Part 1: Hypothesis Tests of the Mean

A researcher randomly selects a group of 9 students from a population with a mean IQ of 100 ($\mu = 100$) and standard deviation of 15 ($\sigma = 15$). The 9 students are given an intensive "Get Smart" training and then administered an IQ test. After the training, the sample mean IQ is 113 ($\overline{X} = 113$) and the sample standard deviation is 10 (s = 10). The researcher wants to know if the training resulted in a change in IQ score. Given this information, complete the following:

1. a. State the appropriate null and alternative hypotheses associated with the researcher's question. (2 pts)

   H_0: $\mu_{IQ} = 100$

   H_A: $\mu_{IQ} \neq 100$

   b. State in words what the hypotheses mean in terms of the population. (2 pts).

The null hypothesis means that the mean or average of all the IQs in the population being studied will be equal to 100. The alternative hypothesis means that the mean or averages of all the IQs in the population being studied will be different from 100. The ability for the "Get Smart" training program to change IQ is being tested in this study.

2. a. Based on the hypotheses, should the researcher conduct a one-tailed or two-tailed test? Explain. (1 pt).

The researcher should conduct a two-tailed test because they are looking to see if there is a change in IQ scores. The researcher is not looking for a specific direction (increasing or decreasing) in the change of IQ scores and therefore would use a non-directional or two-tailed test.

   b. What type of test should be conducted (i.e., z test, one sample t test, independent samples t test, dependent samples t test)? Provide justification for your response. (1 pt)

A z-test should be conducted since the population variance is known. The population standard deviation has been calculated from the population variance and provided in the description of the problem. The population standard deviation will be used to calculate the standard error of the mean which will allow for calculating a z-score.

3. If the researcher chooses to use a .05 level of significance, what would be the critical values associated with the test he will conduct? (2 pts)

The critical value for a .05 level of significance in a two-tailed z-test is $z = \pm 1.96$

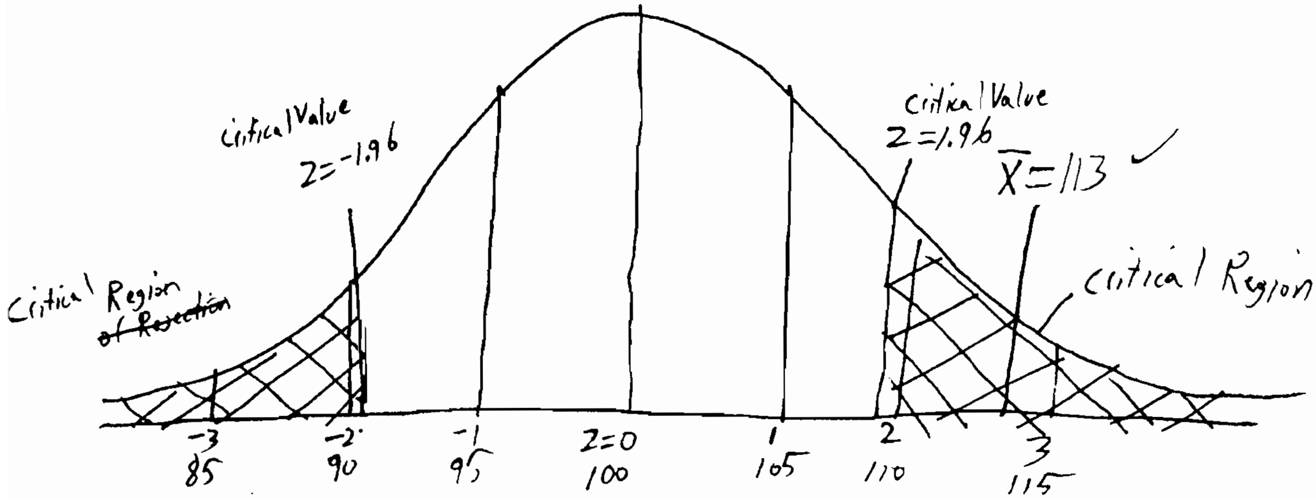
4. a. Compute the standard error of the mean ($\sigma_{\bar{x}}$) to ***two decimal places***. (2 pts)

$$\sigma_{\bar{x}} = \frac{\sigma X}{\sqrt{n}} = \frac{15}{\sqrt{9}} = \frac{15}{3} = 5.00$$

b. Draw and label your sampling distribution for means from samples of size n = 9 (including both raw scores and z-scores for the hypothesized population mean and $\pm$ three standard deviations). Shade in the critical region (the part or parts past your critical value or values). (2 pts)

-0 Well done!

c. Plot the sample mean on the drawing of the sampling distribution. (1 pt)



5. Compute the appropriate test statistic to ***two decimal places***. (1 pt)

$$z = \frac{(\bar{x}-\mu)}{\sigma_{\bar{x}}} = \frac{(113-100)}{5} = \frac{13}{5} = 2.60$$

6. Determine the two-tailed p-value (***to three decimal places***) by determining the area between your mean and the nearest tail, and then double that. (2 pts).

