MARGINAL COMPETITION: FIRST-ORDER GAINS, SECOND-ORDER COSTS

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Abstract

It is well known that a monopolist's price is a private optimum, so small price cuts generate only second-order losses in monopoly profit. It is also well known that deadweight loss rises quadratically as price departs from the competitive benchmark. This paper emphasizes a less appreciated implication of these two facts: a marginal price reduction produces a first-order reduction in deadweight loss while imposing only a second-order cost on the monopolist. At the monopoly optimum, the social gain from a small price cut therefore dominates the private loss by an arbitrarily large margin. This observation reframes how we should think about competition and antitrust and points to the value of marginal competition as well as unconventional welfare-improving interventions, including subsidizing marginal price reductions by monopolists and jawboning. We also prove a useful statistic: at the monopoly price, the marginal deadweight loss reduction from a \$1 price cut equals the quantity sold.

Section 1. Introduction

A monopolist's price is a private optimum, so by the envelope theorem, small price reductions cost the firm almost nothing—profit losses are second-order in the size of the price cut. But deadweight loss is not optimized by the monopolist, so small price reductions yield first-order welfare gains. The ratio of social gain to private loss therefore diverges as the price approaches the monopoly price: at the profit-maximizing price, a monopolist can be induced to reduce deadweight loss at arbitrarily small cost to itself. This paper develops this simple observation and draws out its implications for antitrust, philanthropy and the political economy of jawboning.

Figure 1 plots monopoly profit, deadweight loss, and the absolute value of the ratio of their derivatives with respect to quantity.[1] This ratio measures the marginal social gain from an incremental change in output relative to the monopolist's marginal private loss; I refer to it as the Harberger ratio. At the profit-maximizing output, the derivative of profit with respect to quantity is zero, while the derivative of deadweight loss is strictly positive, implying that the Harberger ratio is infinite. More generally, near the monopoly optimum, a small increase in output (equivalently, a small price reduction) yields a first-order increase in social welfare at the cost of only a second-order reduction in private profit.

The logic implies a convenient and surprising sufficient statistic: At the monopoly price, the marginal deadweight loss reduction from a \$1 price cut equals the quantity sold.

$$\left.\frac{d\,DWL(p)}{dp}\right|_{p_m} = Q(p_m).$$

Therefore, for a small price reduction $\delta > 0$,

$$DWL(p_m) - DWL(p_m - \delta) \approx Q(p_m)\,\delta.$$

. *Date*: January 5, 2026.

. I thank Robin Hanson and Garret Jones for comments. All equations were checked with ChatGPT5.2.

