Week 3 Recitation Problems

MATH:113, Recitations 304 and 305

Limit Problems

1. $\lim_{x\to -3} \frac{6+4t}{t^2+1}$
2. is the function $g(z) = \frac{6}{z^2-3z-10}$ continuous at
   (a) $z = -2$?
   (b) $z = 0$?
   (c) $z = 5$?
3. for the function
$$f(x) = \begin{cases} 7-4x & x < 1 \\ x^2+2 & x \geq 1 \end{cases}$$
   evaluate the limits
   (a) $\lim_{x\to -6} f(x)$
   (b) $\lim_{x\to 1} f(x)$
   (c) *(challenge!)* $\lim_{x\to\sqrt{3}} f(f(x))$

. . . . . .

What errors can you find?

$$\begin{aligned}
\lim_{t\to\pi} \frac{1-\cos^2(t)}{\sin(-t)} &= \lim_{t\to\pi} \frac{-\sin^2(t)}{\sin(-t)} \\
&= \lim_{t\to\pi} \frac{-\sin^2(t)}{-\sin(t)} \\
&= \frac{\lim_{t\to\pi} -\sin^2(t)}{\lim_{t\to\pi} -\sin(t)} \\
&= \frac{0}{0} \\
&= \text{DNE}.
\end{aligned}$$

Solutions

1. Plug in values! Does $\lim_{t\to -3} t^2-1$ exist? (yes!) so,

$$\lim_{t\to -3} \frac{6+4t}{t^2+1} = \frac{\lim_{t\to -3} 6+4t}{\lim_{t\to -3} t^2+1}$$

$$= \frac{6+4(-3)}{(-3)^2+1}$$

$$= -\frac{3}{5}.$$

2. Factor the denominator!

$$z^2-3z-10 = (z-5)(z+2)$$

and our numerator is constant, so

(a) no, (b) yes, (c) no

3.

$$\lim_{x\to -6} f(x) = \lim_{x\to -6} 7-4x$$
$$= 7-4(-6)$$
$$= 7+24$$
$$= 31$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} x^2+2$$
$$= (1)^2+2$$
$$= 3$$

$$\lim_{x\to \sqrt{3}} f(f(x)) = f\left(\lim_{x\to\sqrt{3}} f(x)\right)$$
$$= f(\sqrt{3}^2+2)$$
$$= f(5)$$
$$= 27$$

4. Errors are:

① $1-\cos^2(t) \neq -\sin^2(t)$

② $\lim_{t\to\pi} -\sin(t)$ doesn't exist, so we can't write it as a fraction

③ answer's wrong!

$$\lim_{t\to\pi} \frac{1-\cos^2(t)}{\sin(t)} = \lim_{t\to\infty} \frac{\sin^2(t)}{-\sin(t)}$$
$$= \text{"} \quad \text{"} \quad -\sin(t)$$
$$= 0$$