

Week 6 Recitation Problems

MATH:113, Recitations 304 and 305

Names: SOLUTIONS

Fact 1: the power rule.

For $n \neq 0$,

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

Fact 2: the product rule.

For f, g differentiable,

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$$

Fact 3: linearity of derivatives.

For f and g differentiable,

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x), \quad \frac{d}{dx}c \cdot f(x) = c \cdot \frac{d}{dx}f(x)$$

Round	Attempted	Correct
1		
2		
3		
Totals		

Bonus problem: Prove that

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

Hints: let $t = \frac{n}{x}$, then use the fact that $e = \lim_{t \rightarrow 0} (1+t)^{1/t}$.

Proof 1: Derivative of the natural logarithm.

$$\begin{aligned} \lim_{n \rightarrow 0} \frac{\ln(x+n) - \ln(x)}{n} &= \lim_{n \rightarrow 0} \frac{\ln\left(\frac{x+n}{x}\right)}{\frac{n}{x}} \quad \text{ln}a - \ln b = \ln \frac{a}{b} \\ &= \lim_{n \rightarrow 0} \frac{1}{n} \ln\left(\frac{x+n}{x}\right) \\ &= \lim_{n \rightarrow 0} \frac{1}{n} \cdot \ln\left(1 + \frac{n}{x}\right) \\ t = \frac{n}{x} \Rightarrow \frac{1}{n} &= \frac{1}{xt} \\ &= \lim_{t \rightarrow 0} \frac{1}{xt} \cdot \ln(1+xt) \\ &= \lim_{t \rightarrow 0} \frac{1}{x} \cdot \ln((1+xt)^{1/t}) \quad a \cdot \ln b = \ln b^a \\ &= \frac{1}{x} \lim_{t \rightarrow 0} \ln((1+xt)^{1/t}) \\ &\approx \frac{1}{x} \cdot \ln(e) \\ &= \frac{1}{x} . \end{aligned}$$

WARM-UP

$$\frac{d}{dx} (x^2 + 3x + 1) = \underline{2x+3}$$

(linearity, power rule)

$$\begin{aligned}\frac{d}{dx} (3x^{e+4}) &= 3(e+4)x^{(e+4)-1} \\ (\text{linearity, power rule}) &= 3(ex^{e+3} + 4x^{e+3}) \\ &= \underline{3ex^{e+3} + 12x^{e+3}}\end{aligned}$$

$$\frac{d}{dx} \frac{1}{x^2} = \frac{d}{dx} x^{-2}$$

$$\begin{aligned}(\text{power rule}) &= -2x^{-3} \\ &= -\frac{2}{x^3}\end{aligned}$$

$$\frac{d}{dx} e^x = \underline{e^x} \quad (\text{identity})$$

ROUND 1

$$\begin{aligned}\frac{d}{dx} \sqrt{5x + \frac{\sqrt{7}}{x}} &= \sqrt{5} \frac{d}{dx} x + \sqrt{7} \frac{d}{dx} x^{-1} \\ &= \sqrt{5}(1) + \sqrt{7}(-x^{-2}) \\ &= \underline{\sqrt{5} - \frac{\sqrt{7}}{x^2}}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} (e^x - x^e) &= \frac{d}{dx} e^x - \frac{d}{dx} x^e \\ &= \underline{e^x - ex^{(e-1)}}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} \frac{5}{x^3} &= 5 \frac{d}{dx} x^{-3} \\ &= 5(-3 \cdot x^{-4}) \\ &= \underline{-\frac{15}{x^4}}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} \sqrt[4]{x} - 4e^x &= \frac{d}{dx} (x^{\frac{1}{4}}) - 4 \frac{d}{dx} e^x \\ &= \left(\frac{1}{4} \cdot x^{-\frac{3}{4}}\right) - 4e^x \\ &= \underline{\frac{1}{4\sqrt[4]{x^3}} - 4e^x}\end{aligned}$$

ROUND 2

$$\frac{d}{dx} (x+1)^2 = \frac{d}{dx} x^2 + 2x + 1$$

$$= 2x + 2$$

$$\frac{d}{dx} \frac{x^6}{5} = \frac{1}{5} \frac{d}{dx} x^6$$

$$= \frac{1}{5} (6x^5)$$

$$= \frac{6x^5}{5}$$

$$\frac{d}{dx} (6x^3 - x)(10 - 20x)$$

$$= -480x^3 + 180x^2 + 40x - 10$$

$$\frac{d}{dx} \left(\frac{1}{x} \right) = \frac{d}{dx} x^{-1}$$

$$= -\frac{1}{x^2}$$

ROUND 3

1. calculate function @ endpoints:

$$\underline{-2(0)^2 + 16(0)} = 0, \text{ and } \underline{16(12 - (12))^2} = 0$$

and check for equality of limits (or values, as these are polynomials) at "breaks"

2. it levels off - taking derivatives at $t = \frac{3}{4}, t = \frac{4}{5}, t = \frac{5}{6}$, etc.
we find that these derivatives are decreasing and, at $t=1$,
the derivative is 0

3. $t = \frac{1}{3}$ gives $\underline{-4(\frac{1}{3}) + 16} = \frac{44}{3}$ ($\cdot 1000$ feet/hour)

$t = 3$ gives $\underline{0}$ ($\cdot 1000$ ft/hour)

$t = 11$ gives $\underline{16(2(11) - 24)} = -32$ ($\cdot 1000$ ft/hour)

4. 5 mins is $t = \frac{144}{12} - \frac{1}{12} = \frac{143}{12}$, gives $\underline{-16(\frac{1}{6})} = \frac{8}{3}$ ($\cdot 1000$ ft/hr)

1 min is $t = \frac{720}{60} - \frac{1}{60} = \frac{719}{60}$, gives $\underline{-16(\frac{1}{30})} = \frac{8}{15}$ ($\cdot 1000$ ft/hr)