Week 5 Recitation Problems

MATH:114, Recitations 309 and 310

Curve Length and Surface Area

1. Given a function $f(x)$, how might we **approximate** the length of $f(x)$ on the closed interval $[a, b]$? Draw an annotated picture or write a few words to explain, and include relevant geometric formulas or ideas. *(Hint 1: use the Euclidean distance formula, which you are free to look up. Hint 2: break the curve up into chunks!)*

2. Using your strategy from Problem 1, translate your approximation into an exact continuous calculation (that is, one which uses an integral). Draw an annotated picture or write a few words to explain, and include relevant calculus theorems or geometric ideas. *(Hint: think about the rectangle or trapezoid methods for estimating the area under a curve, which you are free to look up.)*

3. Let

$$f(x) = \frac{x^3}{6} + \frac{1}{2x}.$$

Find the length of $f(x)$ when $1 \leq x \leq 3$.

4. The formula

$$S = 2\pi \int_a^b g(x) \cdot \sqrt{1 + g'(x)^2}\, dx$$

describes how to find the surface area of the solid generated by the curve $g(x)$ on the closed interval $[a, b]$. What is familiar about this formula? Using annotated pictures or a few words, describe the geometric ideas at work here.

5. Let $g(x) = \sqrt{4 - x^2}$, and $-1 \leq x \leq 1$. Find the surface area of the solid generated by rotating $g(x)$ around the x axis.