1. The demand curve is linear of the form A-bQ with A=10 and b=1. Marginal cost is constant at c (=2).

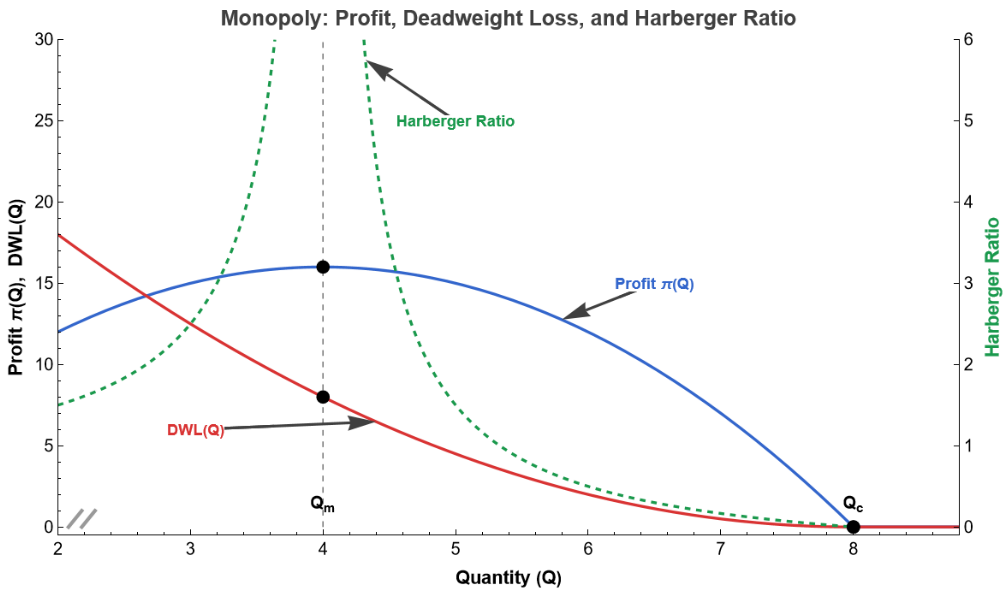

FIGURE 1. The graph shows monopoly profit, deadweight loss and the Harberger ratio (the marginal welfare loss from profit). Note that at the profit maximum the Harberger ratio is infinite and it drops off rapidly as we move from the monopoly quantity (price) towards the competitive quantity (price).

For example, if $Q(p_m) = 100$ million units per year, then the marginal deadweight-loss reduction is about \$100 million per \$1 decrease in price (equivalently, about \$1 million per cent).

The result follows directly from the first-order conditions of the firm's optimization problem and holds for any demand curve. The intuition is that at the monopolist's optimum we can ignore the effect of a price change on profits. Thus, the decline in deadweight loss from a small price reduction is equivalent to the increase in consumer surplus. But (thinking horizontally) the increase in consumer surplus from a small change in price is just the quantity at that price.[2]

2. Here are two proofs. First, let demand be $Q(p)$ with inverse demand $P(\cdot)$ and constant marginal cost c. Define deadweight loss at price p as the welfare gap relative to the efficient outcome $p = c$:

$$DWL(p) \equiv \int_{Q(p)}^{Q(c)} \big(P(q) - c\big)\, dq,$$

which is the standard Harberger measure of allocative loss from supracompetitive pricing. By Leibniz's rule,

$$\frac{d\,DWL}{dp} = -\big(P(Q(p)) - c\big)\, Q'(p) = -(p-c)Q'(p).$$

The monopolist's profit is $\pi(p) = (p-c)Q(p)$, so the first-order condition at p_m is

$$0 = \pi'(p_m) = Q(p_m) + (p_m - c)Q'(p_m) \quad \Rightarrow \quad -(p_m - c)Q'(p_m) = Q(p_m).$$

Substituting into $dDWL/dp$ yields $dDWL/dp|_{p_m} = Q(p_m) > 0$. Hence for a small price cut $\delta > 0$,

$$DWL(p_m) - DWL(p_m - \delta) \approx \left.\frac{d\,DWL}{dp}\right|_{p_m} \delta = Q(p_m)\delta.$$

Because DWL is defined as a welfare gap, this change captures a pure efficiency gain and excludes inframarginal transfers between consumers and producers.

Alternatively, note that that at the optimum the envelope theorem applies and we can ignore the effect of a change in price on profits. In this case, the reduction in deadweight loss from a price decline is equivalent to the gain in consumer surplus. Since $CS(p) = \int_p^{\bar{p}} Q(t)\, dt$ then using Leibniz's rule directly we have $\frac{dCS}{dp} = -Q(p) \cdot 1 + Q(\bar{p}) \cdot 0 = -Q(p)$

Section 2. A General Statement from the Envelope Theorem

More generally, let x be a firm's strategic choice (price, output, quality, exclusion intensity). Let $\pi(x)$ be private payoff and $W(x)$ be social welfare. Assume π is twice differentiable and strictly concave near its interior maximizer x_m, so $\pi'(x_m) = 0$ and $\pi''(x_m) < 0$.

Lemma 1 (Marginal competition)**.** *If $W'(x_m) \neq 0$, then for a small deviation Δ in the welfare-improving direction,*

$$\pi(x_m) - \pi(x_m + \Delta) = O(\Delta^2), \tag{1}$$

$$W(x_m + \Delta) - W(x_m) = O(\Delta), \tag{2}$$

and therefore the welfare gain per dollar of private profit loss diverges as $\Delta \to 0$.

