HW 1

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average. Typically, each assignment will consist of from 5 to 10 parts (but could be a little more or a little less).

Note: Five of the of the eight parts below will be graded, with each graded part worth 4 points. (I won't specify which parts will be graded until after the papers have been submitted.)

Please try to do the problems below using only the material covered during the first and second lectures. If instead you use conditional probabilities (which aren't introduced until Ch. 3), I'm going to be very picky when grading your solutions.

1) Alice, Bob, and 10 other people will be randomly placed in 12 seats at a round table. What is the probability that Alice and Bob will be seated next to one another? (As a check of your work, and to get practice working with various methods, try to obtain the answer in more than one way (but only submit the solution that you want me to grade). One way to attack this problem is to consider a sample space consisting of 12! possible outcomes, but it can also be solved by considering a much smaller sample space consisting of the possible pairs of seats that Alice and Bob are assigned to.)

2) Suppose that Eddie decides to bet on two horse races, and in each race horses numbered 1 through 8 are entered. Further suppose that Eddie knows nothing about any of the horses, jockeys, or trainers, and decides to pick the winner of the 1st race by randomly picking a number from the first eight integers. Then he'll randomly select a number from the seven remaining integers to pick a horse to finish second, and then randomly choose a number from the 6 remaining integers to predict the third place finisher. Having made his win, place, and show picks for the first race, he'll then start with the eight integers again, and randomly make win, place, and show selections for the second race in the same manner. What is the probability that all six of his selections will be correct? (*Note*: It's not necessary to assume that all eight horses are equally capable of winning.)

3) Suppose that a group of eight people will be randomly selected from a group consisting of 10 married couples (so 20 people in all). What is the probability that the randomly chosen group of eight will contain exactly two of the couples (and four other people, but no couples among those four)?

4) Consider four 20-sided dice, which are such that each has the numbers 1 through 20 printed on its 20 faces, and are designed so that when randomly rolled, each face is equally likely to be the upward face. If these four dice are randomly rolled, what is the probability that the sum of the four numbers on their upward faces will equal 10? (Consider a sample space of $20^4 = 160,000$ equally-likely outcomes.)

5) Suppose that three red blocks, three white blocks, and three blue blocks will be randomly ordered, from left to right. What is the probability that the three blue blocks will be next to one another?

6) Consider events A and B for which $P(A) = 0.2$ and $P(B) = 0.4$. Can A^C and B^C be mutually-exclusive events? Answer *yes* or *no* and explain your answer.

7) Consider three men and three women, with no two of these people being the same height. If the women are randomly paired with the men to form three couples, what is the probability that the shortest man is paired with the tallest woman, or the tallest man is paired with the shortest woman?

8) Suppose that A, B, and C are events for which

$P(A) = 0.17$,
$P(B) = 0.37$,
$P(C) = 0.19$,
$P(A \cap B) = 0.07$,
$P(B \cap C) = 0.11$,
$P(A \cap B \cap C) = 0.03$, &
$P(A \cup B \cup C) = 0.48$.

What is the value of $P(A \cup C)$?