Week 10 Recitation Problems

MATH:113, Recitations 304 and 305

Names: ____

Today, we'll be dissecting *two* related rates problems. **Do not attempt to solve the problems right away. Work through the questions in order, and discuss with your group. We'll check in periodically as a class.** Here's Problem #1:

> *Suppose, for a certain magical rectangle $\mathscr{R}$, that its length is always exactly three times its height. Describe $\mathscr{R}$'s length by y and its height by x. If the height of $\mathscr{R}$ is decreasing at a rate of 2 inches per minute, how quickly is the length of $\mathscr{R}$ shrinking? If the height of $\mathscr{R}$ is exactly 6 inches and is decreasing at a rate of 2 inches per minute, how quickly is the area of $\mathscr{R}$ shrinking?*

1. **Concepts.** In your group, discuss the following:
   - (i) What *is* x? Is it a variable, a function, or both? If x is a function, what might it take as input?
   - (ii) Answer the same questions, but for y.
   - (iii) What is the relationship between x and y? That is, which depends on which?
2. **Setup.**
   - (i) How can we describe the *area* of $\mathscr{R}$?
   - (ii) How are the "rates of decrease" from the prompt related to y and x?
   - (iii) Why are these problems called *related rates*?
   - (iv) What derivative tools might we need to use here?
3. **Computation.**
   - (i) Determine how quickly the length of $\mathscr{R}$ is changing when the height of $\mathscr{R}$ is decreasing at a rate of 2 inches per minute.
   - (ii) Determine how quickly the area of $\mathscr{R}$ is changing when the height of $\mathscr{R}$ is 6 inches, and decreasing at a rate of 2 inches per minute.

Here's Problem #2: as before, **do not attempt to solve it right away.**

> *Imagine that you're a clown with a particularly good set of lungs. You're in charge of making balloon animals at your friend's wife's brother's friend's kid's birthday party — unfortunately, your clown school didn't have the accreditation for awarding Balloon Animal Certificates, so you've only been trained to make extremely spherical "animals."*
>
> *With your impressive wind power generation, you blow up a balloon $\mathfrak{B}$ so that its volume increases at a rate of 100 cubic inches per second (which is crazy fast). How quickly is the radius of $\mathfrak{B}$ increasing when the diameter is exactly 50 inches?*

1. **Setup.**
   (i) What function(s) can we use to describe the volume of $\mathfrak{B}$? What about the radius? What geometric relationships are involved?
   (ii) Pay attention to the derivative of the function which describes the volume of $\mathfrak{B}$: what familiar geometric formula is it?
   (iii) How is the "rate of increase" from the prompt related to our function(s) for the volume?
   (iv) What derivative tools might we need to use here?
2. **Computation.**
   (i) Determine how quickly the radius of $\mathfrak{B}$ is increasing when its diameter is 50 inches.
   (ii) **Bonus:** how long would it take for $\mathfrak{B}$ to have the same volume as the earth?

If you've completed the above problems, here's a bonus problem:

> *Find the linear, quadratic, and cubic approximations for the function $f(x) = e^x$.*