic moments and total energies of each configuration. We also performed virtual crystal (VC) calculations to confirm if the rigid band model is reliable in this study. We kept lattice parameters the same as that of TbMn<sub>6</sub>Sn<sub>6</sub> since the lattice parameter change is less than 0.5% up to 33% doping (18).

A realistic tight-binding (TB) Hamiltonian was constructed using the maximally localized Wannier functions (MLWFs) method (33–35) implemented in Wannier90 (36) after the self-consistent DFT calculations were performed using Wien2k. To circumvent complications associated with the 4f state of the Tb atom while preserving the main physics, we used an open-core method for the 4f state. A set of 118 Wannier functions (WFs) consisting of Tb-5d, Mn-3d, and Sn-s,p orbitals offers an effective representation of the electronic structure near the Fermi level ( $E_F$ ). The intrinsic AHC can be calculated by integrating the Berry curvature over the BZ (37)

$$\sigma_{\alpha\beta} = -\frac{e^2}{\hbar} \int_{\text{BZ}} \frac{d\vec{k}}{(2\pi)^3} \sum_n f(E_{n\vec{k}}) \Omega_{n,\alpha\beta}(\vec{k}) \quad (2)$$

where  $f(E_{n\vec{k}})$  is the Fermi-Dirac distribution,  $\Omega_{n,\alpha\beta}(\vec{k})$  is the contribution to the Berry curvature from state  $n$ , and  $\alpha, \beta = \{x, y, z\}$ . We implemented Eq. 2 within an in-house ab initio TB framework (38). A dense  $256^3$   $k$ -point mesh is used for the AHC calculations in TB to ensure good convergence.

## Supplementary Materials

### This PDF file includes:

Notes S1 to S6

Figs. S1 to S12

Tables S1 to S6

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#### Acknowledgments

**Funding:** This work is mainly supported by the Air Force Office of Scientific Research under

grant FA9550-21-1-0068, the David and Lucile Packard Foundation, and the Gordon and Betty Moore Foundation's EPIQS Initiative, grant no. GBMF6759 to J.-H.C. This material is based upon work supported by the National Science Foundation Graduate Research Fellowship Program under grant no. DGE-2140004. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. O.P. acknowledges support from the University of Washington Mary Gates Research Scholarship. The TEM work was supported by NSF through the University of Washington Molecular Engineering Materials Center, a Materials Research Science and Engineering Center (DMR-2308979). The material synthesis was partially supported by NSF Partnerships for Research and Education in Materials (PREM) grant DMR-2121953. Part of this work was conducted at the Molecular Analysis Facility, a National Nanotechnology Coordinated Infrastructure site at the University of Washington, which is supported in part by the National Science Foundation (grant NNCI-1542101), the University of Washington, the Molecular Engineering & Sciences Institute, the Clean Energy Institute, and the National Institutes of Health. STEM imaging was conducted as part of a user project at the Center for Nanophase Materials Sciences (CNMS), which is a US DOE, Office of Science User Facility at Oak Ridge National Laboratory. Crystal structure images were generated with VESTA (39). The theory work was supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, Materials Sciences and Engineering Division. The software implementation was supported by the USDOE Early Career Research Program. Ames Laboratory is operated for the U.S. Department of Energy by Iowa State University under contract no. DE-AC02-07CH11358. I.I.M. acknowledges support from the National Science Foundation under award no. DMR-2403804. **Author contributions:** J.M.D., E.R., O.P., and K.H. grew the samples and performed the magnetization and magnetotransport measurements. G.R. performed the STEM experiments and analyzed the data with guidance from J.C.I.Y.L., Z.N., L.K., and I.I.M. performed DFT calculations and contributed to the theoretical analysis. J.-H.C. oversaw the project. J.M.D., E.R., and J.-H.C. wrote the manuscript with input from all authors. **Competing interests:** The authors declare that they have no competing interests. **Data and materials availability:** All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplementary Materials. The data for the plots in the main paper can be found at <https://zenodo.org/records/15677103>.