Proof. By Taylor expansion around x_m,

$$\pi(x_m + \Delta) = \pi(x_m) + \pi'(x_m)\Delta + \tfrac{1}{2}\pi''(x_m)\Delta^2 + O(\Delta^3).$$

Since $\pi'(x_m) = 0$, the private loss is $-\frac{1}{2}\pi''(x_m)\Delta^2 + O(\Delta^3) = O(\Delta^2)$. Similarly,

$$W(x_m + \Delta) = W(x_m) + W'(x_m)\Delta + O(\Delta^2),$$

and $W'(x_m) \neq 0$ implies the welfare change is $O(\Delta)$. □

Lemma 1 is an envelope-theorem implication: profit is locally second-order sensitive around an interior optimum, whereas welfare need not be.

Corollary (Harberger ratio at a private optimum). Under the conditions of Lemma 1, define deadweight loss as the welfare gap relative to the welfare maximizer x^*, $DWL(x) \equiv W(x^*) - W(x)$, and for a small welfare-improving deviation Δ define the Harberger ratio

$$HR(\Delta) = \frac{W(x_m + \Delta) - W(x_m)}{\pi(x_m) - \pi(x_m + \Delta)}.$$

Then $HR(\Delta) \to \infty$ as $\Delta \to 0$. More precisely, if $W'(x_m)\Delta > 0$,

$$HR(\Delta) = \frac{2W'(x_m)}{-\pi''(x_m)} \cdot \frac{1}{\Delta} + o\left(\frac{1}{\Delta}\right),$$

so the allocative gain per unit of profit forgone is unbounded at the margin around an interior private optimum.

For example, if a monopolist optimizes quality, then small quality increases yield first-order welfare gains at second-order profit cost. The point here is different from the Spence model. Spence 1975 shows that a monopolist may choose to over or under provide quality relative to the optimum depending on whether the average consumer's valuation of quality is lower or higher than the marginal consumer's valuation of quality. The point here is that regardless of the direction of the quality deviation–improving it can be done at very low cost to the monopolist and significant gain to consumers.

Thinking about quality in this way reframes the question from under or over provision relative to an unknown optimum to the costs of changing quality on the margin. We give some examples further below.

Blanchard and Kiyotaki 1987 offer a general-equilibrium analogue of the paper's core geometry: with monopolistic competition, privately optimal markups depress aggregate activity because each firm does not internalize the effect of its pricing on economy-wide demand. As in our partial-equilibrium argument, small reductions in markups can generate first-order welfare gains even when the private incentive to change is second-order.

Section 3. From Qualitative to Quantitative

The Harberger ratio approaches infinity at the monopoly quantity, confirming that marginal welfare gains come at vanishing private cost. But total welfare gains depend on how far from the optimum a practical intervention can push the market. Three factors govern this: scale, curvature and pass-through.

First, scale: from $DWL(p_m) - DWL(p_m - \delta) \approx Q(p_m)\,\delta$ we know dWL is proportional to $Q(p)$, so high-volume markets generate large dollar reductions in deadweight loss even for small price changes. This is what makes the sufficient statistic practical, Q(pm) is often directly observed, requiring no structural estimation.

Second, curvature: the profit function's second derivative determines how quickly private losses accumulate as we move away from the monopoly optimum. A flatter profit function permits larger welfare-improving deviations at modest cost.

Third, pass-through: for interventions that operate through costs—taxes, subsidies, or entry—the induced price change depends on how firms transmit cost shifts to prices. Weyl and Fabinger (2013) show that pass-through summarizes this mapping under imperfect competition and is tightly linked to demand curvature.

Curvature and pass-through are notoriously difficult to estimate in general. But we develop intuition by analyzing a specific marginal intervention in a well known model. Namely, one additional entrant—in a standard Cournot model.

