CS 330-002 Formal Methods and Models

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(sample) Final, Spring 2018

Name: _____

Q	Score	
1		20
2		20
3		20
4		20
5		20
6		20
Tot.		100

Question 1 (true/false) is required. On the remaining questions, **your best 4 out of 5 scores will be used to determine your total grade** (i.e. you may freely skip one of the last five questions).

This test is governed by the GMU Honor Code. The paper you turn in must be your sole work. Help may be obtained from the instructor to understand the description of the problem but the solution must be the student's own work. The exam is closed book and closed notes, with the exception of a single sheet of note paper. Electronic devices are not permitted. Any deviation from this is considered an Honor Code violation.

1. (*20pts*) Answer **T** or **F** for each of the following questions. No explanation or justification is required for your answers.

   (a) Let
   $L_1 = \mathcal{L}((01 + 11)^*(00 + 10)^*)$,
   $L_2 = \{\Lambda, 0, 1, 01, 011, 0011, 1100, 0110, 1001, 0101, 1010, 0111\}$.
   Then $L_1 \cap L_2 = \{\Lambda, 01, 1100, 0110, 0101, 0111\}$.

   T ***F***

   (b) If L_1 is a language generated by some regular expression, and L_2 is a language generated by some finite automaton, then the language $L_3 = L_1 \cap L_2$ might not be a regular language.

   T ***F***

   (c) If a language is infinite, then it cannot be a regular language.

   T ***F***

   (d) If i is a non-negative integer, then 0^i10^i can be a string of only 1s.

   T ***F***

   (e) If for some deterministic finite automaton, $\delta^*(q_0, aba) = q_1$ and $\delta(q_1, a) = q_2$, then it follows that $\delta^*(q_0, abaa) = q_2$.

   T ***F***

2. (*20pts*)

(a) Give a regular expression for the language
$L = \{x \in \{0,1\}^* \mid$ the first two characters of x are the same as the last two characters of $x\}$.
Note 1: $0101 \in L$ but $0110 \notin L$.
Note 2: L includes strings shorter than 4 and longer than 4.

(b) Convert the regular expression $(a^*b + c)$ into a regular grammar with unit productions.

3. (*20pts*) Convert the following regular grammar into a regular expression. For full credit, eliminate symbols in the order S then A then B.

$\{S \to aS,\ S \to bB,\ A \to \Lambda,\ A \to aS,\ A \to bB,$
$B \to aA,\ B \to bB,\ B \to \Lambda\}$

4. (*20pts*)

(a) Eliminate the unit productions from the following grammar:
$\{S \to A,\ S \to aS,\ S \to bB,\ A \to C,\ A \to \Lambda,\ A \to bB,$
$B \to aA,\ B \to S,\ B \to bB,\ C \to aA,\ C \to \Lambda\}$

(b) Convert the folling grammar to a deterministic regular grammar:
$\{S \to aS,\ S \to aA,\ S \to bB,\ A \to \Lambda,\ A \to bB,\ A \to aA,$
$B \to aA,\ B \to bS\}$

(c) Convert the grammar from part (b) into a (non-deterministic) finite automaton.

5. (*20pts*)

(a) Give a deterministic finite automaton which will generate the language
$L = \{x \in \{a, b\}^* \mid$ the first character of x must appear at least twice more in $x\}$.

(b) Given pattern $P = aabaa$, give a deterministic finite automaton which generate language $L(P)$, that is, the set of strings over $\Sigma = \{a, b\}$ which contain P as a substring.

6. (*20pts*)

(a) The language L of strings of the form $a^i b^j$, where $i \geq j$ can easily be generated by a context free grammar, but not by a regular grammar. Show why L is not a regular language.

(b) Give a context free grammar for the language $L = \{a^i b^j a^j c^k \mid i, j, k \geq 0 \wedge k > i\}$.