
MATH 301

Number Theory

MW 9:00-10:15 am

West 1008

Instructor: Claire Merriman

Email: emerrima@gmu.edu

Office: Exploratory Hall 4405

Office hours:

See Canvas

Grading:

Reading and Preparation: 5%

In Class Activities: 6%

Homework: 10%

Quizzes: 9%

Midterms: 45% (15% each)

Final Exam: 20%

Grading Scale:

A+ 97-100 A 93-96 A- 90-92

B+ 87-89 B 83-86 B- 80-82

C+ 77-79 C 70-76

D 60-69 F 0-59

Midterms:

September 24, November 5,
December 3

Final Exam

December 15 7:30-10:15 am

Classroom Expectations

Learning mathematics requires you *do* mathematics. This means that you will spend part of class working on math problems, and you will have times that you struggle to solve them. The goal is for this to be a **productive struggle**, where you emerge with a greater understanding of the concepts.

It is also important that you come to class on time having completed the reading assignment for the day and stay until the end of class.

Course Information

The primary topics are in Chapters 1 through 6 of the Strayer textbook, as well as part of Chapter 8. These include properties of the integers and related sets, including divisibility properties, prime numbers and their distribution, congruences, Diophantine equations, arithmetic functions, primitive roots, quadratic residues, and cryptology.

The first three weeks of the course will cover introduction to proofs along with Chapter 1 of the Strayer textbook. Additional proof techniques will be introduced throughout the course.

This course also introduces methods of proof and emphasizes writing clear mathematical arguments. The standard for writing and presentation will be higher than in a calculus class, including on homework.

Textbook and Software

Elementary Number Theory by James K. Strayer

[Introduction to Proof via Inquiry-Based Learning](#) by Dana Ernst. The [author's website](#) has various options for free pdf versions.

Assignments will be graded and returned on **Gradescope**. Gradescope is available from Canvas, but it is easier to access if you also create an account with your GMU email address.

This course uses **Canvas** for course links, grades, and announcements. Rough course notes are available on Canvas.

Learning Outcomes

Upon successful completion of the course, students will be able to:

- Understand primes and factorization
- Understand modular arithmetic and some methods for solving systems of congruences
- Connect modular arithmetic to integer valued functions
- Solve problems using quadratic reciprocity
- Understand several techniques for solving Diophantine (integer solution) equations
- Increase student confidence in their ability to solve difficult math problems by using previous results, trying different methods, asking questions, and working with others
- Increase student confidence in their ability to judge when a solution is "done."
- Improve student's mathematical communication skills

I expect this course to be a place where you will be treated with respect. All members of this class are expected to contribute to a respectful, welcoming, and inclusive environment for every other member of the class. Students are expected to abide by [Mason's Non-Discrimination Policy](#) affirm our belief that student diversity is an invaluable asset for enriching the learning that happens in our course.

Assignment Descriptions

Preparation

You will have short assignments due before the start of each class to get familiar with the topics we will cover in class. Sometimes these questions will be short calculations, but they will often ask you to explain steps in the examples given in the textbook. These questions will be graded on a mix of completion and accuracy with the following rubrics:

- 0 points: Missing or minimal effort, such as only writing down the problem or a definition.
- 1 point: Contains a reasonable attempt to answer at least one question with significant mathematical errors or omissions (such as only answering one of two questions).
- 2 points: Contains a reasonable attempt to answer all questions with some mathematical errors.
- 3 points: Answers to all questions demonstrate understanding of the mathematical concepts, possibly with some minor errors.

Preparation assignments for the week will be available on the preceding Thursday. These assignments are due at the start of class and will **not be accepted more than 5 minutes after the start of class**.

The lowest 2 grades will be dropped.

In class activities

You will have problems to work on in groups throughout class time. These will range from calculation problems that check for understanding to more in-depth problems that benefit from group discussion. We may also do some other activities throughout class.

I will only grade one submission per group, so you should all work together. These problems typically require some amount of discussion. Often homework problems are continuations of the group questions.

In class activities are primarily graded on a mix of accuracy and completions out of 10 points. Since the lecture portion of class is important for being able to contribute to group discussion, excessive tardiness or leaving early will result in a reduced individual score:

- 5 to 10 minutes late or leave 5 to 10 minutes early: 3 points deducted from score.
- 10 to 15 minutes late or leave 10 to 15 minutes early: 5 points deducted from score.
- More than 15 minutes late, leave more than 15 minutes early, or unexcused absence: 0 points.

The instructor reserves the right to deduct additional points for distracting behavior, creating a hostile group environment, or lack of active participation in group work.

