HW 10

STAT 544, Fall 2015

Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: Five of the of the six parts below will be graded, with each graded part worth 4 points. (I won't specify which parts will be graded until after the papers have been submitted.)

1) In the board game *Napoleon's War* the following can possibly occur:

- The French player moves two of his cannons within range of two British cannons.
- The two British cannons "fire" at the two French cannons. This is done by rolling one 6-sided die for each British cannon. For each die that results in one or two spots on it's upwards face, a French cannon is eliminated.
- Then each French cannon which has not been eliminated can "fire" at the British cannons. This is done by rolling one 6-sided die for each French cannon which wasn't eliminated by the British. For each die that results in one or two spots on it's upwards face, a British cannon is eliminated.

Let X be the number of French cannons eliminated in the scenerio described above, and let Y be the number of British cannons eliminated in the scenerio described above. (So, for example, if the British player obtains 3 spots on one die and 2 spots on the other die, X takes the value 1. Then if the French player, who only rolls one die because one of his cannons was eliminated, obtains 4 spots on his die roll, Y takes the value 0. However, if instead the British played rolls a five and a six, and then the French player rolls a one and a two, X assumes the value 0 and Y assumes the value 2.) Give the conditional pmf of Y given that $X = 0$. (*Note*: One can make use of the joint pmf of X and Y requested for HW 9 (and if you do it this way, then make use of the solution to that HW 9 problem — there is no need to derive those results again for this assignment). Alternatively, one can simply give some thought to the situation that occurs if $X = 0$ and determine that Y has a fairly simple well-known distribution is such a case.)

2) Let X_1 and X_2 be independent random variables having pmf

$$p_X(x) = (1-p)p^x \, I_{\{0,1,2,\ldots\}}(x).$$

Give the pmf of $V = X_1 + X_2$. (*Hint*: You might want to try doing something similar to what is done in the example on the top portion of p. 6-11 of the course notes.)

3) Let X and Y have the following joint density:

$$f_{X,Y}(x,y) = \begin{cases} y/x^2, & 0 < y < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(*Note*: This joint pdf was also used in HW 8 and HW 9.)

(a) Give the cdf of $W = Y/X$.

(b) Give the value of $E(W) = E(Y/X)$. (*Note*: There are two good ways to obtain the desired expectation. You can use the given joint pdf to obtain $E(Y/X)$ by integrating y/x times the pdf over the x-y plane (which is a method that I'll cover when I lecture about the first portion of Ch. 7 prior to giving the 2nd midterm exam). Alternatively, you can use the cdf obtained for part (a) to (easily) obtain the pdf of W, and then get the value of $E(W)$ by integrating w times $f_W(w)$. Neither way should be terribly hard, but you may find this 2nd way easier for this particular problem (provided that you got part (a) correct). Still, I encourage you to try it both ways in order to get some additional practice with probability methods. (But just turn in the solution that you want me to grade).)

4) Let X and Y have the following joint density:

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & 0 < x < 2,\ 0 < y < 2,\ 0 < x + y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(*Note*: This joint pdf is nonzero on a triangular region of the 1st quadrant of the x-y plane having area 2.)

(a) Give the pdf of $V = \min\{X, Y\}$.

(b) Give the value of $E(X+Y)$. (*Note*: There are a variety of ways to obtain the desired expectation. You can use the given joint pdf to obtain $E(X+Y)$ by integrating $x+y$ times the pdf over the x-y plane (which is a method that I'll cover when I lecture about the first portion of Ch. 7 prior to giving the 2nd midterm exam). Alternatively, you can note that $E(X+Y) = E(X) + E(Y)$, and obtain the two marginal densities from the joint pdf, and then use the marginal densities to obtain $E(X)$ and $E(Y)$. And a third approach is to let $W = X+Y$, use the "cdf method" to obtain the pdf of W, and then use the pdf of W to obtain $E(W) = E(X+Y)$. I encourage you to try it all of these ways in order to get some additional practice with probability methods. (But just turn in the solution that you want me to grade).)