Math 114: Practice Exam 4

This is a 50 minute exam.

1. For $f(x) = 2^x$:

(a) Complete the following table and use it to find a formula for $f^{(k)}(0)$. Recall $\frac{d}{dx}(2^x) = 2^x \ln(2)$.

k	$f^{(k)}(x)$	$f^{(k)}(0)$
0		
1		
2		
3		

(b) What is the Taylor series representation for $f(x)$ centered at 0?

(c) Find the second degree Taylor polynomial for $f(x)$ centered at 0, and use it to approximate $\sqrt{2}$. Use $\ln(2) \approx \ln(e) = 1$ in your approximation.

(d) How large could the error be in the above approximation? Again, use the fact that $\ln(2) < \ln(e) = 1$.

(e) What is the smallest degree Taylor polynomial necessary to guarantee accuracy within 10^{-2}?

2. Find the power series representation centered at 0 for $\dfrac{-2x}{(1-x^2)^2}$ and find its interval and radius of convergence. Hint: First find the power series representation centered at 0 for $\dfrac{1}{1-x^2}$.

3. Use Taylor series to evaluate $\lim\limits_{x\to 0} \dfrac{x^2/2 - 1 + \cos(x)}{x^4}$.

4. Eliminate the parameter in $x(t) = te^t + 1$ and $y(t) = t^2e^{2t} + 4te^t$ for $0 \le t \le 1$ to obtain an equation in x and y.

5. Graph the function from 4 and include orientation.

6. Find the tangent line to the above graph at $t = 0$.

7. Convert $\frac{x^2}{4} + \frac{y^2}{9} = 1$ to polar coordinates.

8. Convert $r = \tan\theta$ to Cartesian coordinates.