Section 4. Cournot oligopoly: the first competitor does most of the work

One way that price might decrease (quantity increase) "marginally" is via competition. We show the effect of marginal competition for linear demand and for CES ($Q(p) = K\,p^{-\varepsilon}, \varepsilon > 1$) in each case with n symmetric Cournot competitors, each with marginal cost c.

In the appendix we show the following ratios of DWL as a function of the number of competitors, n, for the linear and CES cases respectively:

$$\frac{DWL_n^{\text{lin}}}{DWL_1^{\text{lin}}} = \frac{4}{(n+1)^2}.$$

$$\frac{DWL_n^{\text{ces}}}{DWL_1^{\text{ces}}} = \frac{g(n,\varepsilon)}{g(1,\varepsilon)},$$

where

$$g(n,\varepsilon) = \frac{\varepsilon}{\varepsilon-1}\Big(1 - r_n^{(\varepsilon-1)/\varepsilon}\Big) - (1 - r_n), \qquad r_n = \left(1 - \frac{1}{\varepsilon n}\right)^{\varepsilon}.$$

In the CES case, for $\varepsilon = 2$ the expression reduces to the $\frac{1}{n^2}$ which suggests correctly that DWL falls even faster in the CES case than in the linear case.

Table 1 shows the decline in DWL with the number of competitors and also the profit loss and Harberger ratio. It's worth noting that the Harberger ratio in the linear and CES cases respectively have the nice forms

$$HR_n^{\text{lin}} = \frac{n+3}{n-1} \qquad HR_n^{\text{CES}} = \frac{n+1}{n-1}$$

From the table we see that DWL falls tremendously with the first competitor–the first competitor reduces deadweight loss by 50-75%–and the addition of a 3rd and 4th competitor adds much less to welfare. Adding the first competitor does cut profits but, as shown by the Harberger ratio, significantly less than DWL suggesting both the desirability of antitrust or other action and the political possibility, as we discuss further below.

As competition increases more of the gains to consumers are simple transfers from the monopolist and deadweight loss declines only marginally. Table 1 thus captures a central policy implication of this benchmark: the social gains from competition, as opposed to potential distributional gains, decline rapidly after the first few competitors.

	DWL reduction (% of monopoly DWL)				Profit loss (% of monopoly profit)		Harberger ratio	
n	Linear	CES ($\varepsilon = 2$)	CES ($\varepsilon = 3$)	CES ($\varepsilon = 5$)	Linear	CES ($\varepsilon = 2$)	Linear ($\frac{n+3}{n-1}$)	CES ($\varepsilon = 2$) ($\frac{n+1}{n-1}$)
1	0.0	0.0	0.0	0.0	0.0	0.0	–	–
2	55.6	75.0	71.4	69.0	11.1	25.0	5.00	3.00
3	75.0	88.9	86.8	85.2	25.0	44.4	3.00	2.00
4	84.0	93.8	92.4	91.4	36.0	56.3	2.33	1.67
5	88.9	96.0	95.1	94.4	44.4	64.0	2.00	1.50
6	91.8	97.2	96.6	96.1	51.0	69.4	1.80	1.40

Table 1. DWL reduction, profit loss, and Harberger ratios as the number of Cournot firms increases. "Profit loss" is the percent decline in total industry profit relative to monopoly. The Harberger ratio is the ratio of the percent DWL reduction to the percent profit loss. For space, profit loss and Harberger ratios are reported only for the linear benchmark and for CES with $\varepsilon = 2$.

Section 5. Implications for antitrust triage

The preceding results suggest a practical implication for enforcement priorities. Interventions that plausibly move an outcome even modestly off a monopoly optimum—for example by weakening exclusionary practices, enabling a small entrant, or reducing switching costs—can have high returns because they reduce deadweight loss to first order while the incumbent's profit reduction is second order in the policy change.

