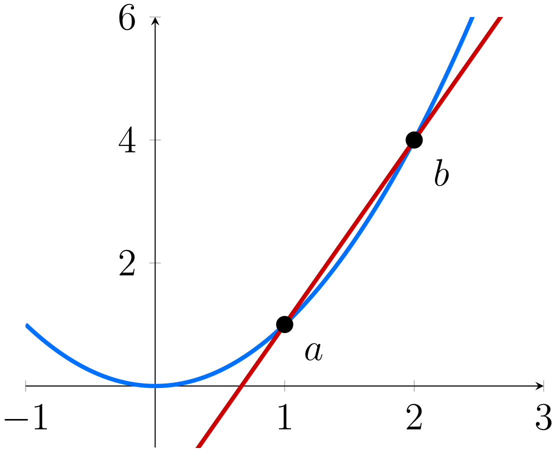
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

secant lines, tangent lines, and etymology

secant (n.) 1590s, from Latin *secantem* "a cutting," present participle of *secare* "to cut"; from PIE root ***sek**, "to cut."

related words: dissect, hacksaw, section, sector, scythe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-1
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3

$$m = \frac{f(b) - f(a)}{b - a}$$

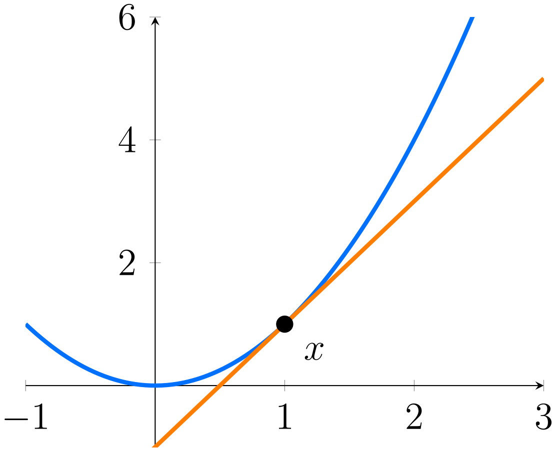
$$m = \frac{\overbrace{f(b) - f(a)}^{\text{rise}}}{\underbrace{b - a}_{\text{run}}}$$

secant lines tell us about *averages.*

... *but what if we want exactness?*

tangent (adj.) 1590s, “meeting at a point without intersecting,” from Latin *tangere* “to touch,” from PIE root ***tag-**, “to touch, handle.”

related words: tactile, tangible, (to) taste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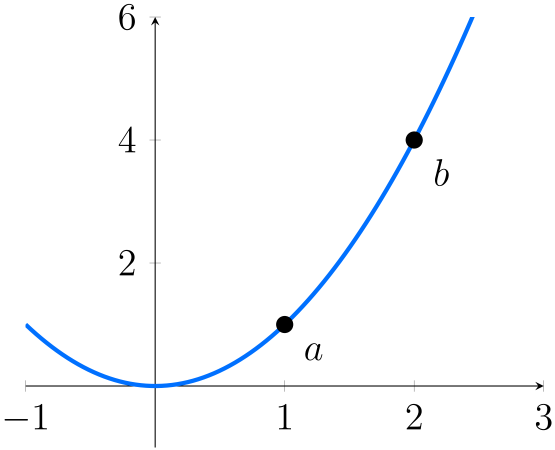
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$$x = a,$$

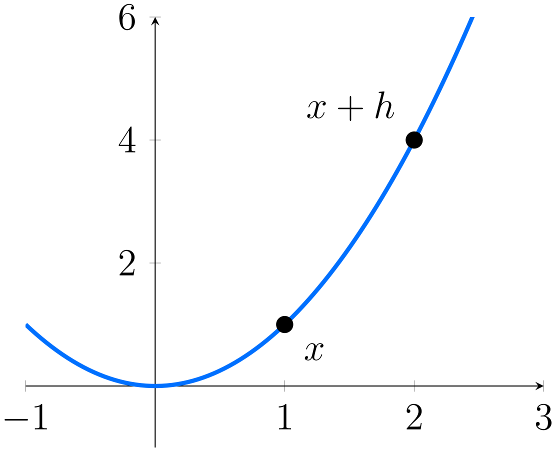
$$x = a, \; b = x + \overbrace{h}^{\text{"horizontal" distance from } x}$$

