HW 12

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: 5 of the 8 parts below will be graded, with each graded part worth 4 points.

1) Suppose that the 10 people of 5 married couples are randomly seated around a round table. Letting X be the number of wives who are seated next to their husband, give the value of $Var(X)$. (*Note*: In Problem 3 of HW 11 you were requested to obtain the value of $E(X)$. For your solution to this problem, you can use any results given in the solution to that problem without deriving them.)

2) Suppose that a subset of 4 balls will be randomly selected from a collection of 3 amber, 3 blue, and 5 green balls. Letting X be the number of amber balls in the selected subset, and letting Y be the number of blue balls in the selected subset, give the value of $Cov(X, Y)$. (*Notes*: (*i*) In Problem 2 of HW 11 you were requested to obtain the value of $E(X - Y)$. For your solution to this problem, you can use any results given in the solution to that problem without deriving them. (*ii*) Although you could obtain the value of) $E(XY)$ in a direct "brute-force" manner, another way to obtain the value would be to make use of $E(XY) = E(E(XY|X))$.)

3) Let X and Y have the following joint density:

$$f(x, y) = \begin{cases} 3(x + y), & 0 < x < 1,\, 0 < y < 1,\, 0 < x + y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(*Note*: The joint density is positive on a triangular-shaped region in the x–y plane.) Give the value of $E(Y|X = 1/4)$.

4) Suppose that someone will flip a fair coin until he gets an outcome of *heads*. If he gets *heads* on the first flip of the coin, he'll roll one ordinary 6-sided die, if he gets his first outcome of *heads* on the second flip of the coin, he'll roll two ordinary 6-sided dice, if he gets his first outcome of *heads* on the third flip of the coin, he'll roll three ordinary 6-sided dice, etc. Given that he will flip the coin until he gets *heads* and then roll the prescribed number of dice, let X be the number of dice that will result in having 6 spots showing on their upward faces.

(a) Give the value of $E(X)$.

(b) Give the value of $Var(X)$.

5) Let X be a random variable having pdf

$$f(x) = \begin{cases} 2e^{(1-2x)}, & x > 1/2, \\ 0, & x \le 1/2. \end{cases}$$

(a) Obtain the mgf of X, $M_X(t)$, being sure to indicate what values of t are applicable.

(b) Obtain the value of $E(X)$.

6) Consider N $(1, 16)$ random variables $X_1, X_2, \ldots, X_{25}$ and N $(0, 25)$ random variables $Y_1, Y_2, \ldots, Y_{100}$, with these 125 random variables being independent. Give the value of $P(\overline{X} > \overline{Y})$. (*Hints*: First determine the distributions of $\overline{X}$ and $\overline{Y}$. Then get the distribution of $\overline{Y} - \overline{X}$ and use it to obtain the desired probability.)