Lowest 3 grades will be dropped.

Homework

Weekly homework assignments will cover the material from the past week's lecture and reading assignments. You are encouraged to work together on the problems; however, **you must write up your own solutions and submit them individually**. Assignments should be legible—you should work out problems then write up a new, final version. Homework should be on a separate page or in a separate file from your course notes. I will provide TeX files for each homework assignment, for those who wish to type their assignments, but that is not required.

Each homework assignment will have two sections: "Proofs and Writing" and "Calculations and Algorithms". "Proofs and Writing" questions are always worth 4 points. The total point value will vary each week, but each

assignment counts as 1% of your final course grade. In order for grades to be calculated correctly, the score may need to be rescaled for Canvas.

Homework is due on **Gradescope** at the beginning of class on Wednesdays. Late homework assignments will receive a 20 percentage-point penalty per day.

After the homework is graded, **you may revise your answers to the Proofs and Writing question based on the feedback and resubmit for a new grade** due one week after the original due date. You may only resubmit problems where you had a reasonable attempt at a solution on the first submission. A reasonable attempt includes enough information to receive feedback. If the graded assignments are not returned by the class before the resubmission deadline, this deadline will be adjusted.

The lowest homework grade will be dropped.

Weekly Quizzes

There will be a 10 minute, 1-3 question quizzes at the start of class each Wednesday. These questions will be similar to the "Calculations and Algorithms" questions on the Homework. The total point value will vary each week, but each quiz counts towards 1% of your final course grade. In order for grades to be calculated correctly, the score may need to be rescaled for Canvas.

Lowest 2 scores are dropped.

Midterm Exams

There will be four midterm exams taken in class for the entire class period. Each midterm is worth 10% of your final grade.

A missed exam counts as a zero unless a valid excuse from a physician, the Dean's Office, or other official source is presented to your instructor and accepted. In rare circumstances, an extension might be granted for communicable illnesses that do not require a physician. You need to contact your instructor as soon as possible regarding your extenuating circumstance for consideration of an exception.

You may use a four-function calculator on the exams. You are not permitted to use any outside materials, resources, or other electronic devices (including but not limited to other calculators, mobile phones, smartwatches, etc.) on the exams. Any violation of this policy is a violation of the university's academic integrity policy.

Your lowest midterm exam score will be replaced with the final exam score, as long as it is higher.

Final Exam

The final exam in this course is cumulative and will follow the same format as the midterms. You may use a four-function calculator on the exam. You are not permitted to use any outside materials, resources, or other electronic devices (including but not limited to other calculators, mobile phones, smartwatches, etc.) on the exams. Any violation of this policy is a violation of the university's academic integrity policy.

Ways to get help

Office Hours and Tutoring

The best way to succeed in this course is to ask for help before you fall behind. This includes working with your classmates and asking questions during class. Here are other ways to get help:

- **Office hours:** Office hours are a time I am in my office to help you with the material in the course. They can also be a time to meet or work with other students in the class.
- **Email:** Please put "MATH 301" in the subject line. I will try to respond to emails within 24 hours on weekdays. Keep in mind that some math questions might be difficult to ask or answer via email—photos of your work help, but office hours are often more productive.

- **The Math Tutoring Center:** no appointment necessary tutoring provided by the mathematics department. See [Tutoring Center website](#) for information about how to access the Tutoring Center and for the current schedule.

Other course policies

Academic Honesty

George Mason has established institutional academic standards. Three fundamental principles to follow at all times are that:

1. all work submitted be your own, as defined by the assignment;
2. when you use the work, the words, or the ideas of others, including fellow students or online sites, you give full credit through accurate citations; and
3. if you are uncertain about the ground rules on a particular assignment or exam, ask for clarification. No grade is important enough to justify academic misconduct.

Statement on AI usage

You may not use generative AI, such as ChatGPT, Copilot, or Gemini, in this course. You may use standard spelling and grammar checking tools, which may or may not utilize AI, for spelling and grammar. Be aware that mathematical writing conventions may vary from standard English.

Late and missing work

For lecture participation, recitation worksheets, and homework, the dropped scores account for any missing work due to routine reasons (illness, commuting issues, work scheduling conflict, etc). In extreme, unavoidable situations that require missing more than a week of class, a makeup plan may be formulated upon the receipt of proper documentation. You need to contact your instructor as soon as possible regarding your extenuating circumstance for consideration of a makeup plan. Generally, makeup work for these assignments will only be permitted for documented reasons that require more than a week of absences.