$$x = a,\ b = x + \overbrace{h}^{\text{“horizontal” distance from } x}$$

$$m = \frac{f(b) - f(a)}{b - a}$$

$$x = a,\ b = x + \overbrace{h}^{\text{“horizontal” distance from } x}$$

$$m = \frac{f(b) - f(a)}{b - a} = \frac{f(x+h) - f(x)}{(x+h) - x}$$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$x + h$
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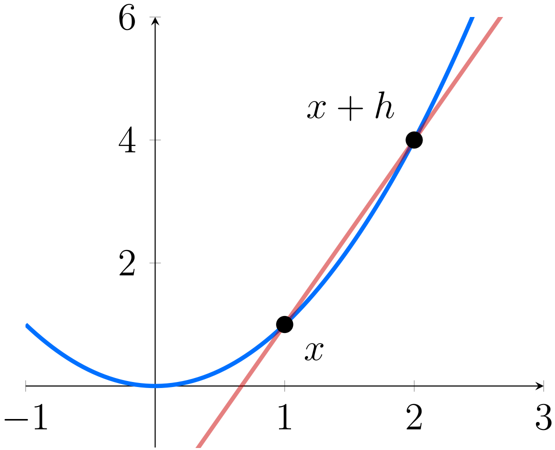
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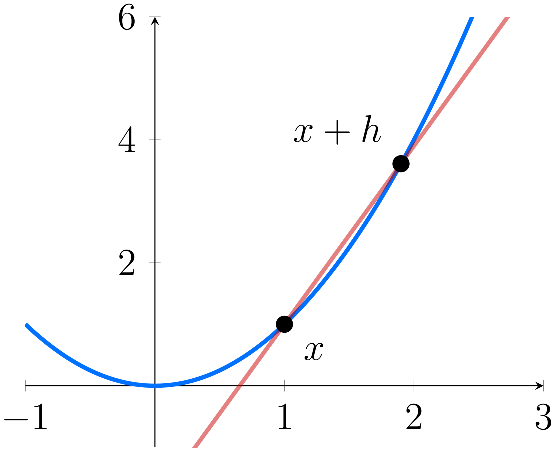
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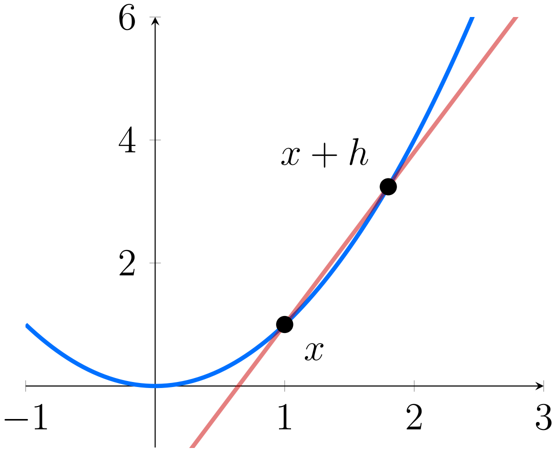
