# Week 11 Recitation Problems MATH:113, Recitations 304 and 305 

Names: $\qquad$

## Linearization

1. Concepts. Discuss the following with your group.
(i) What's the goal of linear approximation? What are we trying to achieve?
(ii) Given a function $f: \mathbb{R} \rightarrow \mathbb{R}$ (i.e. a function which takes real numbers as input and gives real numbers as output), what information do we need about $f$ to construct a linear approximation near some anchor point $a \in \mathbb{R}$ ? (Note: the " $\in$ " symbol means "in," so $a \in \mathbb{R}$ means " $a$ belongs to the set of real numbers $\mathbb{R}$.")
(iii) How accurate are these approximations?

## 2. Setup.

(i) Without looking at your notes or the internet, write down the general equation for the linear approximation $L(x)$ of a differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$ at the point $x=a$.
(ii) Annotate the equation you wrote down for $L(x)$. What information does each term give you?
(iii) Re-write the approximation $L(x)$ in terms of differentials. How do you interpret this?

## 3. Computation.

(i) Approximate the function $f(x)=\sqrt[4]{x}$ at the point $x=2$.
(ii) Using your approximation, guess the values of $\sqrt[4]{3}$ and $\sqrt[4]{10}$.
4. Bonus: if you had to tell a computer how to find the value of a derivative at a point, how would you do it?
5. Bonus: if you wanted to get a better approximation of $f$, how would you do it?

## Critical points and curve sketching

1. Concepts. Discuss the following with your group.
(i) What is a critical point?
(ii) What is an inflection point?
(iii) What information do critical and inflection points give us?
(iv) Given a differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$, two theorems give us guarantees about certain values of $f$ or its derivative $f^{\prime}$ on a given interval. What are these theorems? What properties of $f$ or the interval $[a, b]$ are required for the theorems to hold?

## 2. Computation.

(i) Consider the function $f(x)=(x-1)^{2}(x+1)^{2}$ on the interval $[-3,3]$. What are its critical and inflection points?
(ii) Without using the internet, plot critical and inflection points. Note the sign of the first and second derivatives to the left and right of each point. Can you tell (approximately) how the function looks based just on these points? Based on this information, sketch $f(x)$.