Submitted 24 March 2025

Accepted 31 July 2025

Published 29 August 2025

10.1126/sciadv.adx4671

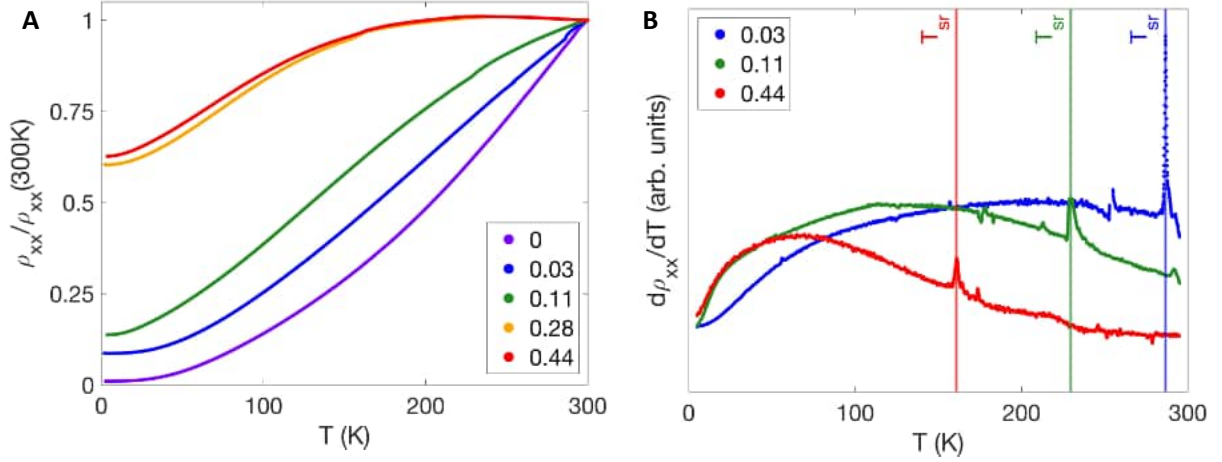


FIG. S1. **Resistivity as a function of temperature of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$ .** **A**, Normalized resistivity as a function of temperature for several  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  samples. **B**,  $\frac{d\rho_{xx}}{dT}$  as a function of temperature for  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  with  $x = 0.44$ .  $T_{sr}$  is marked with a vertical line.

x	config	total	Tb	Mn	Cr	$\Delta E$ (eV/cell)
0	1	-6.98	6.33	-2.42		
$\frac{1}{6}$	1	-7.43	6.30	-2.55	-1.92	
	1	-4.92	6.28	-2.63	-0.75	0.000
$\frac{1}{3}$	2	-7.43	6.29	-2.68	-1.89	0.121
	3	-7.49	6.28	-2.61	-2.06	0.145
	4*	-3.37	6.26	-2.60	0.11	-0.046
	1	-3.23	6.27	-2.56	-0.85	0.000
$\frac{1}{2}$	2	-4.35	6.27	-2.78	-0.99	0.013
	3	-4.74	6.27	-2.73	-1.16	0.059
	4*	-3.33	6.26	-2.69	-0.71	-0.075

TABLE S1. Spin magnetic moments of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  are presented for various configurations. The total spin magnetic moment is given in units of  $\mu_B/\text{cell}$ . The magnetic moments of Mn and Cr atoms are averaged across all sites within the given configuration and are reported in units of  $\mu_B/\text{atom}$ . To account for the orbital contribution,  $3.0 \mu_B$  should be added for each Tb atom. Configuration 4\* includes antiferromagnetic (AFM) interactions between Cr atoms. Total energies are compared between configurations with the same Cr concentrations.

atom in  $x = \frac{1}{3}$  is located the top of Mn4 while other two (Mn1, Mn3) are the top of two Cr atom. Fig. S3 present the spin structure of these two lowest energy configuration.

