Week 3 Recitation Problems

MATH:113, Recitations 304 and 305

Continuity

1. Is
$$f(t) = \begin{cases} \cos(t) + 1 & t \leq 0 \\ 2 - 3t & t > 0 \end{cases}$$
continuous at $t = 0$? How do you know?

2. Is
$$g(x) = \begin{cases} e^x & x < 0 \\ 9x^2 + x + 1 & x \geq 0 \end{cases}$$
continuous? Where?

3. Suppose we draw a ray from the origin of the plane until it hits the unit circle at the point $P = (p_x, p_y)$. Let t be the counterclockwise angle from the x-axis to the ray. Now, draw a line L parallel to the y-axis that passes through P and intersects the x-axis at the point $Q = (q_x, 0)$. Finally, define functions $A(t)$, $O(t)$, and $R(t)$ on the unit circle where
   - $A(t)$ is the length of the line segment from the origin to the point $(q_x, 0)$,
   - $O(t)$ is the length of the line segment from the point $(q_x, 0)$ to the point P, and
   - $R(t)$ is the ratio of the lengths of the line segments.

   Are $A(t)$ and $O(t)$ continuous? Is $R(t)$ continuous?

Solutions

1. We can check the left- and right limits to ensure they're the same:

$$\lim_{t \to 0^-} f(t) = \lim_{t \to 0^-} \cos(t)+1$$
$$= 2,$$

$$\lim_{t \to 0^+} f(t) = \lim_{t \to 0^+} 2-3t$$
$$= 2$$

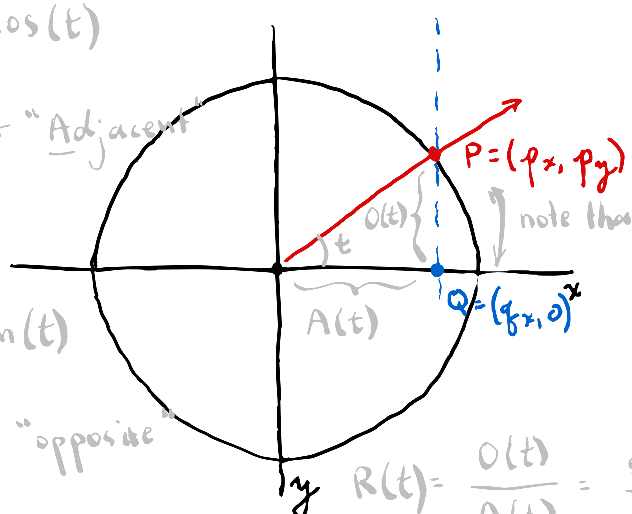
and that $f(0)=2$ (which it does). Thus, it's cont's.

2. Same as #1, but limit is 1 instead.

3. These are just the trig functions in disguise!

$A(t) = \cos(t)$

"A" for "Adjacent"



$O(t) = \sin(t)$

"O" for "opposite"

$R(t) = \dfrac{O(t)}{A(t)} = \dfrac{\sin(t)}{\cos(t)} = \tan(t)$

"R" for "Ratio"

By drawing different rays, we can experiment and determine that A(t) and O(t) can take on all values in $[-1,1]$ and are thus cont's. But we should recognize that, as $t \to \frac{\pi}{2}$, or $t \to \frac{3\pi}{2}$, the denominator of $R(t)$ goes to 0 while the numerator doesn't. In other words,

$$\lim_{t \to \frac{\pi}{2}} \frac{O(t)}{A(t)} = \text{"} \frac{\sin(t)}{\cos(t)} \qquad \text{(and same for } \tfrac{3\pi}{2}.\text{)}$$

$$= \frac{\sin(\frac{\pi}{2})}{0}$$

$$\overset{!}{=} \text{ONE} \qquad \text{so } R(t) \text{ is } \underline{\text{not}} \text{ cont's.}$$