HW 4

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average. Typically, each assignment will consist of from 5 to 10 parts (but could be a little more or a little less).

Note: Five of the of the seven parts below will be graded, with each graded part worth 4 points. (I won't specify which parts will be graded until after the papers have been submitted.)

1) Suppose that 100 points are equally spaced around a circular path, and at 99 of these points there are sheep which do not move, and at the other point there is a wolf who will randomly move. Suppose that each time the wolf moves, he will be equally likely to move clockwise by one point or counterclockwise by one point, and if a sheep is at his new location he will eat it. If the wolf continues moving randomly until all of the sheep are eaten, what is the probability that the sheep who is located directly opposite the wolf's starting point will be the last one eaten? (*Comments*: (*i*) I suggest that you try to make use of one of the "gambler's ruin" results either covered in class, or given in the text. (You don't have to derive a new result. Rather, just figure out how you can make use of one of the existing results.) It can be noted that the situation treated on pp. 84-86 of the text, with one player starting with i and another player starting with $N - i$, and them playing until one of them reaches N and the other 0, is equivalent to the situation I addressed in the class notes, with a gambler starting with i and playing until he either reaches N or reaches 0; and so the formulas given on pp. 84-86 of the book also apply to the situation I addressed in class. (*ii*) One might guess that being directly opposite the wolf's starting position is a better than average location (with regard to surviving the longest), since every other sheep is initially closer to the wolf. So one might guess that the desired probability is certainly less than 1/99 (which would be the probability if all of the sheep are equally likely to be the last one eaten). However, this turns out not to tbe the case; the probability is *not less than* 1/99.)

2) Suppose that a subset of 3 balls will be randomly selected from an urn containing 2 amber, 2 blue, and 2 green balls. Letting X be the number of colors obtained, give the value of $E(X)$. (*Note*: X will either assume the value 2, or the value 3. If one ball of each color is included in the random subset, then X assumes the value 3, but otherwise X will be 2.)

3) Let X_n be the number of times the outcomes *heads* will occur when a fair coin is flipped n times, where n is an even positive integer. It can be shown (using calculus) that $n/2$ is the most likely outcome for X_n (i.e., $P(X_n = x)$ is maximized when $x = n/2$). However, as n increases, this maximum probability gets smaller and smaller, and using Stirling's formula (for the approximation of $n!$ (and $(n/2)!$)), it can be shown that $P(X_n = n/2)$ is asymptotically equivalent to $c/\sqrt{n}$, where c is a positive constant. Give the value of this constant. (Show some work, but basically you can just make use of this particular variation of Stirling's formula: $n! \sim \sqrt{2\pi n}(n/e)^n$.)

4) Suppose that a player will repeatedly play roulette, betting \$10 on green each time he plays. (Each time a green outcome does not occur he will lose \$10, and each time green occurs he will gain \$170. Assume an ideal roulette wheel having 38 equally-likely outcomes, of which two are green.)

(a) Letting Y be the amount (in dollars) he wins or loses if he plays until he wins for the first time and then quits, give the variance of Y. (*Hint*: Express Y as a linear function of a geometric random variable.)

(b) Letting W be the number of times he has to play in order to win 4 times, give the value of $P(W = 6)$. (Express in scientific notation, using three significant digits.)

(c) Letting W be the number of times he has to play in order to win 4 times, give the value of $P(W \leq 60)$. (Express in scientific notation, using three significant digits.)

5) Consider an urn, initially containing 3 white balls and 1 black ball, and suppose balls will be drawn, one-by-one, and without replacement, from this urn until the black ball is obtained. Letting X be the number of draws required, give value of $Var(X)$.