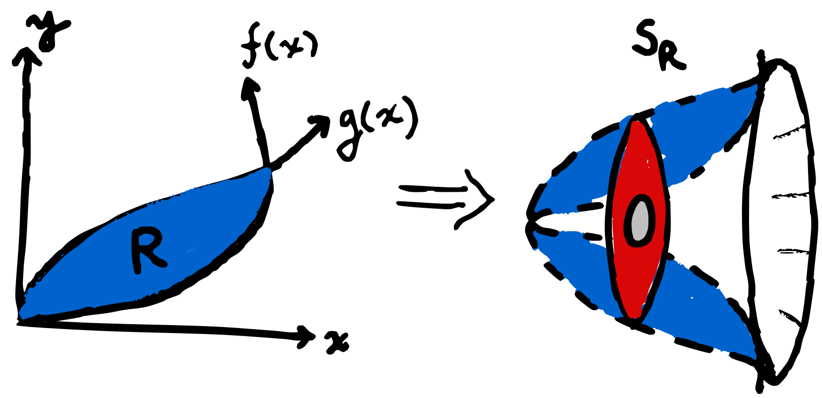
Week 5 Recitation Problems

MATH:114, Recitations 309 and 310

Volumes

1. Suppose the functions $f(x)$ and $g(x)$ bound a closed region R in the plane. Rotate R around the x axis to get a solid of rotation S_R. How does the **washer** method find the volume of S_R? Use words or pictures to explain, including relevant geometric formulas or ideas.

the washer method (or, equivalently, the disk method for two functions) finds the differences between (2d) areas of circles and sums those (2d) areas to get a (3d) volume.



difference of circle areas

$\pi R^2 - \pi r^2$

R = top func, r = bottom func

$\hookrightarrow \pi(g(x))^2 - \pi(f(x))^2$

2. Let $f(x) = x^2$ and $g(x) = x + 2$, and let R be the closed region bounded by $f(x)$ and $g(x)$. Find the volume of the solid generated by rotating R around the x axis.

$x^2 = x + 2$

$0 = x^2 - x - 2$

$= \underbrace{(x-2)}_{2}\underbrace{(x+1)}_{-1}$

intersections at $(2,4)$ and $(-1,1)$

$$V = \int_{-1}^{2} \pi \Big(\underbrace{g(x)}_{\text{outer}}\Big)^2 - \Big(\underbrace{f(x)}_{\text{inner}}\Big)^2 \, dx$$

$$= \pi \int_{-1}^{2} (x+2)^2 - (x^2)^2 \, dx$$

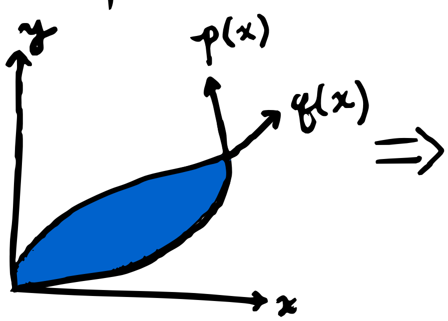
$$= \pi \int_{-1}^{2} x^2 + 4x + 4 - x^4 \, dx$$

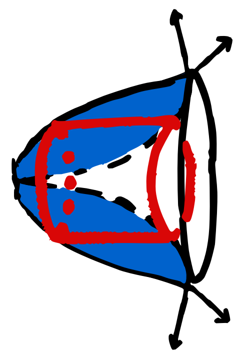
$$= \pi \left[\frac{1}{3}x^3 + 2x^2 + 4x - \frac{1}{5}x^5 \right]_{-1}^{2}$$

$$= \frac{72\pi}{5}$$

3. Let the functions $p(x)$ and $q(x)$ bound a closed region C in the plane. Rotate C around the x axis to get a solid of rotation S_C. How does the **shell** method find the volume of S_C? Use words or pictures to explain, including relevant geometric formulas.

the shell method uses cylinders parallel to the axis of rotation and intersects them with the solid; finding the (2d) surface area of all the cylinders and adding those areas up gives us (3d) volume.





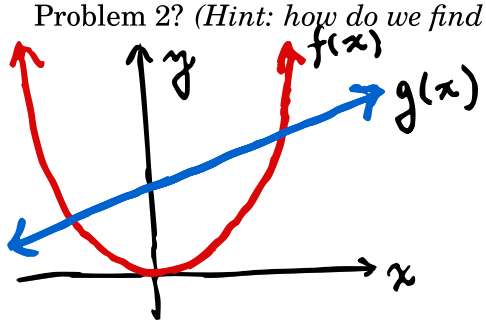
surface area of cylinder

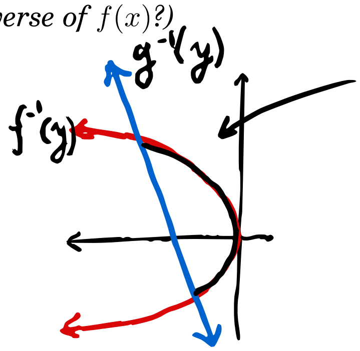
$2\pi r h$

radius is distance from x-axis, height is diff btwn inverse functions:

$$2\pi(y)\left(p^{-1}(y)-q^{-1}(y)\right)$$

4. Why might it be difficult to use the shell method with the functions $f(x)$ and $g(x)$ from Problem 2? *(Hint: how do we find the inverse of $f(x)$?)*



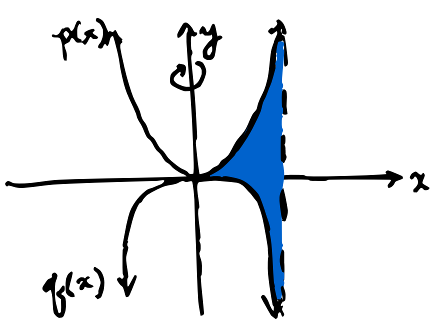


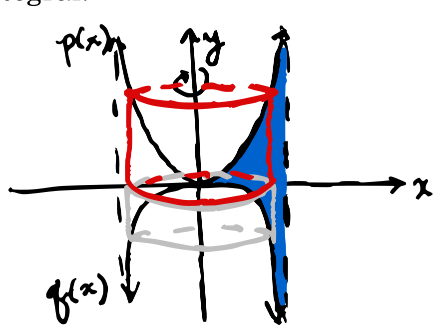
is this curve a function?

NO!

so we have to be careful with the shell method.

5. Let $p(x) = x^2$ and $q(x) = -x^4$. Set up an integral to find the volume of the solid found by rotating the region bounded by $p(x)$, $q(x)$, and the vertical line $x = 1$ around the y axis. If you have time, compute this integral!





using two cylinders:

$$V = \int_0^1 2\pi \cdot x \cdot p(x)\,dx$$

$$= 2\pi \int_0^1 x(x^2)\,dx$$

$$V = \int_0^1 2\pi \cdot x \cdot q(x)\,dx$$

$$= 2\pi \int_0^1 x(-x^4)\,dx$$

re-write as one cylinder:

$$V + V = 2\pi \int_0^1 -x^5\,dx + 2\pi \int_0^1 x^3\,dx$$

$$= 2\pi \left(\int_0^1 x^3 - x^5\,dx \right)$$

$$\left(= \frac{\pi}{6} \right)$$