Week 3 Recitation Problems

MATH:113, Recitations 304 and 305

Continuity

1. Is
$$f(t) = \begin{cases} \cos(t) + 1 & t \leq 0 \\ 2 - 3t & t > 0 \end{cases}$$
continuous at $t = 0$? How do you know?

2. Is
$$g(x) = \begin{cases} e^x & x < 0 \\ 9x^2 + x + 1 & x \geq 0 \end{cases}$$
continuous? Where?

3. Suppose we draw a ray from the origin of the plane until it hits the unit circle at the point $P = (p_x, p_y)$. Let t be the counterclockwise angle from the x-axis to the ray. Now, draw a line L parallel to the y-axis that passes through P and intersects the x-axis at the point $Q = (q_x, 0)$. Finally, define functions $A(t)$, $O(t)$, and $R(t)$ on the unit circle where
   - $A(t)$ is the length of the line segment from the origin to the point $(q_x, 0)$,
   - $O(t)$ is the length of the line segment from the point $(q_x, 0)$ to the point P, and
   - $R(t)$ is the ratio of the lengths of the line segments.

   Are $A(t)$ and $O(t)$ continuous? Is $R(t)$ continuous?