Week 9 Recitation Problems

MATH:114, Recitations 309 and 310

Names: ____

An **improper integral** is the definite integral of a function where **one or both of the limits of integration approach infinity** or **the function is discontinuous somewhere on the interval of integration.**

1. Using what you've covered in lecture, fill out the table below. How are you thinking about these answers?

$p =$	Integral	Finite or Infinite?
1	$\int_1^\infty \frac{1}{x}\,dx$	
$1/2$	$\int_1^\infty \frac{1}{\sqrt{x}}\,dx$	
2	$\int_1^\infty \frac{1}{x^2}\,dx$	

2. If $p \neq 1$, compute the improper integral

$$\int_1^\infty \frac{1}{x^p}\,dx.$$

What are the conditions on p that determine the convergence of the integral? Why do they make sense?

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Let's find out if $\int_3^\infty \ln(x)/\sqrt{x}\,dx$ is convergent.

3. Draw and compare the graphs of $f(x) = \ln(x)$ and $g(x) = 1$. When $x \geq 3$, which of the functions is greater than the other?

4. Using the result from Problem 4, what can you say about the functions $\ln(x)/\sqrt{x}$ and $1/\sqrt{x}$ when $x \geq 3$? If we integrate them as $\int_3^\infty \ln(x)/\sqrt{x}\, dx$ and $\int_3^\infty 1/\sqrt{x}\, dx$, which integral should be bigger?

5. Compute the integral $\int_3^\infty 1/\sqrt{x}\, dx$. Based on your result, what can you say about $\int_3^\infty \ln(x)/\sqrt{x}\, dx$?

6. Suppose we have two functions $f(x)$ and $g(x)$, and let $f(x) \geq g(x) \geq 0$ where $x \geq a$. If...

$$\begin{aligned}
\textstyle\int_a^\infty f(x)\, dx \text{ diverges} &\implies \textstyle\int_a^\infty g(x)\, dx \text{ ____________} \\
\textstyle\int_a^\infty f(x)\, dx \text{ converges} &\implies \textstyle\int_a^\infty g(x)\, dx \text{ ____________} \\
\textstyle\int_a^\infty g(x)\, dx \text{ diverges} &\implies \textstyle\int_a^\infty f(x)\, dx \text{ ____________} \\
\textstyle\int_a^\infty g(x)\, dx \text{ converges} &\implies \textstyle\int_a^\infty f(x)\, dx \text{ ____________}
\end{aligned}$$

Note: the symbolic phrase $a \implies b$ means "if a, then b."

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7. Think about the integral

$$\int_2^\infty \frac{\cos^2(t)}{t^2}\, dt.$$

Do you think that this integral converges or diverges? Why?

8. What is a good function to compare the above integrand to? Write an inequality to justify your answer.