Week 2 Recitations

MATH:113, Recitations 304 and 305

Names: solutions!

1. What are the limits of these *sequences*? Discuss, and plot at least one sequence on a whiteboard. *(Hint: the limit of Q is a special number that shows up often!)*

$$P = \{1, 1/2, 1/3, 1/4, \ldots\} \qquad Q = \left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty} \qquad R = \{\sin(n)\}_{n=0}^{\infty}$$

as n gets large, $1/n$ gets really small. eventually, $1/n$ gets so small that we can't distinguish it from 0.

this one's tricky: it's Euler's constant e, but it's hard to see. using Desmos here helps! by the end of this week, we'll be able to prove that $\lim Q = e$.

sin is a function with period 2π; that is, its values repeat at every multiple of 2π. (for example, $\sin(0) = \sin(2\pi) = \sin(4\pi) = \cdots$) since its values repeat forever, this sequence has no limit.

BONUS: what's the limit of $R = \{\sin(1/n)\}_{n=0}^{\infty}$?

2. Find the domain and range of three of the following functions. On a whiteboard, sketch the curves for two of your chosen functions.

a $f(x) = 1/x$ b $g(x) = \ln(x)$ c $h(x) = x^2$ d $p(x) = x^2 + 1$ e $q(x) = x^3$

a
domain: $\underbrace{\mathbb{R} - \{0\}}_{\text{reals minus the point } x=0}$
range: $\mathbb{R}$

b
domain: $\mathbb{R}^+$ (all positive reals)
range: $\mathbb{R}^+$

c
domain: $\mathbb{R}$
range: $\mathbb{R}^{\geq 0}$ (all nonnegative reals

d
domain: $\mathbb{R}$
range: $\{x \in \mathbb{R} : x \geq 1\}$

e
domain: $\mathbb{R}$
range: $\mathbb{R}$

Note that

$$\lim_{x\to p^+} f(x)$$

reads "the limit of $f(x)$ as x approaches p *from above*" — this means that x is always bigger than p, but is getting smaller as it gets closer to p. Similarly,

$$\lim_{x\to p^-} f(x)$$

reads "the limit of $f(x)$ as x approaches p *from below*" — this means that x is always smaller than p, but is getting bigger as it gets closer to p.

3. Find these limits, and sketch a curve on a whiteboard for each limit you find.

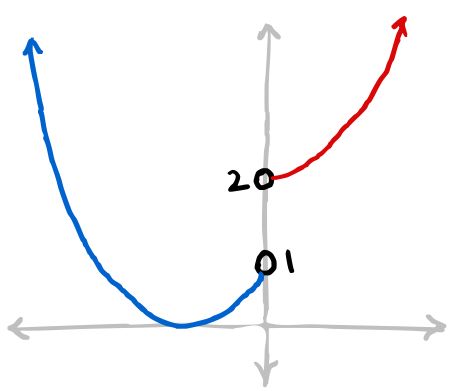
$$\lim_{x\to\infty} 1/x \qquad\qquad \lim_{x\to 0} 1/x$$

$$\lim_{x\to\infty} 1/x = \frac{1}{\lim_{x\to\infty} x} \text{ "}= \frac{1}{\infty}\text{" } = 0$$

$$\lim_{x\to 0} 1/x = \frac{1}{\lim_{x\to 0} x} \stackrel{?}{=} \frac{1}{0} = \text{DNE!}$$

$$f(x) = \begin{cases} (x+1)^2 & x < 0 \\ x^2 + 2 & x > 0 \end{cases}$$

$$\lim_{x\to 0^-} f(x) \qquad \lim_{x\to 0^+} f(x) \qquad \lim_{x\to 0} f(x)$$



... but because $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$, we say that f does not have a limit at 0.

$\lim_{x\to 0^-} f(x) = 1$, as we're approaching 0 from below and $(x+1)^2$ for $x=0$ is $(0+1)^2 = (1)^2 = 1$.

$\lim_{x\to 0^+} f(x) = 2$, as we're approaching 0 from above and x^2+2 for $x=0$ is $(0)^2+2 = 0+2 = 2$.