HW 7

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: Five of the of the six parts below will be graded, with each graded part worth 4 points. (I won't specify which parts will be graded until after the papers have been submitted.)

1) Consider a random variable U having pdf

$$f_U(u) = 6\, I_{(-1/3,\,-1/6)}(u).$$

(*Note*: In the 8th lecture of the semester, I'll cover that U is an example of a uniform random variable.)

(a) Give the pdf of $V = 1/U$. (*Comment*: Since we're dealing with a monotone function, you can make use of Theorem 7.1 on p. 209 of the text if you want. Alternatively, you can use the "cdf method." Of course, doing it both ways can provide a check of your work (but if you get two different values, be sure to clearly indicate which one you want me to base your score on).)

(b) Considering the random variable V defined in part (a), give the value of $E(V) = E(1/U)$. (*Comment*: You can obtain the desired value using either the pdf of U or the pdf of V. Of course, doing it both ways can provide a check of your work (but if you get two different values, be sure to clearly indicate which one you want me to base your score on).)

2) Letting X be a gamma random variable having mean 2 and variance 1, give the value of $E(1/X)$. (*Hint:* Use an integration "trick" similar to what I do on p. 5-16 of the notes.)

3) Consider a Poisson process for which events occur at a rate of 1.5/hr, and let T be the time in hours to the 3rd event. What is the value of $P(T \le 3)$? (*Hint:* Instead of integrating the pdf of T over an appropriate interval, compute the probability of an equivalent event involving a Poisson random variable.)

4) Consider a random variable X having pdf

$$f_X(x) = \begin{cases} \frac{24}{x^4}, & x > 2, \\ 0, & x \le 2, \end{cases}$$

and consider $U \sim \text{unif}\,(0,\,1)$. Give a function of U which has the same distribution as X.

5) Give the value of $P(Z^2 \le 2.25)$, rounded to the nearest thousandth, where $Z \sim \mathrm{N}\,(0, 1)$. (*Hint:* There is no need to determine the pdf of Z^2. The easy way to obtain the desired probability is to make use of an equivalent event having the form $-c \le Z \le c$, for some constant c.)