CS 330-002 Formal Methods and Models

Ivan Avramovic, George Mason University
(sample) Midterm, Spring 2018

Name: ________________________________

Q	Score	
1		20
2		20
3		15
4		15
5		15
6		15
Tot.		100

This test is governed by the GMU Honor Code. The paper you turn in must be your sole work. Help may be obtained from the instructor to understand the description of the problem but the solution must be the student's own work. The exam is closed book and closed notes, with the exception of a single sheet of note paper. Electronic devices are not permitted. Any deviation from this is considered an Honor Code violation.

1. (*20pts*) Answer **T** or **F** for each of the following questions. No explanation or justification is required for your answers.

   (a) Let $S = \{1, \{1\}, \{\{1\}\}, \{\{\{1\}\}\}, ...\}$. It follows that $S \cap 2^S = \emptyset$.

   T ***F***

   (b) $p \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q \vee \neg r)$ is an expression in *conjunctive normal form*.

   T ***F***

   (c) In an inference rule proof, if you succeed in deriving a line with a standalone $TRUE$ on it, then you've succeeded in proving your proposition.

   T ***F***

   (d) Evaluate: $\neg \forall n \in \mathcal{I}^+ : \exists m \in \mathcal{I}^+ : n \times m \neq n! \times m$

   T ***F***

   (e) In any loop in which k is guaranteed to have a well-defined numerical value, the expression $k + (k + 1) = 2k + 1$ is a provable loop invariant.

   T ***F***

2. (*20pts*) Prove that $(\neg b \lor \neg a) \to (b \land \neg c) \equiv b \land (c \to a)$.

(a) Use a truth table. For full credit, there must be at least one additional column in the table for each operator besides negations.

(b) Use algebra. For full credit, you must label your steps with the name of the laws you use.

3. (*15pts*) Prove that $(p \rightarrow (q \vee r)) \leftrightarrow ((p \wedge \neg q) \rightarrow r)$ using rules of inference (you may not use the *Substitution* rule). For full credit, you must annotate each step with the name of the rule you used.
Hint: the expression $q \vee \neg q$ may be introduced on any line as a *Tautology*.

4. (*15pts*) Given a string of characters S and an integer constant k, write a predicate logic formula which asserts that some k character long substring occurs at least twice in the string, in possibly overlapping but not identical positions. For example, if $S = banana$ and $k = 3$, then the substring *ana* occurs twice.

5. (*15pts*) Prove that $\sum_{i=0}^{n}(i+2)2^i = (n+1)2^{n+1}$ for all $n \geq 0$. For full credit, you must use a **formal** inductive proof form.

6. (*15pts*) Consider the following segment of code. Assume that i, x, and n are integers, and that $n \geq 1$.

$i \Leftarrow 1$
$x \Leftarrow 1$
while $i < n$ **do**
    $x \Leftarrow x + 2^i$
    $i \Leftarrow i + 1$

(a) State a loop invariant. For full credit, it must be a non-trivial invariant which gives meaningful information about the variables i and x.

(b) Prove the loop invariant you stated in part (a). You may use either the inference rule or the inline comment style of proof. Also, state what must be true at the end of the proof (show work). Note: a correct proof using an incorrect invariant may receive full credit.