Reducing the cost of an entrant may be an especially attractive intervention because when the fixed cost of entry F is slightly above the equilibrium threshold, a small reduction in entry costs can unlock a discrete efficiency gain. Hence the benefit-cost ratio of reducing F can be arbitrarily large near the entry margin, even if the socially optimal number of firms is not necessarily larger in general.

Contestability theory suggests that actual entry may not be necessary for price discipline (Baumol, Panzar, and Willig 1982). In the limiting case of perfectly contestable markets—costless entry and exit and no incumbent advantages—the threat of entry can constrain price toward average cost. While perfect contestability is an idealization, the logic extends: reducing barriers to entry (or reducing switching costs) can shift incumbent behavior even if entry does not occur in equilibrium.

Interoperability and portability policies can increase contestability by reducing switching costs and weakening network-based lock-in, thereby raising the elasticity of demand faced by the incumbent at a given price. Even if large-scale entry does not occur immediately, the credible threat of customer migration or rapid competitive response can discipline markups and improve allocative efficiency. Examples include mobile number portability (allowing customers to keep their phone number when changing carriers), open-banking portability (mandated account-data portability and APIs), and reputation portability in two-sided platforms (allowing users to move their reputation across platforms). In each case, a little bit of competition or threat of competition can pay large rewards in reducing deadweight loss.

As another example let ω be the weight assigned to "predicted virality" vs. "source credibility" on a platform such as X (formerly Twitter) or Facebook. Platforms maximize ω for engagement and often resist changing this because they fear losing revenue. But the last incremental unit of "virality optimization" adds almost zero marginal profit. Yet that last unit may cause the most social damage (amplifying extreme toxicity). A regulator doesn't need to ban algorithms. They can mandate a "Marginal Down-ranking" of ω. The platform's internal A/B tests would likely confirm that a 1% reduction in toxicity weighting results in a statistically insignificant drop in ad revenue but a measurable improvement in user sentiment.

Section 6. Marginal Philanthropy

The marginal-competition geometry suggests a philanthropic analogue: to do well in the world shade your maximizing decisions slightly towards the social optimum (Hanson 2012). Monopolists may be willing to do this for reasons of altruism–and because the costs are low. Philanthropists may want to consider how marginal payments may improve welfare substantially.

Philanthropists could attempt to induce lower monopoly prices either directly, by compensating incumbents for price reductions, or indirectly, through reputational rewards such as favorable publicity. A direct payment to reduce prices could be thought of as a marginal version of a patent buyout (Kremer 1998) albeit one raising issues of credibility and enforcement.

Reputational rewards such as publicity for lowering prices for low-income customers are often dismissed as "charity washing" or "reputation laundering," on the grounds that the incumbent's private sacrifice is small. The Harberger-ratio logic developed here cautions against that inference. A small private sacrifice does not imply a small social gain: when prices are set at or near a private optimum, even modest price reductions can generate first-order reductions in deadweight loss. Moreover, if we focus on realistic reforms, small sacrifices may be a feature not a bug.[3]

Philanthropists need not approach the monopolist directly. Another practical approach is to fund contestability inputs that lower effective entry and switching costs: compatibility testing and certification, data migration and portability tools, open standards, regulatory filings and compliance infrastructure, litigation that targets exclusionary conduct, and distribution access for entrants. Such interventions can create or sharpen an outside option, which can discipline markups even if the entrant remains small.

The point is not that philanthropy should replace competition policy. Rather, the same local logic that makes "monopoly to duopoly" unusually valuable can create scope for targeted private interventions when formal enforcement is slow, jurisdictionally constrained, or politically blocked.

Section 7. The Political Economy of Jawboning

Profit changes can proxy for the political-economy cost of reform: thus another reason why marginal competition may be beneficial is that it is less likely to be strongly opposed by monopoly interests. In the limit, marginal reform reduces deadweight loss at negligible cost to the monopolist and so will not be opposed.

3. Note also that if the cost of moving price by δ is $O(\delta^2)$ and the welfare gain is $O(\delta)$, donors face a convex cost function with linear benefits. This implies that a philanthropist should spread funds across many markets rather than concentrating because small interventions dominate large ones (per dollar) (Landsburg 1997).

