# Week 7 Recitation Problems

MATH:113, Recitations 304 and 305

**What errors can you find?**

1.

2.

3.

4.

5.

6.

**Definition 1: the chain rule™**

**Fact 1:** $\frac{\mathrm{d}}{\mathrm{d}t}\sin t$

**Fact 2:** $\frac{\mathrm{d}}{\mathrm{d}t}\cos t$

**Fact 3:** $\frac{\mathrm{d}}{\mathrm{d}t}\tan t$

**Find at least four of the following derivatives.**

$(2t^3 + \cos(t))^{50}$ $\qquad$ $\sin(3x^2 + x)$ $\qquad$ $\sqrt{5x + \tan(4x)}$

$\cos^4(t) + \cos(4t)$ $\qquad$ $e^{w^4 - 2w^2 + 9}$ $\qquad$ $\cos(x^2 e^x)$

**For the following problem, come up with a solution to present to one of the instructors.**

At $t$ tenths of a second, the position of a piston in a Lycoming IO-360-B4A V4 internal combustion propeller engine is measured by the function $P(t) = -\sin(t^2) + 1$. During the first three-tenths of a second of flight, on what intervals of time is the first piston in the intake phase (moving down, to suck in air) and compression phase (moving up, to compress the air-fuel mixture)?

**Complete this proof of the product rule by filling in the blanks.**

Using the limit definition of the derivative, we know that

$$\frac{\mathrm{d}}{\mathrm{d}x}(fg)(x) = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

Then, using an algebraic manipulation, we can say that

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(fg)(x) &= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x+h) + f(x+h)g(x) - f(x+h)g(x) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)(\qquad\qquad)}{h} + \lim_{h\to 0} \frac{g(x)(\qquad\qquad)}{h} \\
&= \lim_{h\to 0} f(x+h)\frac{\qquad\qquad}{h} + \lim_{h\to 0} g(x+h)\frac{\qquad\qquad}{h} \\
&= \lim_{h\to 0} f(x+h) \cdot \lim_{h\to 0} \frac{\qquad\qquad}{h} + \lim_{h\to 0} g(x+h) \cdot \lim_{h\to 0} \frac{\qquad\qquad}{h} \\
&= \lim_{h\to 0} f(x+h) \cdot \underline{\quad} + \lim_{h\to 0} g(x+h) \cdot \underline{\quad} \\
&= f(x)g'(x) + f'(x)g(x),
\end{aligned}$$

as desired. ■

**The above proof is missing a critical assumption. What is it?**

**Bonus: prove that differentiability implies continuity.**

■