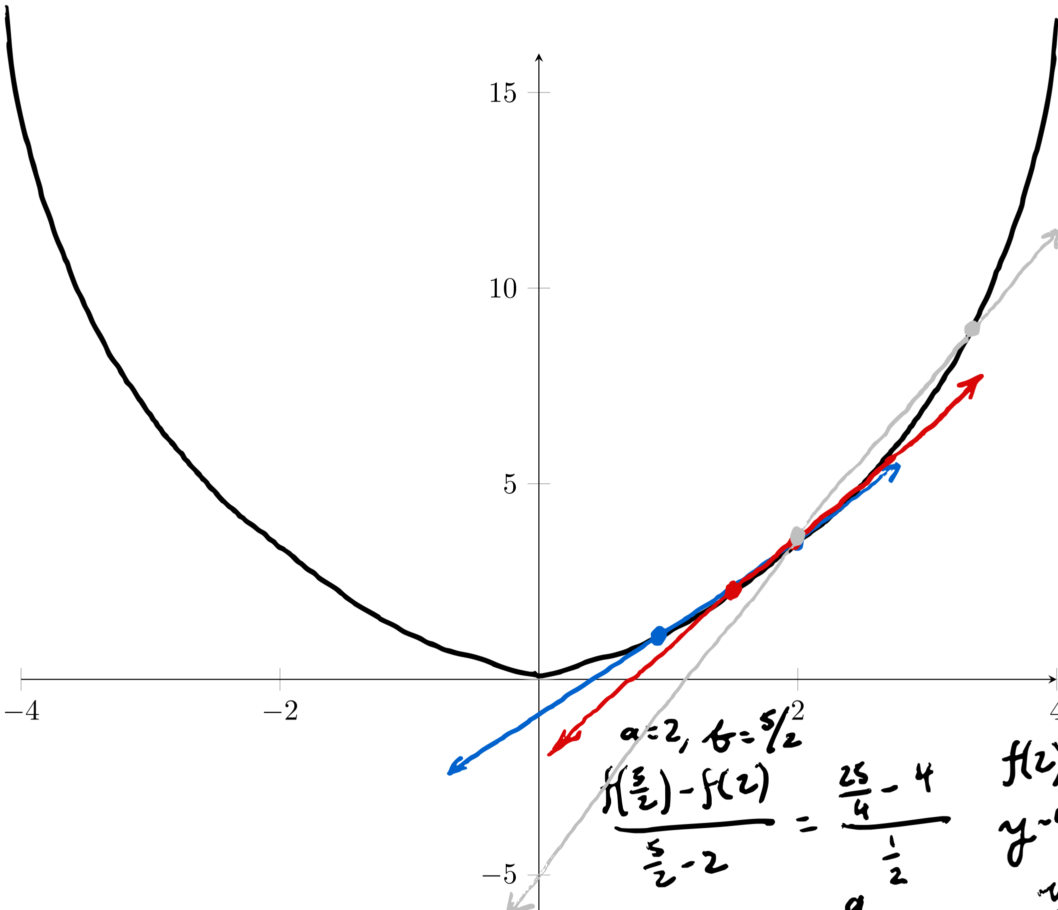
Week 5 Recitation Problems

MATH:113, Recitations 304 and 305

Names: SOLUTIONS



$a=2,\ b=5/2$

$$\frac{f(\frac{5}{2})-f(2)}{\frac{5}{2}-2} = \frac{\frac{25}{4}-4}{\frac{1}{2}} = \frac{9}{2}$$

$f(2)=4,$

$y-4=\frac{9}{2}(x-2)$

$y=\frac{9}{2}x-5$

1. Plot the function $f(x)=x^2$. Next, find <u>and plot</u> the secant lines for $a=2$ and: $b=1$, $b=3/2$, and $b=5/2$.

$a=2,\ b=1$

$$\frac{f(1)-f(2)}{1-2} = \frac{(1)^2-(2)^2}{1-2} = \frac{-3}{-1} = 3$$

$f(2)=4,$

$y-4=3(x-2)$

$y=3x-2$

$a=1,\ b=\frac{3}{2}$

$$\frac{f(\frac{3}{2})-f(2)}{\frac{3}{2}-2} = \frac{\frac{9}{4}-4}{-\frac{1}{2}} = \frac{7}{2}$$

$f(2)=4,$

$y-4=\frac{7}{2}(x-2)$

$y=\frac{7}{2}x-3$

2. Which line best approximates the slope of $f(x)$ at $x=2$?

both $y=\frac{9}{2}x-5$ and $y=\frac{7}{2}x-3$ are both good candidates – the first is too steep, and the second not quite steep enough.

3. Write down the limit definition for the derivative of a function $f(x)$: explain what each variable represents.

Definition 1: derivative of a function.

$$f'(x) = \lim_{\underbrace{h \to 0}_{\text{distance between anchor and test points is getting infinitely smaller!}}} \frac{f(x+h) - f(x)}{h}$$

} vertical distance between inputs

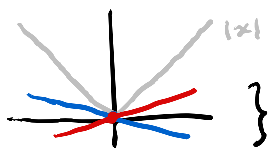
} distance from "anchor point" x and "test point" $x+h$

4. Why does the above expression give us the *exact* slope?

As h shrinks, the distance between x and $x+h$ gets smaller; eventually, this distance is indistinguishable from 0, so the average slope between the anchor point x and $x+h$ is <u>the</u> slope at x.

5. Are there any scenarios where a function *doesn't* have a derivative? If so, give an example.

yes! consider the absolute value function $|x|$ at $x=0$:



checking the left- and right hand limits (i.e. $h \to 0^-$, $h \to 0^+$),

} there are multiple tangent lines to pick from!

6. Find the derivatives of the functions below. After finding each derivative, find the value of the derivative at the input value $x = 4$.

$$f(x) = \sqrt{5x-8}$$

$$\lim_{h \to 0} \frac{\sqrt{5(x+h)-8} - \sqrt{5x-8}}{h}$$ } rationalize the numerator!

$$h(x) = \frac{1}{x}$$

$$\lim_{h \to 0} \frac{\left(\frac{1}{x+h}\right) - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\frac{-h}{x^2+hx}}{h}$$

$$= \lim_{h \to 0} \frac{-1}{x^2+hx} = -\frac{1}{x^2}$$

$$g(x) = 7x^2 + 5x$$

$$\lim_{h \to 0} \frac{(7(x+h)^2 + 5(x+h)) - (7x^2+5x)}{h}$$

$$= \lim_{h \to 0} \frac{7x^2 + 14xh + 7h^2 + 5x + 5h - 7x^2 - 5x}{h}$$

$$= \lim_{h \to 0} \frac{14xh + 7h^2 + 5h}{h}$$

$$= \lim_{h \to 0} 14x + 7h + 5$$

$$= 14x + 5$$

$$L(x) = \frac{x}{x+1}$$

$$\lim_{h \to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} = \lim_{h \to 0} \frac{\frac{(x+1)(x+h) - (x+h+1)(x)}{(x+h+1)(x+1)}}{h}$$

$$= \lim_{h \to 0} \frac{1}{x^2+hx+2x+h+1} \cdot \frac{1}{h}$$

$$= \frac{1}{(x+1)^2}$$