Table S2 presents the magnetic moments of individual Mn and Cr atoms in configurations where AFM interactions occur between certain Cr atoms. It shows the variation in magnetic moments, which depend on the details of neighboring atomic arrangements. The Mn atom with the largest magnetic moment in both  $x = \frac{1}{3}$  and  $x = \frac{1}{2}$  is located on the same plane as two AFM Cr atoms. Con-

versely, the Mn2 atom, which has the smallest magnetic moment among Mn atoms at  $x = \frac{1}{3}$ , is positioned directly above Mn4, while the other two Mn atoms (Mn1 and Mn3) are located above two Cr atoms. These two lowest-energy configurations are visualized in Fig. S3.

x	config	Mn1	Mn2	Mn3	Mn4	Cr1	Cr2	Cr3
$\frac{1}{3}$	4*	-2.58	-2.49	-2.61	-2.74	-2.10	2.12	
$\frac{1}{2}$	4*	-2.65	-2.69	-2.74		-2.18	2.08	-2.03

TABLE S2. The spin magnetic moments of Mn and Cr atoms in the lowest-energy configuration are presented. For the  $x = \frac{1}{3}$  ( $\frac{1}{2}$ ) composition, there are 4 (3) Mn sites and 2 (3) Cr sites. The magnetic moments exhibit variations depending on the specific atomic sites.

### C. Supplementary Note 3: Temperature dependence of $\mu_0 H_c$ near $x^*$

Fig. S4 presents the full temperature dependence of  $\mu_0 H_c$  for samples near  $x^*$ . Panels A-E show  $\mu_0 H_c$  as a function of temperature for several samples with blue data points indicating  $\mu_0 H_c < 14$  T and red data points indicating  $\mu_0 H_c > 14$  T. Panel F presents these same data along with the full 2 K  $x$  dependence by plotting  $\mu_0 H_c$  as a function of  $x$  at a variety of temperatures. It can be seen that  $\mu_0 H_c$  grows in all samples as the temperature decreases. There is no obvious change in  $\mu_0 H_c$  near  $T^*$  as sometimes seen in compensated ferri-magnets (41, 42).

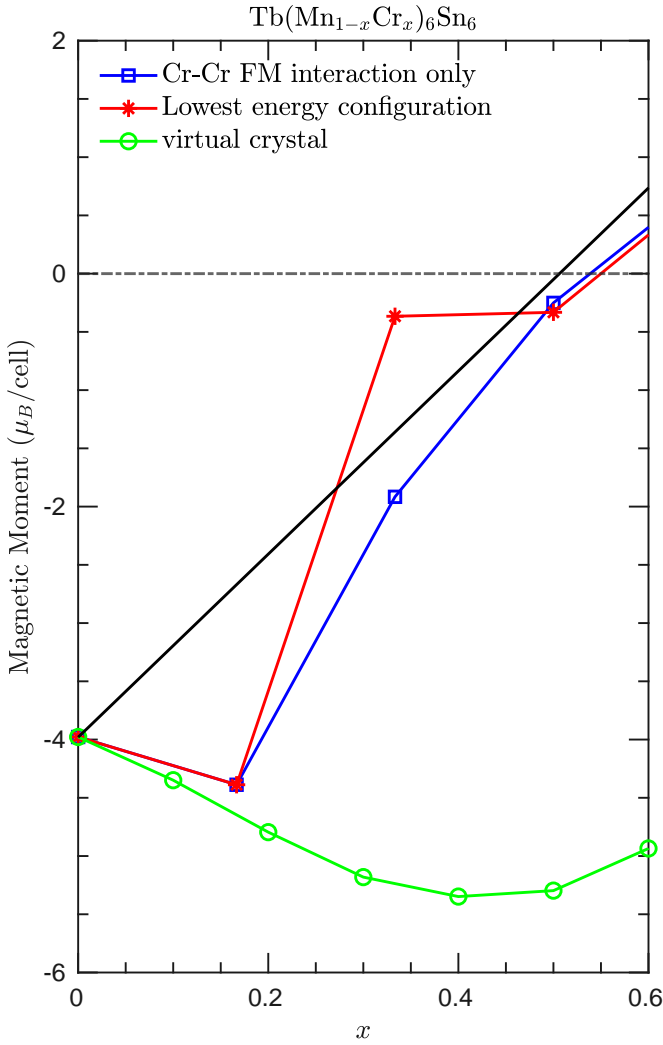


FIG. S2. **Calculated magnetic moments as a function of doping.** The total magnetic moment of  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  as a function of  $x$  is shown. It is well established that the interactions between in-plane Mn atoms are strongly FM (40), but the nature of the interactions between Cr atoms remains unclear. Two cases are separately presented: one assuming only FM interactions between Cr atoms (Blue) and the other including AFM interactions between Cr atoms (Red). The black solid line represents the calculation under the assumption that, with Cr doping, the magnetic moments at specific atomic sites remain constant at 9.33, -2.42, -1.11, and 1.21  $\mu_B$  for Tb, Mn, Cr, and the sum of all other contributions, respectively. In the VC calculation (Green), all interactions between the 3d transition metal atoms are assumed to be FM.

