HW 3

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: Five of the of the seven parts below will be graded, with each graded part worth 4 points. (I won't specify which parts will be graded until after the papers have been submitted.)

1) Suppose that the 26 letters `A`, `B`, `C`, …, `Y`, and `Z` are randomly ordered, from left to right. Letting E_1 be the event that `A` is placed to the left of `C`, and E_2 be the event that `B` is placed to the left of `C`, use the definition of independent events to show that either E_1 and E_2 are independent, or E_1 and E_2 are not independent.

2) Suppose that 6 balls will be randomly put into 4 urns (with each ball being equally likely to be put into each of the urns, independently of which urns the other balls are put in). What is the probability that each urn gets at least one ball put into it?

3) Suppose that 100 people are lined up to board an airplane with 100 seats, and each seat has been assigned to exactly one of the passengers. However, suppose that the first passenger in line decides to randomly choose a seat to sit in (with all 100 seats being equally likely to be chosen). Then each subsequent passenger sits in his or her assigned seat if it is empty, but otherwise randomly picks an empty seat to sit in. What is the probability that the last passenger in line gets to sit in his or her assigned seat? (*Comments*: Try to obtain the answer in a clever way, and avoid a messy derivation. Some things to think about:

Can passenger 100 wind up in his originally assigned seat?
Can he wind up in the seat originally assigned to the first passenger?
Can he wind up in any of the other seats?

What has to happen for him to wind up in his originally assigned seat?)

4) Consider the circuit shown in part (a) of Figure 3.4 on p. 103 of the text, only *ignore the switch at location 5* (or else, equivalently, assume that it is always closed). If the switches at 1, 2, 3, and 4 are independently closed or open with probabilities 0.1 (for closed) and 0.9 (for open), what is the probability of a closed path between A and B?

5) Consider a discrete random variable X having cdf

$$F_X(x) = \begin{cases} 1, & x \geq 6, \\ 5/6, & 3 \leq x < 6, \\ 1/2, & 2 \leq x < 3, \\ 0, & x < 2. \end{cases}$$

(a) What is the value of $P(X > 4)$? (*Hint*: Consider the relationship between the event that X exceeds 4 and the event that X is less than or equal to 4 (which is the event associated with $F_X(4)$).)

(b) What is the value of $P(X = 3)$? (*Hint*: Consider the relationship between the cdf and the pmf of a discrete random variable (see the bottom half of p. 4-4 of the class notes).)

(c) What is the value of $P(X = 4)$?