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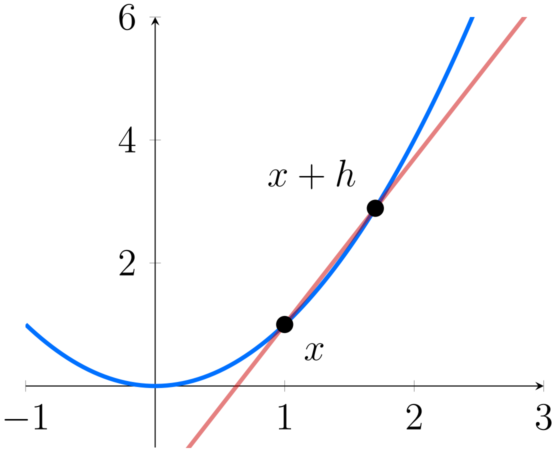
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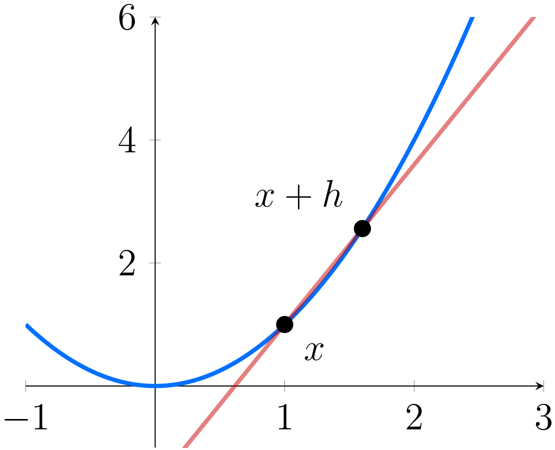
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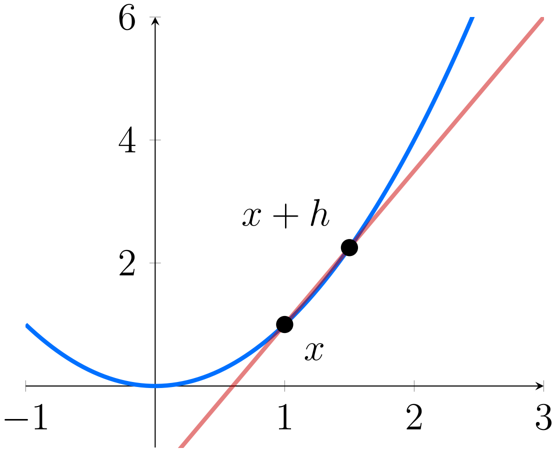
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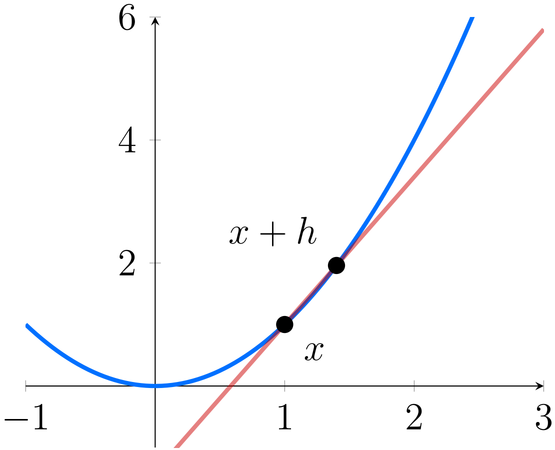
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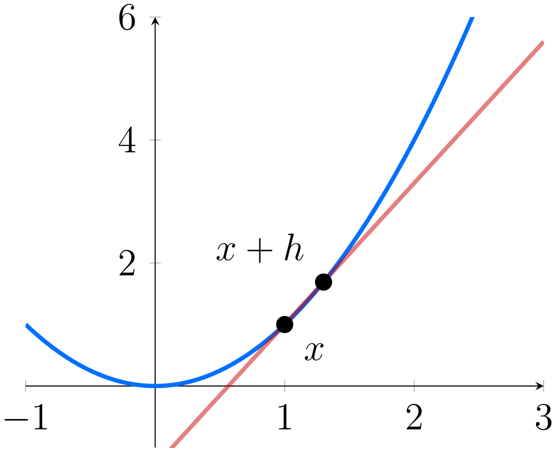
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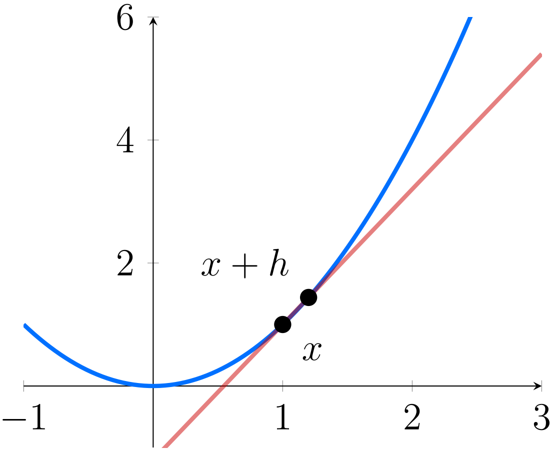

