Week 5 Recitation Problems

MATH:114, Recitations 309 and 310

Logarithms and Exponential Change

1. Evaluate

$$\int \frac{1}{x\ln(x)}\, dx$$

2. Evaluate

$$\int \frac{x}{x^2+4}\, dx$$

If a function $y(t)$ is increasing or decreasing at an exponential rate, we can say it is **exponentially growing** or **exponentially decreasing**, and this rate of change is proportional to its value at a time t. In other words, $y(t)$ is **proportional to its own derivative** $y'(t)$, so

$$\frac{d}{dt}\, y(t) = k \cdot y(t). \qquad (*)$$

Writing this just in terms of our function y, and treating it like a variable, the following expressions are equivalent:

$$\frac{dy}{dt} = k \cdot y(t) \qquad y'(t) = k \cdot y(t) \qquad y' = k \cdot y$$

A **differential equation** is when a function is equated to its own derivative(s), in an expression like the ones above. An **initial value problem** arises when you are given k and the value of $y(t)$ at a "starting value" or **initial condition** t_0 on its domain — like $t_0 = 0$, so $y(t_0) = y(0) = C$, where C is some constant — and we are tasked with recovering the function $y(t)$. The above types of initial value problems have one specific solution:

$$y(t) = Ce^{kt},$$

where $k > 0$.

3. Check that the function $y(t) = Ce^{kt}$ satisfies the equation in $(*)$.

4. Find the general solution for the initial value problem where $k = 1/4$ and $y(t_0) = y(0) = 200$.

5. If $y(t)$ from Problem 4 describes the population of mosquitoes, when will we triumph over our pestilent insect overlords and vanquish their population? (In other words, at what time t does $y(t) = 0$?)