limits

sequences

a *sequence* is a list of mathematical objects.

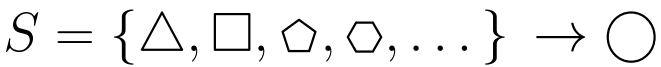
$$S = \{\triangle, \square, \pentagon, \hexagon, \dots\}$$

definition 1:

definition 1:

the *limit of a sequence* S is the value that S's terms approach.

$$S = \{\triangle, \square, \pentagon, \hexagon, \dots\}$$



in our class, a *sequence* is an ordered list of *real numbers*.

symbolically, we'll write sequences as

$$S = \{a_1, a_2, \ldots, a_n, \ldots\}$$

where a_n is the n^{th} term in the sequence.

we can also write sequences using shorthand:

$$S = \{a_n\}_{n=1}^{\infty}$$

where a_n is the n^{th} term in the sequence.

$$S = \{3, 3.1, 3.14, 3.141, 3.1415, \ldots$$

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what's the n^{th} term of S?

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what's the n^{th} term of S?

what's the limit of S?

definition 2:

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the *limit of a sequence* S is the value that the n^{th} term approaches as n gets really, really big.

... can we *more precisely* define a limit?

definition 3:

definition 3:

a sequence *has a limit at* L if the entries a_n get *arbitrarily close to* L as $n \to \infty$.

if L is the limit of a sequence as $n \to \infty$, we write

$$\lim_{n \to \infty} a_n = L.$$

take **five minutes** to complete **problem 1** on your worksheets.

functions

a *function* is a relationship between a set of *inputs* and a set of *outputs*.

functions are a special kind of relationship:
each possible input has *exactly* one output.

definitions:

the *domain* of a function is the set of all inputs which have an output. the *range* (or *image*) of a function is the set of all possible outputs.

$$f(x) = \sqrt{x}$$

take **ten minutes** to complete **problem 2** on your worksheets.

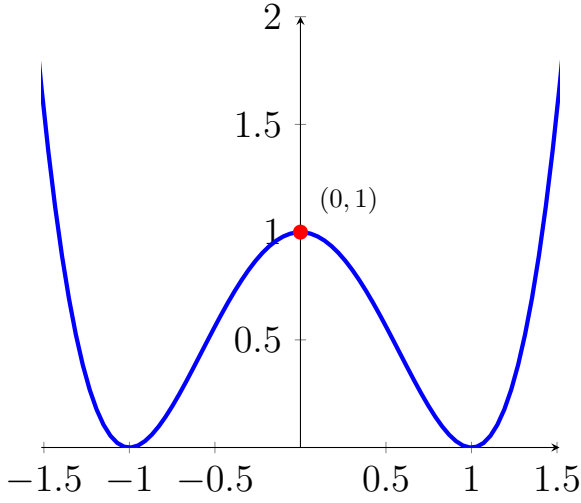
functions can have limits, too.

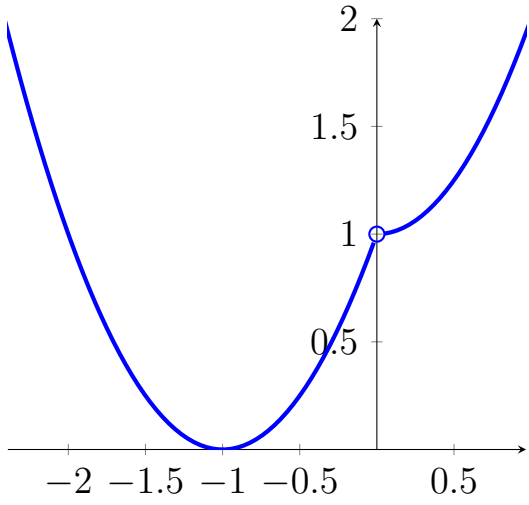
definition:

a function *has a limit* L *as* x *approaches the* p if $f(x)$ gets really close to L as we make x really close to p.

if L is the limit of a function as $x \to p$, we write

$$\lim_{x \to p} f(x) = L.$$


2
1.5
(0, 1)
1
0.5
−1.5
−1
−0.5
0.5
1
1.5


2
1.5
1
0.5
−2
−1.5
−1
−0.5
0.5

take **the rest of class** to complete **problem 3** on your worksheets.