Indeed, marginal reform suggests a theory of "jawboning," Jawboning is typically seen as effective only when it is implicitly backed by a threat of more serious action. If the cost of action to the monopolist is small, however, then jawboning can meaningfully succeed even without threat.

Consider the example of a monopolist's profit-maximizing choice of quality. In particular, consider privacy is a quality dimension. Marginal users (those nearly indifferent to joining a platform) likely care less about privacy than average users (who value the network but would appreciate more privacy). So under-provision of privacy is predicted (Spence 1975). At the platform's chosen privacy level, small privacy improvements yield first-order welfare gains (benefiting all users, plus externalities) at second-order profit cost (the platform is at its optimum). Thus on the margin significant improvements in quality may be possible at relatively low cost perhaps even at the cost of jawboning.

This reframes regulatory debates: instead of asking "should we force the monopolist to provide higher quality?" (a large intervention with uncertain effects), ask "what's the smallest quality improvement we could induce, and how large is its welfare benefit?" The answer, near any private optimum, is: arbitrarily large benefit-cost ratios for sufficiently small interventions.

Section 8. Conclusion

At a private optimum, small deviations reduce profit only to second order, whereas welfare can change to first order. The consequences of this simple result are perhaps underappreciated.

The traditional question in antitrust and regulation is "how far is the market from the social optimum?"—a question that requires estimating demand systems, cost structures, and counterfactual equilibria. The marginal competition perspective asks instead: "how cheaply can we move the market a little?" Near a monopoly optimum, the answer is: very cheaply, with large welfare returns. The sufficient statistic $\frac{d\,DWL(p)}{dp}\Big|_{p_m} = Q(p_m)$ makes this operational without full structural estimation.

The implications extend beyond price. Any margin the firm optimizes—quality, durability, interoperability, privacy—exhibits the same local geometry. Debates about whether monopolists over- or under-provide along these dimensions remain important, but they are secondary to a simpler point: whatever the direction of distortion, correcting it is locally cheap.

Finally, the logic suggests that small reforms deserve more attention than they typically receive. Grand antitrust interventions and structural breakups attract scrutiny and opposition. Marginal interventions—enabling one entrant, reducing one barrier, inducing one small price cut—face less resistance precisely because they cost incumbents little. That is not a weakness. The Harberger ratio tells us that where private costs are smallest, social returns can be largest.

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Appendix A. Derivations for Table 1

This appendix derives the Cournot equilibrium outcomes and the implied deadweight loss (DWL) and total industry profit needed to construct Table 1. Throughout, DWL is measured relative to the efficient benchmark $p = c$:

$$DWL \;\equiv\; \int_{Q_n}^{Q(c)} \big(P(q) - c\big)\, dq,$$

where $P(\cdot)$ is inverse demand and $Q(c)$ is the efficient quantity.

Subsection A.1. **Linear inverse demand.** Assume inverse demand is $P(Q) = a - bQ$ with $b > 0$ and constant marginal cost $c < a$. Let $A \equiv a - c$.

Cournot equilibrium. Firm i chooses q_i to maximize

$$\pi_i(q_i, q_{-i}) = (P(Q) - c)q_i = (A - bQ)q_i, \qquad Q \equiv q_i + \sum_{j \neq i} q_j.$$

The first-order condition is

$$\frac{\partial \pi_i}{\partial q_i} = A - bQ - bq_i = 0.$$

In a symmetric equilibrium $q_i = q$ for all i, so $Q = nq$ and the FOC becomes

$$A - b(nq) - bq = 0 \quad \Rightarrow \quad q = \frac{A}{b(n+1)}.$$

Hence

$$Q_n = \frac{nA}{b(n+1)}, \qquad p_n = P(Q_n) = a - bQ_n = a - \frac{nA}{n+1} = c + \frac{A}{n+1}.$$