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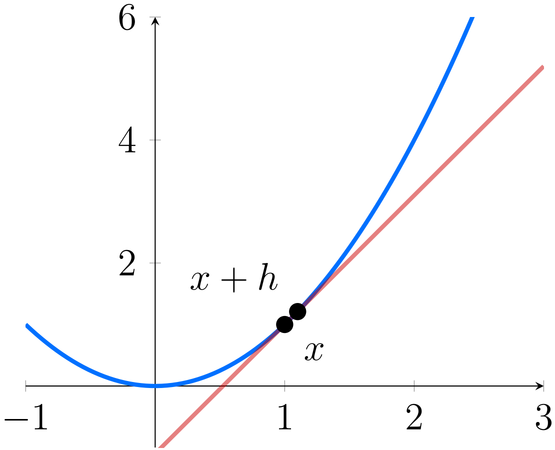
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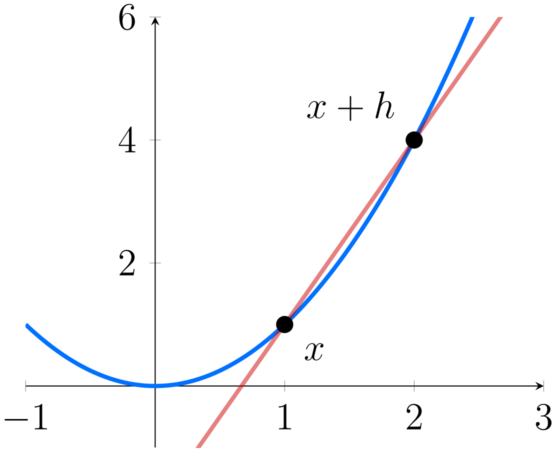
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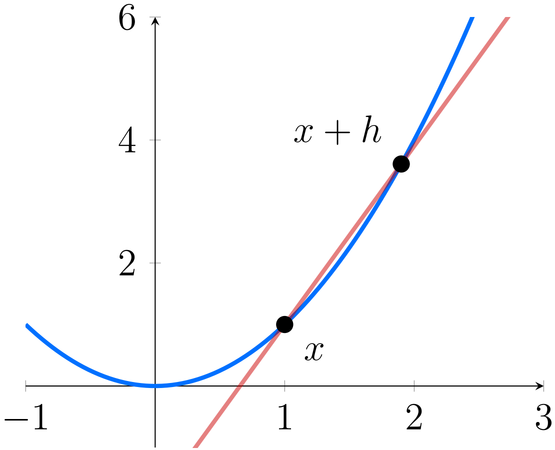
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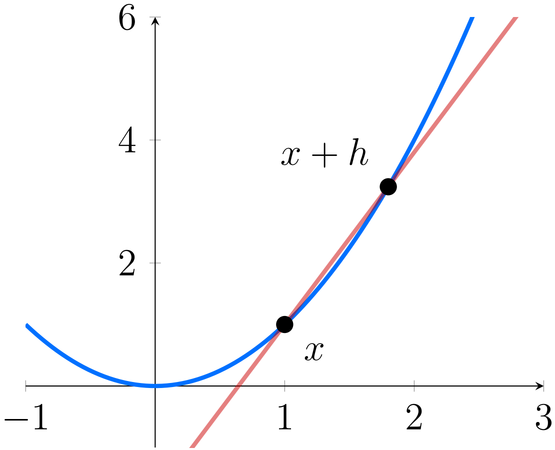
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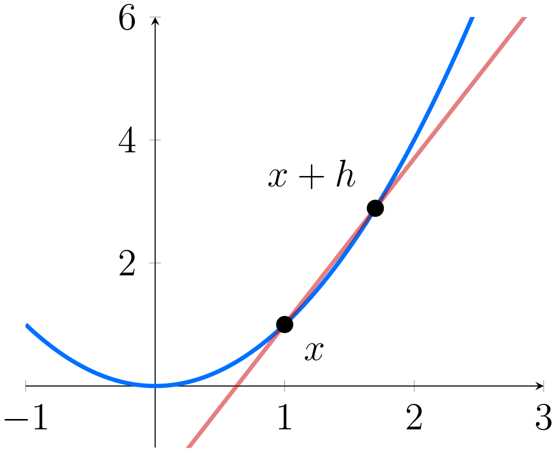
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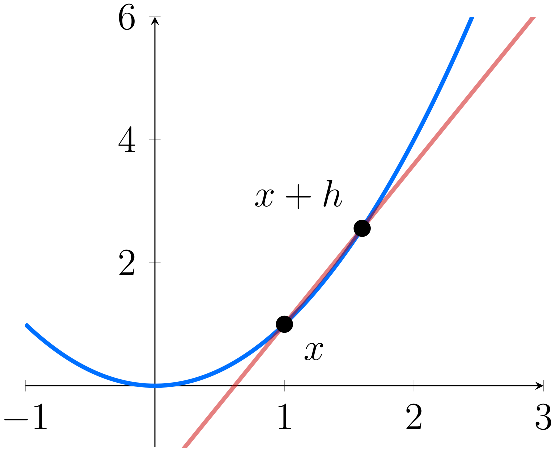
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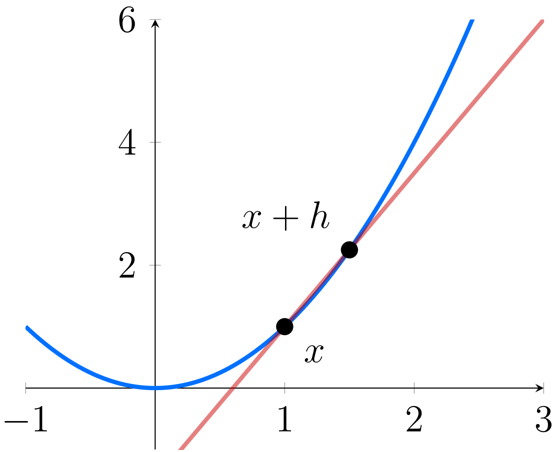
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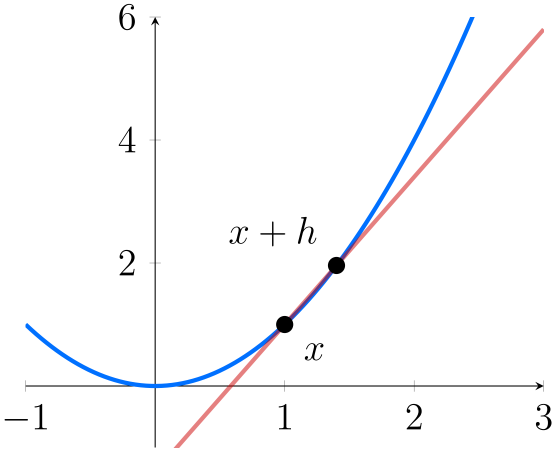
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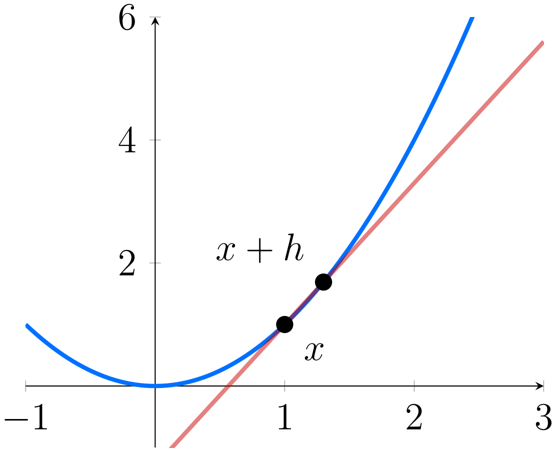

