THE GREATEST THING YOU WILL DO ALL WEEK#11

MATH 114 - CALCULUS II - SPRING 2020

Professor/TA : _____ **Sec:** _____

FULL NAME: _____ **Partners:** _____

Approximating functions using polynomials.

(A) Let us first do LINEAR approximations. We want to approximate the function $g(x) = e^x$ about $x = 0$.

1) Deliberate with your group about how you might approximate the function $g(x) = e^x$ about $x = 0$. Explain your thinking.

(2) Use one of the discussed methods to approximate the function using a line $L(x) = a_0 + a_1 x$. Draw the linear approximation $L(x)$ on the graph.

(3) Find a formula for the tangent line of an arbitrary function $f(x)$ about at the point $x = 0$.

(B) Let us now tackle QUADRATIC approximations.

1) Let us find the quadratic $Q(x) = a_0 + a_1 x + a_2 x^2$ that approximates the function $g(x) = e^x$ around $x = 0$.

(i) Match function values:

(ii) Match tangents:

(iii) Match concavity:

2) Finally, $Q(x) =$ $\boxed{}$

Use these results to graph the quadratic Q(x) (make sure to match location, slope and concavity!).

3) Find a formula for the quadratic approximation of an arbitrary function $f(x)$ about the point $x = 0$.

(C) Complete the following table. Which method provides the best approximation? Why?

Functions	Value at $x = 1$	Error
$g(x) = e^x$	2.71828	
$L(x) =$		
$Q(x) =$		

(D) Find a cubic $[C(x) = a_0 + a_1x + a_2x^2 + a_3x^3]$ approximation for $g(x) = e^x$ at $x = 0$.

(E) Extra time: Discuss how you could generalize this to a polynomial of degree n $\left[P(x) = \sum_{i=0}^{n} a_i x_i\right]$ approximation at $x = 0$.