Total industry profit. Markup is $p_n - c = \frac{A}{n+1}$. Each firm's profit is

$$\pi_i = (p_n - c)q = \frac{A}{n+1} \cdot \frac{A}{b(n+1)} = \frac{A^2}{b(n+1)^2},$$

so total industry profit is

$$\Pi_n = n\pi_i = \frac{nA^2}{b(n+1)^2}.$$

Under monopoly ($n = 1$), $\Pi_1 = \frac{A^2}{4b}$, so

$$\frac{\Pi_n}{\Pi_1} = \frac{4n}{(n+1)^2}, \qquad \text{Profit loss fraction} \;=\; 1 - \frac{\Pi_n}{\Pi_1} = 1 - \frac{4n}{(n+1)^2}.$$

Deadweight loss. The efficient quantity solves $P(Q) = c$, hence $Q(c) = \frac{a-c}{b} = \frac{A}{b}$. With linear demand and constant marginal cost, DWL is the Harberger triangle:

$$DWL_n = \frac{1}{2}\,(p_n - c)\,(Q(c) - Q_n) = \frac{1}{2} \cdot \frac{A}{n+1} \cdot \left(\frac{A}{b} - \frac{nA}{b(n+1)}\right) = \frac{A^2}{2b(n+1)^2}.$$

Under monopoly, $DWL_1 = \frac{A^2}{8b}$, so

$$\frac{DWL_n}{DWL_1} = \frac{4}{(n+1)^2}, \qquad \text{DWL reduction fraction} \;=\; 1 - \frac{DWL_n}{DWL_1} = 1 - \frac{4}{(n+1)^2}.$$

Harberger ratio (as defined in Table 1). Define

$$HR_n \;\equiv\; \frac{\text{DWL reduction fraction}}{\text{Profit loss fraction}} = \frac{1-\frac{4}{(n+1)^2}}{1-\frac{4n}{(n+1)^2}} = \frac{(n+1)^2-4}{(n+1)^2-4n} = \frac{(n-1)(n+3)}{(n-1)^2} = \frac{n+3}{n-1}, \qquad n \geq 2.$$

Subsection A.2. **CES (isoelastic) demand.** Assume demand is isoelastic

$$Q(p) = Kp^{-\varepsilon}, \qquad \varepsilon > 1,$$

so inverse demand is

$$P(Q) = K^{1/\varepsilon}Q^{-1/\varepsilon}.$$

Let $Q(c) = Kc^{-\varepsilon}$ denote the efficient quantity.

Cournot equilibrium. Firm i maximizes $\pi_i = (P(Q)-c)q_i$. The Cournot FOC is

$$0 = \frac{\partial \pi_i}{\partial q_i} = P(Q) - c + P'(Q)q_i.$$

Since $P(Q) = K^{1/\varepsilon}Q^{-1/\varepsilon}$,

$$P'(Q) = -\frac{1}{\varepsilon}K^{1/\varepsilon}Q^{-1/\varepsilon-1} = -\frac{1}{\varepsilon}\frac{P(Q)}{Q}.$$

Imposing symmetry $q_i = Q/n$ yields

$$P(Q) - c - \frac{1}{\varepsilon}\frac{P(Q)}{Q}\cdot\frac{Q}{n} = 0 \quad \Rightarrow \quad P(Q)\Big(1-\frac{1}{\varepsilon n}\Big) = c.$$

Therefore equilibrium price is

$$p_n = c\cdot\frac{\varepsilon n}{\varepsilon n - 1}.$$

Quantity follows from demand:

$$Q_n = Kp_n^{-\varepsilon} = Kc^{-\varepsilon}\Big(1-\frac{1}{\varepsilon n}\Big)^{\varepsilon} = Q(c)\Big(1-\frac{1}{\varepsilon n}\Big)^{\varepsilon}.$$

Define the output ratio

$$r_n \equiv \frac{Q_n}{Q(c)} = \Big(1-\frac{1}{\varepsilon n}\Big)^{\varepsilon}.$$