   Based on the z table $p_{2.6}$ = 0.0047, (0.0047)(2) = 0.009, p = 0.009

7. Calculate a 95% confidence interval (to ***two decimal places***). (2pts)

   95% CI = $\bar{X} \pm z_{\frac{\alpha}{2}}(\sigma_{\bar{x}})$ = 113±1.96(5) = 103.2, 122.8, The 95 % Confidence Interval lies between 103.2 and 122.8. [103.2, 122.8]

8. a. Based on the plot of the sample mean, in item 4c, should the null hypothesis be retained or rejected? Explain. (1 pt)

The null hypothesis should be rejected because the sample mean lies in the shaded area beyond the critical z-value for retaining the null hypothesis.

b. Based on your computed test statistic in item 5, should the null hypothesis be retained or rejected? Explain. (1 pt)

The null hypothesis should be rejected because the calculated z-score (2.6) is greater than the critical z-value (±1.96) for retaining the null hypothesis. This z-score indicates a probability of less than .05 for committing a Type I Error (rejecting the null hypothesis when the null hypothesis is true).

c. Based on your p-value in item 6, should the null hypothesis be retained or rejected? Explain.(1 pt)

The null hypothesis should be rejected because the p-value for the calculated z-score (.009) is less than the alpha level that was set for rejecting the null hypothesis (.05). The p-value represents the probability of ~~making a Type I Error (rejecting the null hypothesis when the null hypothesis is true). Since the probability of making a Type I Error is less than the critical value for rejecting the null hypothesis established by the researcher, the null hypothesis should be rejected.~~ an observed value of 2.60 (z) or 113 ($\bar{X}$) if the population mean is actually 108.

d. Based on your confidence interval in item 7, should the null hypothesis be retained or rejected? Explain. (1 pt)

The null hypothesis should be rejected because the population mean does not fall within the range of the 95% confidence interval that was calculated from the sample mean.

e. What can be inferred about the effects of the training on students in the population? (1 pt).

It can be inferred that the people in the sample have different IQs than the average person. Specifically, the people who underwent the "Get Smart" training course have higher IQs than the average person. Therefore, the "Get Smart" training program can raise IQ scores.

Part II: Hypothesis Tests of the Mean

The Food and Nutrition Board of the National Academy of Sciences state that the RDA of iron for adult females under 51 is 18mg. The following iron intakes during a 24 hour period were obtained from a random sample of 40 women under the age of 51.

15.0	18.1	14.4	14.6	10.9	18.1	18.2	18.3
16.0	12.6	16.6	20.7	19.8	11.6	12.8	15.6
15.3	9.4	19.5	18.3	14.5	16.6	11.5	16.4
14.6	11.9	12.5	18.6	13.1	12.1	10.7	17.3
17.0	6.3	16.8	12.5	16.3	14.7	12.7	16.3

Does the data suggest women under age 51 are getting on average *less than* the RDA of 18mg of iron? To address this question, complete the following:

1. a. State the appropriate null and alternative hypotheses associated with the research question. (2 pts)

H_0: $\mu_{Iron} \geq 18$

H_A: $\mu_{Iron} < 18$

b. State in words what the hypotheses mean in terms of the population. (2 pts).

The null hypothesis states that the mean or average amount of daily iron intake by the population of women under the age of 51 is greater than or equal to 18 mg. The alternative hypothesis states that the mean or average daily iron intake by the population of women under the age of 51 is less than 18mg.

2. a. Based on the hypotheses, should the researcher conduct a one-tailed or two-tailed test? Explain. (1 pt).

The researched should conduct a one-tailed test since the research question suggests a direction in which the alternative hypothesis may be different from the null hypothesis.

b. What type of test should be conducted (i.e., z test, one sample t test, independent samples t test, dependent samples t test)? Provide justification for your response. (1 pt)

A one sample t test should be conducted to test these hypotheses because the population variance is unknown and a sample mean is being compared to a population mean.

3. a. What are the degrees of freedom associated with this problem? (1 pt)

df = n-1 = 40-1 = 39   df = 39

b. If the test is to be conducted with a significance level of .05, what critical value (or values) must be exceeded to reject the null hypothesis? (1 pt)

Using a t-table, the critical value for a one-tailed t-test with 39 degrees of freedom at a significance level of .05 is -1.685

4. Using SPSS, compute the sample mean, variance (s^2) and standard deviation (s) to ***at least two decimal places***. Insert results. (3 pts)