#### D. Supplementary Note 4: Analysis of AHE

Fig. S5 presents  $\rho_{xy}$  as a function of  $\mu_0 H$  at several temperatures for  $x = 0, 0.03, 0.10, 0.28, 0.43,$  and  $0.49$ , respectively. As noted in the main paper, a well-defined anomalous Hall effect can be observed.

Fig. S6 presents  $|\sigma_{AH}|$  as a function of  $\sigma_{xx}$  on a log-

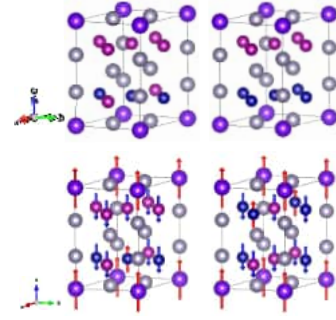


FIG. S3. **Calculated magnetic configurations.** Top: The configurations of the lowest-energy states (config. 1) for  $x = \frac{1}{3}$  (left) and  $x = \frac{1}{2}$  (right), considering only FM interactions between transition metal atoms.

Bottom: The spin configurations of the lowest-energy states (config. 4\*) for  $x = \frac{1}{3}$  (left) and  $x = \frac{1}{2}$  (right) are shown. In both cases, two Cr atoms within the same plane exhibit AFM interaction. The purple, red, blue, and gray spheres represent Tb, Mn, Cr, and Sn atoms, respectively. Red arrows indicate positive magnetic moments, while blue arrows represent negative magnetic moments.

log scale for  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  for a variety of  $x$ . Here  $\sigma_{xx}$  is defined as  $\sigma_{xx} = \frac{\rho_{xx,0}}{\rho_{xx,0}^2 + \rho_{AH}^2}$ . For  $x = 0.49$  data near  $T^*$  are removed as  $\rho_{AH}$  is very small as it crosses zero as a function of temperature and for  $x = 0.43$  only data from  $T > T^*$  are presented as there to a limited number of reliable data points available below  $T^*$  due to the massive  $H_c$  at low temperatures. These data imply that the AHE in  $\text{Tb}(\text{Mn}_{1-x}\text{Cr}_x)_6\text{Sn}_6$  may be largely due to an intrinsic Berry phase contribution when they are compared to the theory from Ref. (43): regardless of  $x$ ,  $\sigma_{xx}$  is between  $3 \times 10^3$  and  $10^5$   $\text{S}\cdot\text{cm}^{-1}$  and  $\sigma_{AH}$  is roughly constant (ie. does not change orders of magnitude) and of the order of  $10^2$   $\text{S}\cdot\text{cm}^{-1}$  as  $\sigma_{xx}$  is varied by over an order of magnitude.

As noted in the main text, fitting of  $\rho_{AH}$  as a function of  $\rho_{xx}$  for  $\text{TbMn}_6\text{Sn}_6$  is non-trivial. The resistivity form of the anomalous Hall fits are shown here to act as a check of the conductivity fits shown in the main paper. The accepted scaling of the anomalous Hall effect in general goes as  $\rho_{AH} = a + c\rho_{xx}^2$  where  $c = \sigma_{AH}^{int}$  (26), yet adding an additional cubic term has been observed to improve the fit quality (13). This cubic term has been argued to be related to the fluctuations of Tb ions (13) as noted in the main text. As seen in Fig. S7, our data agree with this observation of a cubic term enhancing the fit quality in  $\text{TbMn}_6\text{Sn}_6$  just as the  $\frac{d}{\sigma_{xx}}$  term enhances the fit quality of the conductivity data. As such, we present analysis of the AHE using a fitting function of the form

$$\rho_{AH} = a + c\rho_{xx}^2 + d\rho_{xx}^3 \quad (\text{S1})$$

here.