Total industry profit. Total industry profit equals markup times quantity:

$$\Pi_n = (p_n - c)Q_n.$$

Using $p_n - c = c\left(\frac{\varepsilon n}{\varepsilon n-1} - 1\right) = \frac{c}{\varepsilon n - 1}$ and $Q_n = Q(c)r_n$,

$$\Pi_n = \frac{cQ(c)}{\varepsilon n - 1}\,r_n.$$

For $\varepsilon = 2$, this simplifies to

$$r_n = \Big(1-\frac{1}{2n}\Big)^2 = \frac{(2n-1)^2}{4n^2}, \qquad \Pi_n = \frac{cQ(c)}{2n-1}\cdot\frac{(2n-1)^2}{4n^2} = \frac{cQ(c)}{4}\cdot\frac{2n-1}{n^2}.$$

Since $\Pi_1 = \frac{cQ(c)}{4}$ when $\varepsilon = 2$,

$$\frac{\Pi_n}{\Pi_1} = \frac{2n-1}{n^2}, \qquad \text{Profit loss fraction} \;=\; 1 - \frac{2n-1}{n^2} = \frac{(n-1)^2}{n^2}.$$

Deadweight loss. By definition,

$$DWL_n = \int_{Q_n}^{Q(c)} \big(P(q) - c\big)\, dq.$$

Write $B \equiv K^{1/\varepsilon}$ so that $P(q) = Bq^{-1/\varepsilon}$. Since $P(Q(c)) = c$, we have

$$c = B\, Q(c)^{-1/\varepsilon} \quad \Rightarrow \quad B = c\, Q(c)^{1/\varepsilon}.$$

Compute the integral:

$$\int Bq^{-1/\varepsilon}\, dq = B \cdot \frac{\varepsilon}{\varepsilon - 1}\, q^{(\varepsilon-1)/\varepsilon}.$$

Thus

$$\begin{aligned} DWL_n &= B\frac{\varepsilon}{\varepsilon - 1}\Big(Q(c)^{(\varepsilon-1)/\varepsilon} - Q_n^{(\varepsilon-1)/\varepsilon}\Big) \; - \; c\big(Q(c) - Q_n\big) \\ &= cQ(c)\left[\frac{\varepsilon}{\varepsilon - 1}\Big(1 - r_n^{(\varepsilon-1)/\varepsilon}\Big) - (1 - r_n)\right]. \end{aligned}$$

Equivalently, defining

$$g(n, \varepsilon) \equiv \frac{\varepsilon}{\varepsilon - 1}\Big(1 - r_n^{(\varepsilon-1)/\varepsilon}\Big) - (1 - r_n), \qquad r_n = \Big(1 - \frac{1}{\varepsilon n}\Big)^{\varepsilon},$$

we have

$$DWL_n = cQ(c)\, g(n, \varepsilon), \qquad \frac{DWL_n}{DWL_1} = \frac{g(n, \varepsilon)}{g(1, \varepsilon)}.$$

For $\varepsilon = 2$, note that $r_n^{(\varepsilon-1)/\varepsilon} = r_n^{1/2} = 1 - \frac{1}{2n}$. Then

$$g(n, 2) = 2\Big(1 - \Big(1 - \frac{1}{2n}\Big)\Big) - \Big(1 - \Big(1 - \frac{1}{2n}\Big)^2\Big) = \frac{1}{4n^2},$$

so

$$DWL_n = \frac{cQ(c)}{4n^2}, \qquad \frac{DWL_n}{DWL_1} = \frac{1}{n^2}, \qquad \text{DWL reduction fraction} \; = \; 1 - \frac{1}{n^2}.$$

Harberger ratio for $\varepsilon = 2$. Using the fractions above,

$$HR_n^{(\varepsilon=2)} = \frac{1 - \frac{1}{n^2}}{\frac{(n-1)^2}{n^2}} = \frac{(n-1)(n+1)}{(n-1)^2} = \frac{n+1}{n-1}, \qquad n \geq 2.$$