A missed exam counts as a zero unless a valid excuse from a physician, the Dean's Office, or other official source is presented to your instructor and accepted. In rare circumstances, a 24 extension might be granted for midterms in the case of communicable illnesses that do not require a physician. You need to contact your instructor as soon as possible regarding your extenuating circumstance for consideration of an exception.

Important dates

First day of classes: Monday, August 25

Last day to drop with 100% tuition refund: Monday, September 8

Last day to drop with 50% tuition refund: Tuesday, September 16

Last day for unrestricted withdrawal: Tuesday, September 30

Last day for selective withdrawal (W on transcript): Monday, October 27

Last day of classes: Monday, December 8

Final exam period: Wednesday, December 10 through Wednesday, December 17

Tentative course schedule—Adjustments to schedule will be in Canvas

Date	Topic	Assignments Due
August 25	Introduction to the course, Strayer §1.1 Divisibility, Introduction to “Doing” Mathematics	None
August 27	Strayer §1.1 Divisibility, Direct Proofs, Logic, Proof by Contradiction	Prep: Ernst Ch 1 and §2.1; Course survey Homework 1
September 1	No Classes Labor Day	None
September 3	Strayer §1.1 Divisibility Using other proofs as a guide for new theorem, Well-Ordering Principle	Prep: Ernst §2.3 Homework 2
September 8	Strayer §1.2 Prime Numbers, Strayer §1.3 Greatest Common Divisors Mathematical Induction	Prep: Strayer Appendix A: Mathematical Induction
September 10	Strayer §1.4 The Euclidean Algorithm	Prep: Strayer §1.4 Homework 3
September 15	§1.5 The Fundamental Theorem of Arithmetic	Prep: Strayer §1.5
September 17	§1.5 The Fundamental Theorem of Arithmetic	Prep: Strayer §1.5 Homework 4
September 22	Strayer §2.1 Congruences	Prep: Strayer §2.1
September 24	Exam 1 Strayer Chapter 1 and Proof Methods	Exam 1
September 29	Strayer §2.2 Linear Congruences in one variable, Strayer §6.1 Linear Diophantine Equations	Prep: Strayer §2.2
October 1	Strayer §2.3 The Chinese Remainder Theorem	Prep: Strayer §2.3 Homework 5
October 6	Strayer §2.3 The Chinese Remainder Theorem, Strayer §2.4 Wilson’s Theorem	Prep: Strayer §2.4
October 8	Strayer §2.5 Fermat’s Little Theorem, Strayer §2.6 Euler’s Theorem	Prep: Strayer §2.5 Homework 6
October 13	No Classes—Fall Break	Fall Break
October 15	Strayer §3.2 The Euler Phi-Function, Strayer §5.1 The Order of an Integer; Primitive Roots	Prep: Strayer §3.2 Homework 7
October 20	Strayer §5.1 The Order of an Integer; Primitive Roots	Prep: Strayer §5.1

October 22	Strayer §5.2 Primitive Roots for Prime Numbers, Strayer §8.2 Cryptography	Prep: Strayer §5.2 Homework 8
October 27	The RSA Encryption System, Strayer §8.3 Primality Testing	Prep: Uploaded notes
October 29	Strayer §6.2 Nonlinear Diophantine Equations; A Congruence Method, Strayer §6.5 Representations of an Integer as a Sum of Squares (Sum of Two Squares)	Prep: Strayer §6.1 Homework 9
November 3	Strayer §4.1 Quadratic Residues	Prep: Strayer §4.1
November 5	Exam 2: Chapter 2, Sections 3.2, 5.1-5.2, 8.2-8.3, RSA	Exam 2
November 10	Strayer §4.2 The Legendre Symbol, Strayer §6.5 Representations of an Integer as a Sum of Squares	Prep: Strayer §4.2
November 12	Strayer §4.3 Quadratic Reciprocity	Prep: Strayer §4.3 Homework 10
November 17	Strayer §4.3 Quadratic Reciprocity, Strayer §6.5 Representations of an Integer as a Sum of Squares	Prep: Strayer §4.5
November 19	Strayer §6.3 Pythagorean Triples, Infinite Descent	Prep: Strayer §6.3 Homework 11
November 24	Strayer §6.3 Pythagorean Triples	Prep: Strayer §6.3
November 26	No classes Thanksgiving	Thanksgiving
December 1	Strayer §4.6 Fermat's Last Theorem	Prep: Strayer §4.6
December 3	Exam 3 In Class Sections 4.1-4.5 and Sections 6.2 and 6.5	Exam 3
December 8	Catch up or flexible topics	
December 15	Final Exam 7:30-10:15 am	Final Exam

Common Policies Affecting All Courses at George Mason University

These four policies affect students in all courses at George Mason University. This Course Policy Addendum must be made available to students in all courses (see [Catalog Policy AP.2.5](#)).

