

Week 13 — Exam Review

MATH:113, Recitations 304 and 305

IMPLICIT DIFFERENTIATION, RELATED RATES

Names: _____

For each of the following, find $\frac{dy}{dx}$ first by solving directly for y and differentiating directly; then, use implicit differentiation to find $\frac{dy}{dx}$. Check that the derivatives you get are the same.

$$x/y^3 = 1$$

$$x^2 + y^3 = 4$$

$$x^2 + y^2 = 2$$

Implicitly determine the derivatives $\frac{dy}{dx}$ for each of the following expressions. If a point is provided, evaluate $\frac{dy}{dx}$ at that point.

$$e^x - \cos(y) = x$$

$$\tan(x^2 y^4) = 3x + y^2 \quad (\text{warning: very gross})$$

$$x^2 + y^2 = 4 \quad \text{at } (0, 2)$$

$$y^4 + 3xy + x = 2 \quad \text{at } (2, 0)$$

Complete the following related rates problems.

1. The length of one edge of a cube is increasing at a rate of 8 cm/s . How quickly is the volume of the cube growing when this edge is 6cm long?

2. A dinghy carrying a horse (a *horse boat*, patent Dwight Schrute) is pulled toward a dock by a rope. The dock is 6 feet higher than the top of the boat.
 - (a) If we pull in the rope at 2 ft/s , how quickly is the boat approaching the dock if 10 feet of rope are out?
 - (b) How quickly is the *angle from the rope to the boat* changing when 10 feet of rope are out?

This question is directly from your practice exam, save for a small change. Make sure you're comfortable with the concepts, setup, and computation strategies used in this problem.

3. Air is escaping from a perfectly (and impossibly) spherical balloon at a rate of $2\text{ m}^3/\text{min}$. When the radius of the balloon is 1m, how quickly is the *surface area* of the balloon shrinking?