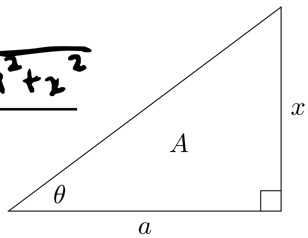
Week 7 Recitation Problems

MATH:114, Recitations 309 and 310

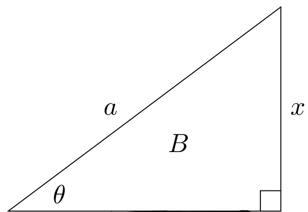
1. Determine the lengths of the missing sides in triangles A, B, and C. You don't need any numbers, just variables!

$c=\sqrt{a^2+x^2}$



$a^2+x^2=c^2$

$\Rightarrow c=\sqrt{a^2+x^2}$

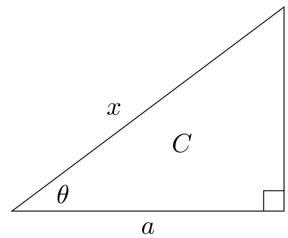


$c=\sqrt{a^2-x^2}$

$c^2+x^2=a^2$

$\Rightarrow c^2=a^2-x^2$

$\Rightarrow c=\sqrt{a^2-x^2}$



$c=\sqrt{x^2-a^2}$

$x^2=a^2+c^2$

$\Rightarrow c^2=x^2-a^2$

$\Rightarrow c=\sqrt{x^2-a^2}$

2. Trigonometric functions define relationships between angles and side lengths. Given an angle θ,

$$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}\qquad\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}\qquad\tan\theta=\frac{\text{opposite}}{\text{adjacent}}$$

For each of the triangles A, B, and C, express x in terms of a trigonometric function.

A

$\tan\theta=\frac{\text{opp}}{\text{adj}}$

$=\frac{x}{a},$

$a\tan\theta=x$

B

$\sin\theta=\frac{\text{opp}}{\text{hyp}}$

$=\frac{x}{a},$

$a\sin\theta=x$

C

$\cos\theta=\frac{\text{adj}}{\text{hyp}}$

$=\frac{a}{x},$

$\frac{1}{\cos\theta}=\frac{x}{a}=\sec\theta,$

$a\sec\theta=x$

3. For each of the triangles A, B, and C, express the length of the missing side using the answers you found in Problem 2. *(Hint: remember your trig identities!)*

A

$\sqrt{a^2+(a\tan\theta)^2}$

$=\sqrt{a^2+a^2\tan^2\theta}$

$=\sqrt{(a^2)(1+\tan^2\theta)}$

$=a\sqrt{1+\tan^2\theta}$

$=a\sqrt{\sec^2\theta}$

$=a|\sec\theta|$

B

$\sqrt{a^2-(a\sin\theta)^2}$

$=\sqrt{a^2-a^2\sin^2\theta}$

$=\sqrt{(a^2)(1-\sin^2\theta)}$

$=a\sqrt{1-\sin^2\theta}$

$=a\sqrt{\cos^2\theta}$

$=a|\cos\theta|$

C

$\sqrt{(a\sec\theta)^2-a^2}$

$=\sqrt{a^2\sec^2\theta-a^2}$

$=\sqrt{(a^2)(\sec^2\theta-1)}$

$=a\sqrt{\sec^2\theta-1}$

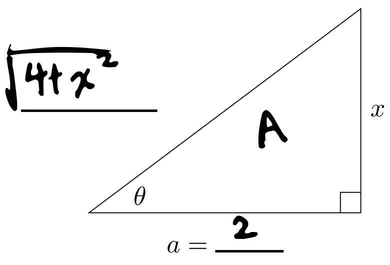
$=a\sqrt{\tan^2\theta}$

$=a|\tan\theta|$

4. Use one of the expressions you found in Problem 3 to set up **but not solve** the integral

$$\int \frac{1}{\sqrt{4+x^2}}dx.$$

You can use the triangle below for reference.



$\tan\theta = \frac{x}{2}$

$\Rightarrow 2\tan\theta = x,$

$\sqrt{4+(2\tan\theta)^2}$

$= \sqrt{4+4\tan^2\theta}$

$= 2\sqrt{1+\tan^2\theta}$

$= 2\sqrt{\sec^2\theta}$

$= 2|\sec\theta|$

$x = 2\tan\theta$

x is a function of θ! so...

$\frac{dx}{d\theta} = 2\sec^2\theta$

$\Rightarrow dx = 2\sec^2\theta \, d\theta$

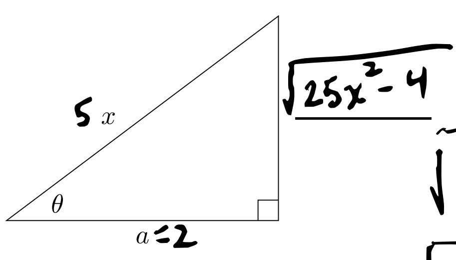
$\Rightarrow \int \frac{1}{\sqrt{4+x^2}}dx = \boxed{\int \frac{2\sec^2\theta}{2\tan\theta} d\theta}$

5. Solve

$$\int \frac{1}{\sqrt{25x^2-4}}dx$$

using the fact that

$$\int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C.$$



$\sec\theta = \frac{5x}{2}$

$\Rightarrow \frac{2}{5}\sec\theta = x$

$\sqrt{25(\frac{2}{5}\sec\theta)^2 - 4}$

$= \sqrt{4\sec^2\theta - 4}$

$= 2|\tan\theta|$

$x = \frac{2}{5}\sec\theta$

x is a function of θ!

$\frac{dx}{d\theta} = \frac{2}{5}\sec\theta\tan\theta$

$\Rightarrow dx = \frac{2}{5}\sec\theta\tan\theta \, d\theta$

$\Rightarrow \int \frac{1}{\sqrt{25x^2-4}}dx = \int \frac{\frac{2}{5}\sec\theta\tan\theta}{2|\tan\theta|} d\theta$

$= \int \frac{1}{5}\sec\theta \, d\theta$

$\boxed{= \frac{1}{5}\ln|\sec\theta + \tan\theta| + C}$

plugging in values:

$\boxed{= \frac{1}{5}\ln\left|\frac{5x}{2} + \frac{\sqrt{25x^2-4}}{2}\right| + C}$