Academic Standards

Academic Standards exist to promote authentic scholarship, support the institution's goal of maintaining high standards of academic excellence, and encourage continued ethical behavior of faculty and students to cultivate an educational community which values integrity and produces graduates who carry this commitment forward into professional practice.

As members of the George Mason University community, we are committed to fostering an environment of trust, respect, and scholarly excellence. Our academic standards are the foundation of this commitment, guiding our behavior and interactions within this academic community. The practices for implementing these standards adapt to modern practices, disciplinary contexts, and technological advancements. Our standards are embodied in our courses, policies, and scholarship, and are upheld in the following principles:

Honesty: Providing accurate information in all academic endeavors, including communications, assignments, and examinations.

Acknowledgement: Giving proper credit for all contributions to one's work. This involves the use of accurate citations and references for any ideas, words, or materials created by others in the style appropriate to the discipline. It also includes acknowledging shared authorship in group projects, co-authored pieces, and project reports.

Uniqueness of Work: Ensuring that all submitted work is the result of one's own effort and is original, including free from self-plagiarism. This principle extends to written assignments, code, presentations, exams, and all other forms of academic work.

Violations of these standards—including but not limited to plagiarism, fabrication, and cheating—are taken seriously and will be addressed in accordance with university policies. The process for reporting, investigating, and adjudicating violations is outlined in the university's procedures. Consequences of violations may include academic sanctions, disciplinary actions, and other measures necessary to uphold the integrity of our academic community.

The principles outlined in these academic standards reflect our collective commitment to upholding the highest standards of honesty, acknowledgement, and uniqueness of work. By adhering to these principles, we ensure the continued excellence and integrity of George Mason University's academic community.

Student responsibility: Students are responsible for understanding how these general expectations regarding academic standards apply to each course, assignment, or exam they participate in; students should ask their instructor for clarification on any aspect that is not clear to them.

Accommodations for Students with Disabilities

Disability Services at George Mason University is committed to upholding the letter and spirit of the laws that ensure equal treatment of people with disabilities. Under the administration of University Life, Disability Services implements and coordinates reasonable accommodations and disability-related services that afford equal access to university programs and activities. Students can begin the registration process with Disability Services at any time during their enrollment at George Mason University. If you are seeking accommodations, please visit <https://ds.gmu.edu/> for detailed information about the Disability Services registration process. Disability Services is located in Student Union Building I (SUB I), Suite 2500. Email: ods@gmu.edu. Phone: (703) 993-2474.

Student responsibility: Students are responsible for registering with Disability Services and communicating about their approved accommodations with their instructor in advance of any relevant class meeting, assignment, or exam.

FERPA and Use of GMU Email Addresses for Course Communication

The Family Educational Rights and Privacy Act (FERPA) governs the disclosure of education records for eligible students and is an essential aspect of any course. Students must use their GMU email account to receive important University information, including communications related to this class. Instructors will not respond to messages sent from or send messages regarding course content to a non-GMU email address.

Student responsibility: Students are responsible for checking their GMU email regularly for course-related information, and/or ensuring that GMU email messages are forwarded to an account they do check.

Title IX Resources and Required Reporting

As a part of George Mason University's commitment to providing a safe and non-discriminatory learning, living, and working environment for all members of the University community, the University does not discriminate on the basis of sex or gender in any of its education or employment programs and activities. Accordingly, all non-confidential employees, including your faculty member, have a legal requirement to report to the Title IX Coordinator, all relevant details obtained directly or indirectly about any incident of Prohibited Conduct (such as sexual harassment, sexual assault, gender-based stalking, dating/domestic violence). Upon notifying the Title IX Coordinator of possible Prohibited Conduct, the Title IX Coordinator will assess the report and determine if outreach is required. If outreach is required, the individual the report is about (the "Complainant") will receive a communication, likely in the form of an email, offering that person the option to meet with a representative of the Title IX office.

For more information about non-confidential employees, resources, and Prohibited Conduct, please see University Policy 1202: Sexual and Gender-Based Misconduct and Other Forms of Interpersonal Violence. Questions regarding Title IX can be directed to the Title IX Coordinator via email to TitleIX@gmu.edu, by phone at 703-993-8730, or in person on the Fairfax campus in Aquia 373.

Student opportunity: If you prefer to speak to someone confidentially, please contact one of Mason's confidential employees in Student Support and Advocacy (SSAC), Counseling and Psychological Services (CAPS), Student Health Services (SHS), and/or the Office of the University Ombudsperson.