HW 5

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: Five of the of the six parts below will be graded, with each graded part worth 4 points.

1) Suppose that a binary message of length 6 will be sent over a noisy channel. (For example, the message could be 001101, representing the number 25.) Suppose that each bit will be corrupted, and received erroneously, with probability 0.2, independently of whether or not any other bit is received erroneously. (So, if `001101` is sent, the probability it is received as `101100`, having errors in the first and sixth bits, is (0.2)(0.8)(0.8)(0.8)(0.8)(0.2).) Further suppose that a 7th bit is added to the message. This "parity check" bit is sent as a `0` if the 6-bit message contains (as it was sent) an even number of `1`s, and it is sent as a `1` if the 6-bit message (as it was sent) has an odd number of `1`s. If the received 6-bit message has an even number of `1`s and the check bit is received as a 1, then one knows that at least one of the 7 bits was received incorrectly. (Either there was an even number of errors (possibly 0 errors) among the 6 bits of the message and the check bit was received incorrectly, or there was an odd number of errors among the 6 bits of the message and the check bit was received correctly.) If the received 6-bit message has an odd number of `1`s and the check bit is received as a 0, then one knows that at least one of the 7 bits was received incorrectly. (Either there was an even number of errors (possibly 0 errors) among the 6 bits of the message and the check bit was received incorrectly, or there was an odd number of errors among the 6 bits of the message and the check bit was received correctly.) It should be noted that the parity check will not detect all messages received in error. If the check bit is consistent with the number of `1`s in the 6-bit message, then it could be that all 7 bits were received correctly, but it could also be that the check bit was received correctly and either 2, 4, or 6 bits of the 6-bit message were received incorrectly, or it could be that the check bit was received incorrectly and an odd number of bits of the 6-bit message were received incorrectly. (For example, if `0011011` is sent, with `001101` being the 6-bit message, and the last `1` being the check bit, and `1111011` is received, then the errors in the first two bits of the message will not be indicated by the parity check. Also, if `0011011` is sent, and `1011010` is received, then the parity check will not indicate that an error was made, since the error in receiving the check bit as 0 masks the error in the first bit of the message, because the error in the first bit of the message created an even number of `1s` in the received 6-bit message, and that result is consistent with the received check bit of `0`.) What is the probability that the parity check will fail to indicate that something is wrong when one or more of the bits in the 6-bit message are received incorrectly? (*Note*: A conditional probability is being requested; given that the 6 bits of the message are erroneously received, what's the chance that the parity check fails to indicate a problem?)

2) Suppose that someone will flip a fair coin until he gets an outcome of *heads*. If he gets *heads* on the first flip of the coin, he'll roll one ordinary 6-sided die, if he gets his first outcome of *heads* on the second flip of the coin, he'll roll two ordinary 6-sided dice, if he gets his first outcome of *heads* on the third flip of the coin, he'll roll three ordinary 6-sided dice, etc. If he flips the coin until he gets *heads* and then rolls the prescribed number of dice, what is the probability that at least one of the dice will result in having 6 spots showing on its upward face? (A good way to solve this problem is to combine a simple, but important, probability law given in Ch. 2 with a major result given in Ch. 3 and one or more of the discrete distributions covered in Ch. 4.)

3) Consider an urn initially containing 3 white balls and 3 black balls, and suppose that balls will be randomly drawn from this urn, one-by-one, and without replacement until a second black ball is obtained. Letting X be the number of draws required to obtain a second black ball, give the pmf of X, expressing it as a piecewise function. (*Comments*: (*i*) See the top of p. 4-4 of the class notes for an example of expressing a pmf as a piecewise function. (*ii*) So if the first ball drawn is black, the next two are white, and the fourth one is black, then X assumes the value 4. But if the first three are all white and then the next two are black, then X assumes the value 5. (*iii*) Before trying to do this problem, it may be good to review the derivation of the negative binomial pmf given in the class notes, and note that for the situation considered here we do not have independent trials (since, for example, the result of the first draw affects the probability that the second draw will result in a black ball).)

4) Consider a random variable X having cdf

$$F_X(x) = \left(1 - \frac{1}{x^2}\right) I_{[1,\infty)}(x).$$

(a) Give the pdf of X.
(b) Give the value of $P(X \geq 4)$.
(c) Give the value of $E(X)$.