

Week 7 Recitation Problems

MATH:113, Recitations 304 and 305

What errors can you find?

- | | | |
|---|--|----|
| 1. line 2: derivative wrt
x should go to 0 | 3. line 8: need to use
product rule here! | 5. |
| 2. line 5: need to use
<u>product and chain rules!</u> | 4. line 10: wrong sign! | 6. |

Definition 1: the chain rule™

For differentiable functions f and g ,

$$\frac{d}{dx} f(g(x)) = g'(x) \cdot f'(g(x))$$

Fact 1: $\frac{d}{dt} \sin t$

$$\cos t$$

Fact 2: $\frac{d}{dt} \cos t$

$$-\sin t$$

Fact 3: $\frac{d}{dt} \tan t$

$$\sec^2 t$$

Find at least four of the following derivatives.

$$\begin{aligned}
 & (2t^3 + \cos(t))^{50} & \left| \right. & \sin(3x^2 + x) & \left| \right. & \frac{5+4\sec^2(4x)}{2\sqrt{5x+\tan(4x)}} \\
 & = 50(2t^3 + \cos(t))^{49} \cdot (6t^2 - \sin(t)) & \left| \right. & = (6x+1) \cdot \cos(3x^2+x) & \left| \right. & \cos(x^2 e^x) \\
 & \quad \cos^4(t) + \cos(4t) & \left| \right. & \quad e^{w^4-2w^2+9} & \left| \right. & \quad = (x^2 e^x \cdot 2x e^x) \cdot \sin(x^2 e^x) \\
 & = -4\cos^3(t)\sin(t) - 4\sin(t) & \left| \right. & = (4w^3 - 4w) \cdot e^{w^4-2w^2+9} & \left| \right. &
 \end{aligned}$$

For the following problem, come up with a solution to present to one of the instructors.

At t tenths of a second, the position of a piston in a Lycoming IO-360-B4A V4 internal combustion engine is measured by the function $P(t) = -\sin(t^2) + 1$. During the first three-tenths of a second of flight, on what intervals of time is the first piston in the intake phase (moving down, to suck in air) and compression phase (moving up, to compress the air-fuel mixture)?

see page 3!

Complete this proof of the product rule by filling in the blanks.

Using the limit definition of the derivative, we know that

$$\frac{d}{dx}(fg)(x) = \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

Then, using an algebraic manipulation, we can say that

$$\begin{aligned}\frac{d}{dx}(fg)(x) &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) + f(x+h)g(x) - f(x+h)g(x) - f(x)g(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{f(x+h)(g(x+h) - g(x))}{h} + \lim_{h \rightarrow 0} \frac{g(x)(f(x+h) - f(x))}{h} \\&= \lim_{h \rightarrow 0} f(x+h) \frac{g(x+h) - g(x)}{h} + \lim_{h \rightarrow 0} g(x+h) \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} f(x+h) \cdot \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} + \lim_{h \rightarrow 0} g(x+h) \cdot \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} f(x+h) \cdot \underline{f'(x)} + \lim_{h \rightarrow 0} g(x+h) \cdot \underline{g'(x)} \\&= f(x)g'(x) + f'(x)g(x),\end{aligned}$$

as desired. ■

The above proof is missing a critical assumption. What is it?

We need to know that both f and g are differentiable (at least at the point x)!

Bonus: prove that differentiability implies continuity.

Suppose f is differentiable at the point a ; then, the following limit exists:

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad (\text{"secant line" form of differentiable})$$

Then,

$$\begin{aligned}\lim_{x \rightarrow a} f(x) - f(a) &= \lim_{x \rightarrow a} f(x) - f(a) \cdot \frac{x - a}{x - a} \\&= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \rightarrow a} (x - a) \\&= f'(a) \cdot 0 \\&= 0,\end{aligned}$$

so we have $\lim_{x \rightarrow a} f(x) - f(a) = 0 \Rightarrow \lim_{x \rightarrow a} f(x) = f(a)$, the definition of continuity! ■

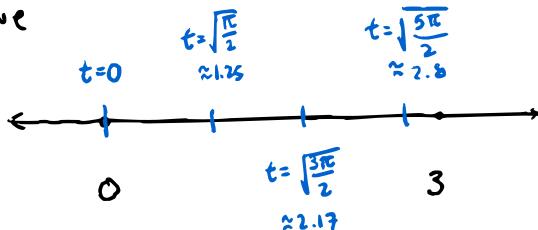
Velocity - directional speed - is given by $P'(t) = -2t \cdot \cos(t^2)$. Then, to find the direction of movement, we find where $P'(t)$ is 0, and test points! So,

$$t=0 \Rightarrow P'(0) = -2(0)^2 \cdot \cos(0^2) \\ = 0,$$

and $\cos(\theta) = 0$ when $\theta = \frac{k\pi}{2}$ for k odd, so

$$t = \pm \sqrt{\frac{k\pi}{2}}, k \text{ odd} \Rightarrow P'(\sqrt{\frac{k\pi}{2}}) = 2\left(\sqrt{\frac{k\pi}{2}}\right) \cdot \cos\left(\sqrt{\frac{k\pi}{2}}^2\right) \\ = " " \cdot 0 \\ = 0.$$

so, we have



and by testing points between the blue points (e.g. $\sqrt{\pi/4}$), we can tell when the velocity is positive or negative. we find that . . .

