# Week 5 Recitation Problems

## MATH:114, Recitations 309 and 310

First, let's talk about **linear approximations.**

1. Let's approximate the function $f(x) = e^x$ around $x = 0$. Discuss a few ideas for this specific approximation.

2. Let $L(x) = a_0 + a_1 x$. What kind of function does $L(x)$ describe? Try to approximate $f(x)$ at $x = 0$ by adjusting $a_0$ and $a_1$, and draw a picture of your approximation.

3. Come up with a general-purpose formula for approximating an arbitrary function $g(x)$ near the point $x$. *(Hint: what does the Mean Value Theorem say?)*

Now, we can talk about **quadratic** and **higher-order** approximations. To do so, we're going to find a quadratic function $Q(x) = a_0 + a_1 x + a_2 x^2$ that approximates $f(x)$ near the point $x$. From the previous page, let

$$f(x) = e^x.$$

4. To construct $Q(x)$, we want the first *and* second derivatives to look a lot like the first and second derivatives of $f(x)$. In each of the following equations, set $f(x)$ (or its derivatives) equal to $Q(x)$ (or its derivatives) and solve for each coefficient.

First, match the values of the functions:

$$f(0) = Q(0) \implies a_0 = \_\_\_\_\_$$

Then, match the values of the first derivatives:

$$f'(0) = Q'(0) \implies a_1 = \_\_\_\_\_$$

Finally, match the values of the second derivatives:

$$f''(0) = Q''(0) \implies a_2 = \_\_\_\_\_$$

5. Using the coefficients you just found, write out $Q(x)$. Can you come up with a general-purpose formula for the quadratic approximation $Q(x)$ for an arbitrary function $g(x)$? What about a cubic approximation?

6. Find the cubic approximation $C(x)$ for $f(x) = e^x$. For each of the linear, quadratic, and cubic approximations, check its value against the true value of $f(x)$ at $x = 0$.