Solutions for HW 1
STAT 346, Spring 2010

1)

(a) Since $C \subset B$, we have that $B \cap C = C$, and so $P(B \cap C \cap F) = P(C \cap F) = 3/52 \doteq 0.0577$ (since 3 of the 52 cards are club face cards).

(b) Since $C \subset B$, we have that $B \cup C = B$, and so $P(B \cup C \cup F) = P(B \cup F) = P(B) + P(F) - P(B \cap F) = 26/52 + 12/52 - 6/52 = 32/52 = 8/13 \doteq 0.615$.

(c) $P((B \cap F)^C) = 1 - P(B \cap F) = 1 - 6/52 = 46/52 = 23/26 \doteq 0.885$ (since 6 of the 52 cards are black face cards). (*Note*: One could also use one of De Morgan's laws as follows: $P((B \cap F)^C) = P(B^C \cup F^C) = P(B^C) + P(F^C) - P(B^C \cap F^C) = (1 - 26/52) + (1 - 12/52) - 20/52 = 46/52 = 23/26$).

2) If we observe how many minutes past 1:00 the bus arrives, the sample space is (0, 30) and the event that a person has to wait at least 10 minutes is the interval [10, 30). Using the formula in the definition on pp. 31-32 of the text, the desired probability is $(30 - 10)/(30 - 0) = 20/30 = 2/3 \doteq 0.667$. (*Note*: The formula in the definition can be applied whether the intervals are open, closed, or half open.)

3) Letting A be the event that the first horse wins and B be the event that the second horse wins, if the odds in favor of the first horse winning are 2 to 5, we have $P(A) = 2/(2 + 5) = 2/7$. Similarly, if the odds against the second horse winning are 7 to 3, the odds in favor of that horse winning are 3 to 7, and we have $P(B) = 3/(3+7) = 3/10$. Using the fact that A and B are mutually exclusive events, the desired probability is $P(A \cup B) = P(A) + P(B) = 2/7 + 3/10 = 41/70 \doteq 0.586$.