HW 11

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: For this assignment, all 5 of the parts below will be graded, with each graded part worth 4 points.

1) Let X_1 and X_2 have the following joint density:

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & x_1 > 0,\, x_2 > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Letting $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$, give the joint pdf of Y_1 and Y_2 (being sure to clearly indicate the region of the y_1-y_2 plane on which the joint pdf is nonzero).

2) Suppose that a subset of 4 balls will be randomly selected from a collection of 3 amber, 3 blue, and 5 green balls. Letting X be the number of amber balls in the selected subset, and letting Y be the number of blue balls in the selected subset, give the value of $E(X - Y)$.

3) Suppose that the 10 people of 5 married couples are randomly seated around a round table. Letting X be the number of wives who are seated next to their husband, give the value of $E(X)$.

4) Let X and Y have the following joint density:

$$f_{X,Y}(x, y) = \begin{cases} 1/2, & 0 < x < 2,\, 0 < y < 2,\, 0 < x + y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(*Note*: This joint pdf is nonzero on a triangular region of the 1st quadrant of the x-y plane having area 2.) Give the correlation of X and Y. (*Note*: This is the same joint pdf that was used in Problem 4 of HW 10. You can use any results given in my solution of that problem for your solution to this problem.)

5) Consider independent random variables X_1, X_2, and X_3 such that X_1 is a random variable having mean 1 and variance 1, X_2 is a random variable having mean 2 and variance 4, and X_3 is a random variable having mean 3 and variance 9. Give the value of the correlation of $Y = X_1 - X_2$ and $Z = X_2 + X_3$. (*Hint*: Use Proposition 4.2 on pp. 305–306 of the text.)