HW 9

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: Five of the of the 7 parts below will be graded, with each graded part worth 4 points. (I won't specify which parts will be graded until after the papers have been submitted.)

1) In the board game *Napoleon's War* the following can possibly occur:

- The French player moves two of his cannons within range of two British cannons.
- The two British cannons "fire" at the two French cannons. This is done by rolling one 6-sided die for each British cannon. For each die that results in one or two spots on it's upwards face, a French cannon is eliminated.
- Then each French cannon which has not been eliminated can "fire" at the British cannons. This is done by rolling one 6-sided die for each French cannon which wasn't eliminated by the British. For each die that results in one or two spots on it's upwards face, a British cannon is eliminated.

Let X be the number of French cannons eliminated in the scenerio described above, and let Y be the number of British cannons eliminated in the scenerio described above. (So, for example, if the British player obtains 3 spots on one die and 2 spots on the other die, X takes the value 1. Then if the French player, who only rolls one die because one of his cannons was eliminated, obtains 4 spots on his die roll, Y takes the value 0. However, if instead the British played rolls a five and a six, and then the French player rolls a one and a two, X assumes the value 0 and Y assumes the value 2.)

(a) Give the joint pmf of X and Y in tabular form by making a 3 by 3 table with the $x = 0$ values of $p_{X,Y}(x, y)$ in the first row, the $x = 1$ values of $p_{X,Y}(x, y)$ in the second row, and the $x = 2$ values of $p_{X,Y}(x, y)$ in the third row. (As a check of your work, you should find that the marginal pmf of X is that of a binomial $(2, 1/3)$ distribution, and also that $P(X > Y) = 11/27$.)

(b) Give the marginal pmf of Y, expressing it as a piecewise function.

(c) Are X and Y independent random variables? Answer *yes* or *no* and provide solid justification for your answer.

2) Consider independent random variables X and Y, where X has pdf

$$f_X(x) = e^{-x}\, I_{(0,\,\infty)}(x),$$

and Y has pdf

$$f_Y(y) = 2e^{-2y}\, I_{(0,\,\infty)}(y).$$

Give the pdf of $V = X + Y$. (*Note*: Since X and Y are independent, one can use the convolution formula. But one can also use the cdf method. I recommend trying it both ways to check your work. (You can also check your work by making sure that your pdf is nonnegative and integrates to 1.))

3) Let X and Y have the following joint density:

$$f_{X,Y}(x, y) = \begin{cases} y/x^2, & 0 < y < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Give the pdf of the conditional distribution of X, given that $Y = 1$ (i.e., $p_{X|Y}(x|1)$). (*Comment*: Your answer should be a relatively simple function of x that is a valid pdf (nonnegative, and integrates to 1). Your answer should not contain y, because the requested pdf is for the case of y equals 1. (Note that this joint pdf was used in Problem 4 of HW 8, and there you were requested to obtain the marginal pdf of Y. Feel free to make use of the answer to part (a) of that HW problem when doing this HW problem.))

4) Consider iid random variables X_1, X_2, X_3, and X_4, having pdf

$$f_X(x) = 8x\, I_{(0,\,1/2)}(x).$$

(a) Give the expected value of the sample maximum, $X_{(4)}$.

(b) Give the pdf of $X_{(3)}$ (the 3rd order statistic).