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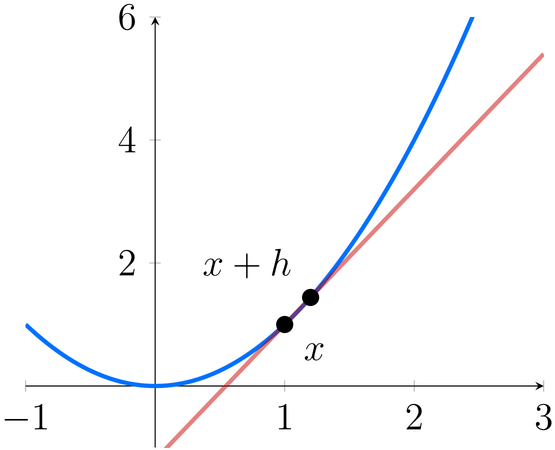
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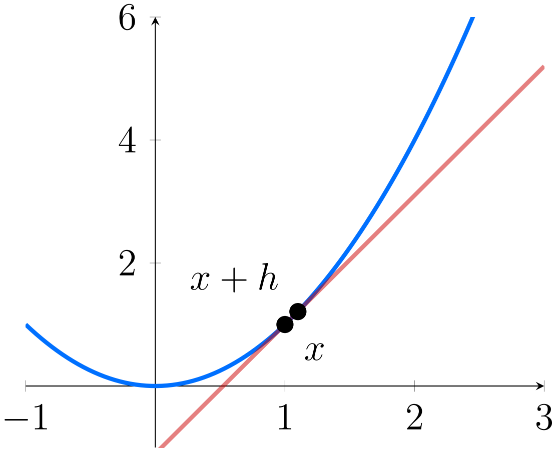
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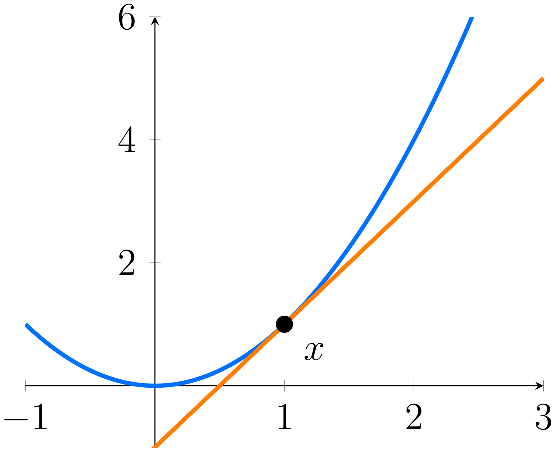

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slopes of tangent lines tell us about *exact local behavior*.

we can generalize this “slope of the tangent line” idea to work with lots of functions:

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we call it a *derivative*.

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$