Week 13 — Exam Review

MATH:113, Recitations 304 and 305

Derivatives

Names: SOLUTIONS

Write down the limit definition of the derivative. What does each term represent?

For a function f continuous at the point x, the derivative is

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

} "rise"
} "run"

Write down the general forms of:

(a) *The power rule.*

For a function $f(x)$ and $f^n(x) = (f(x))^n$,

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n\cdot(f(x))^{n-1}\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x).}$$

(b) *The product rule.*

(c) *The quotient rule.*

(d) *The chain rule.*

(e) *The derivatives of* $\sin(t)$, $\cos(t)$, *and* $\tan(t)$.

(f) *The derivatives of the inverses of the functions in (e).*

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$$

(g) *The derivative of* $\ln(x)$.

$$\frac{d}{dx}(\log_n(x)) = \frac{1}{x\cdot\ln(n)}$$

(h) *The derivative of the exponential function* $e^{u(x)}$.

$$\frac{d}{dx}(e^{u(x)}) = u'(x)\cdot e^{u(x)}$$

chain rule!

Find solutions to the following.

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{6x^2}{2-x}$$

quotient rule gives

$$\frac{d}{dx}\left(\frac{6x^2}{(2-x)}\right) = \frac{6(4x-x^2)}{(2-x)^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\tan(t)\sec(t)$$

power rule gives

$$= \sec^2(t)\sec(t) + \tan(t)\cdot\tan(t)\sec(t)$$
$$= \sec^3(t) + \tan^2(t)\sec(t)$$
$$= \sec(t)\left(\sec^2(t) + \tan^2(t)\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}e^{1-\cos(x)}$$

chain rule gives

$$\frac{d}{dx}\left(e^{1-\cos(x)}\right) = \frac{d}{dx}(1-\cos(x))\cdot e^{1-\cos(x)}$$
$$= \sin(x)\cdot e^{1-\cos(x)}$$

$$\lim_{x\to\infty} x\ln\left(1+\frac{3}{x}\right)$$

try

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x}} \cdot x \cdot \ln\left(1+\frac{3}{x}\right)$$
$$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{3}{x}\right)}{1/x}$$

derivatives + simplification!

$$\overset{H}{=} \lim_{x\to\infty} \frac{3}{1+3/x} = 3$$

$$\lim_{x\to 0}\frac{\sin(10x)}{x}$$

Evaluating at 0 we get $\frac{0}{0}$, so apply L'Hôpital's rule:

$$\lim_{x\to 0}\frac{\sin(10x)}{x} \overset{H}{=} \lim_{x\to 0}\frac{10\cdot\cos(10x)}{1}$$
$$= \frac{10\cdot(1)}{1} = 10$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}(x+4)$$

$$= \frac{1}{\sqrt{1-(x+4)^2}}\cdot\frac{d}{dx}(x+4)$$
$$= \frac{1}{\sqrt{1-(x+4)^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\csc(7t)$$

chain rule gives

$$\frac{d}{dt}\left(\csc(7t)\right) = \frac{d}{dt}(7t)\cdot -\csc(7t)\cot(7t)$$
$$= 7\cdot " \quad "$$
$$= -7\csc(7t)\cot(7t)$$

$$\lim_{x\to 1^+} x^{1/1-x} = L$$

$$\ln(L) = \ln\left(\lim_{x\to 1^+} x^{1/1-x}\right)$$

use the continuity of ln !!!

$$= \lim_{x\to 1^+} \ln\left(x^{1/1-x}\right)$$
$$= \lim_{x\to 1^+} \frac{1}{1-x}\cdot\ln(x)$$
$$\overset{H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}}{1} = 1$$