Descriptive Statistics

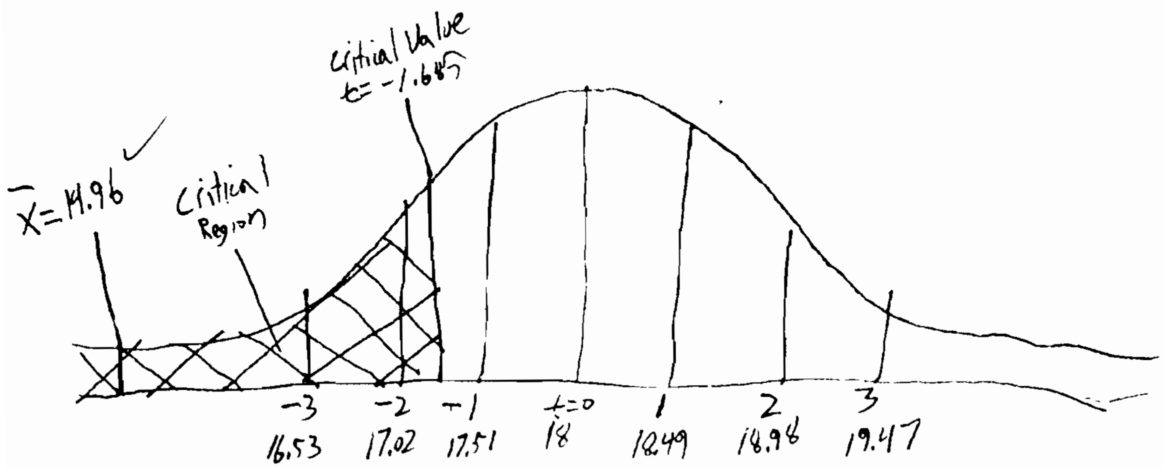
	N	Mean	Std. Deviation	Variance
Iron	40	14.9550	3.12779	9.783
Valid N (listwise)	40			

5. Compute the estimated standard error of the mean ($s_{\bar{x}}$) ***to two decimal places***. Show your work. (2 pts)

$$S_{\bar{x}} = \frac{sx}{\sqrt{n}} = \frac{3.128}{\sqrt{40}} = \frac{3.128}{6.325} = 0.49$$

6. a. Draw and label your sampling distribution for means from samples of size n = 40 (including both raw scores and t values for the hypothesized population mean and ± three standard deviations). Shade in the critical region (the part or parts past your critical value). (2 pts)

b. Plot the sample mean on the drawing of the sampling distribution. (1 pt)



7. Compute the appropriate test statistic *to two decimal places*. (1 pt)

$$t = \frac{(\bar{x}-\mu)}{s_{\bar{x}}} = \frac{(14.955-18)}{0.495} = \frac{(14.955-18)}{0.495} = \frac{(-3.045)}{0.495} = -6.15$$

8. Calculate a 95% confidence interval (*to two decimal places*). (2pts)

95% Confidence Interval = $\bar{X} \pm t_{\frac{\alpha}{2}}(s_{\bar{x}})$ = 14.955 ± 2.023(0.495) = 13.95, 15.96

The 95% Confidence Interval is between 13.95 and 15.96. [13.95, 15.96]

9 a. Based on the plot of the mean, in item 6b, should the null hypothesis be retained or rejected? Explain. (1 pt)

The null hypothesis should be rejected because the sample mean lies in the shaded area beyond the critical value for retaining the null hypothesis.

b. Based on your test statistic in item 7, should the null hypothesis be retained or rejected? Explain. (1 pt)

The null hypothesis should be rejected because the calculated t score (-6.15) is less than the critical value t score (-1.685) for retaining the null hypothesis. This t-score indicates a probability of less than .05 for committing a Type I Error (rejecting the null hypothesis when the null hypothesis is true).

c. Based on your confidence interval in item 8, should the null hypothesis be retained or rejected? Explain. (1 pt)

The null hypothesis should be rejected because the population mean occurs beyond the boundaries of the 95% confidence interval. ✓

d. What can be inferred about the iron intake of women under age 51? (1 pt).

Based on the results of this sample, it can be inferred that women under the age of 51 ingest less than 18 mg of iron in a 24 hour period. Therefore, the report from the Food and Nutrition Board of the National Academy of the Sciences is inaccurate. Well, although the RDA (recommended daily amount) is stated as FDA, the results here indicate that women are not actually consuming that much iron. Perhaps diets need to change to meet the RDA.

10 a. Use SPSS to conduct the t-test you just completed by hand. If your previous calculations do not match, identify the problem and correct it (Note: there may be slight differences at the decimal level due to differences in rounding). Paste the results of you t-test into your homework. (3 pts).

One-Sample Test

	Test Value = 18					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Iron	-6.157	39	.000	-3.04500	-4.0453	-2.0447

✓

b. Based on the two-tailed p-value from your t-test, should you retain or reject the null hypothesis? Explain. State the exact p-value from your printout you are using to make your decision. (1 pt)

The two-tailed p-value = 0.000 in the printout which is less than the level of significance set by the researcher. Since p is less than the level of significance the null hypothesis is rejected. ~~The~~ p-value represents the probability of ~~making a Type I Error (rejecting the null hypothesis when the null hypothesis is true). Since the probability of making a Type I Error is less than the critical value for rejecting the null hypothesis established by the researcher, the null hypothesis should be rejected.~~ a sample mean of 14.95 if the population mean is actually 18.