Midterm Exam II

STAT 346, Spring 2010

Instructions: You can use whatever books, notes, and papers you've brought with you, but you cannot share materials with other students during the exam. You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period. You can use a calculator, but not a computer.

There are 15 five point parts, and I'll count your best 14 (of 15) scores for these problems. There are 4 ten point parts, and I'll count your best 3 (of 4) scores for these problems. For all parts of the exam you are to justify your answers, using notation and terminology properly, and clearly defining any events and random variables that you use (that aren't defined in the statement of the problem).

Express probabilities as exact values (as fractions, or in decimal form), or else round them to the nearest thousandth. Do not express final answers as expressions that need to be evaluated.

Put all of your work on these sheets. If you need more room, direct me to look for additional work on the back of a page. **Draw boxes around your final answers!**

1) Let X be a random variable having cdf

$$F(x) = \begin{cases} 1, & x \geq 4, \\ 3/4, & 1 \leq x < 4, \\ 1/2, & 0 \leq x < 1, \\ 0, & x < 0. \end{cases}$$

(a) (5 points) Give the value of $P(X > 2)$.

(b) (5 points) Give the value of $E(X)$.

2) Suppose that an urn initially contains four white balls and four black balls, and that a ball will be randomly drawn from the urn four times. Let X be the number of times a black ball is drawn.

First suppose that four balls will be drawn without replacement.
(a) (5 points) Give the value of $P(X = 2)$.

(b) (5 points) Give the value of $E(X)$.

Now suppose the four draws will be done *with* **replacement.**
(c) (5 points) Give the value of $P(X = 2)$.

(d) (5 points) Give the value of $E(X)$.

3) (5 points) Let $X \sim N(100, 100)$ and give the value of $P(X > 110.15)$.

4) Let X be a random variable such that

$$E(2X - 3) = 7$$

and

$$Var(2X - 3) = 8.$$

(a) (5 points) Give the value of $E(X)$.

(b) (5 points) Give the value of $Var(X)$.

5) Let X be a random variable having pdf

$$f(x) = \frac{3}{x^4}\, I_{(1,\,\infty)}(x).$$

(a) (5 points) Give the value of $P(X > 3)$.

(b) (5 points) Give the value of $E(X)$.

(c) (5 points) Give the value of $Var(X)$.

(d) (5 points) Give the value of $E(1/X)$.

Continuing with the problem from the previous page, here's the pdf of X again:

$$f(x) = \frac{3}{x^4} I_{(1,\infty)}(x).$$

(e) (10 points) Letting U be a uniform (0, 1) random variable, give a function of U that has the same distribution as X.

(f) (10 points) Give the pdf of $Y = 1/X$.

6) Let $X_1, X_2, X_3, \ldots$ be the lifelengths (in hours) of components that will be put into service sequentially, with a new component being put into service as soon as a component fails (e.g., the second component will be put into service when the first one fails), and suppose that the X_i are independent exponential random variables having mean 300.

(a) (10 points) Letting V be the number of components that will be installed until one of them survives for more than 300 hours, give the expected value of V.

(b) (5 points) Letting Y be the time that the third failure occurs, give the value of $E(Y)$.

(c) (10 points) Give the probability that two or more failures occur in the first 600 hours.

(d) (5 points) Give the value of $E(e^{-X_1/300})$.